Acta Informatica manuscript No.
(will be inserted by the editor)

M/M/1 retrial queue with working vacations

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the date of receipt and acceptance should be inserted later

Abstract In this paper we introduce the new M/M/1 retrial queue with working vacations which is motivated by the performance analysis of a Media Access Control (MAC) function in wireless systems. We give a condition for the stability of the model, which has an important impact on setting the retrial rate for such systems. We derive the closed form solution in equilibrium for the retrial M/M/1 queue with working vacations, and we also show that the conditional stochastic decomposition holds for this model as well.
Keywords: retrial queue, working vacations, conditional stochastic decomposition

1 Introduction

Vacation queues and retrial queues have been intensive research topics in the queueing theory [3–5, 7, 11–13, 15–19]. The M/M/1 queue with working vacations was analyzed and applied for the performance evaluation of Wavelength Division Multiplexing (WDM) optical systems in [16]. However, in the literature there is no published work on queues with both retrials and working vacations.

In this paper we introduce the new M/M/1 retrial queue with working vacations which is motivated by the performance analysis of a Media Access Control (MAC) function in wireless networks [2, 10]. The new queue is characterized as follows. The inter-arrival times of requests are exponentially distributed. Upon the arrival of requests, if the server is busy requests are forced to wait the orbit of infinite size. If the server is not occupied, arriving requests get service immediately. Requests in the orbit try to get service from the server with a constant retrial rate. The single server takes a working vacation at times when requests being served depart from the system and no requests are in the orbit. Each vacation lasts for a duration that has an exponential distribution. During the vacation periods the arriving customers are served with a rate smaller than the normal service rate. At the end of each vacation, the server only takes

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another new vacation if there is no any new request or any repeated request from the orbit.

The rest of the paper is organized as follows. In Section 2 we provide the analysis of the retrial M/M/1 queue with working vacations based on the quasi birth and death (QBD) process and present the closed form expressions for the steady state probabilities. In Section 3, we give the proof for the conditional stochastic decomposition of the new queue. Indeed, an important property of queues with vacations, initially motivated by magnetic secondary memory devices [1], concerns the representation of the waiting time and many other interesting distributions in equilibrium in a decomposed manner, which allows the analysis to be reduced to that of equivalent simpler systems. This property was first formalised for the Poisson arrival case (with general service times) in [9], and proved in a general setting for general independent arrivals in [8]. We shall detail these aspects in Section 3. We present some important equations related to the proof for the conditional stochastic decomposition of the new queue in Appendices.

2 The steady state probabilities of the M/M/1 retrial queue with working vacations

We consider the M/M/1 retrial queue with working vacations. The inter-arrival times of requests are exponentially distributed with parameter λ. Request retrials from the orbit of infinite size follow a Poisson process with rate α. The service rate is μ_b when the system is not on vacation. The single server takes a working vacation at times when requests being served depart from the system and no requests are in the orbit. Vacation durations are exponentially distributed with parameter θ. During the vacation periods arriving customers are served with rate $\mu_v < \mu_b$. At the end of each vacation, the server only takes another new vacation if there is no any new request or repeated request from the orbit.

The system at any time t can be completely described by two integer-valued random variables: $I(t)$ denotes the state of server (the phase of the system) at time t, while $J(t)$ represents the number of customers (the level of the system) in the orbit at time t. There are four possible states of the single server as follows:

(1) the server is on a working vacation at time t and the server is not occupied. When the server is in this state $I(t) = 0$.
(2) the server is on a working vacation at time t and the server is busy. If the server is in this state $I(t) = 1$.
(3) the server is not on a working vacation at time t and the server is not occupied. Whenever the server is in this state $I(t) = 2$.
(4) the server is not on a working vacation at time t and the server is busy. If the server is in this state $I(t) = 3$.

The system is modeled by continuous time discrete state Markov process $Y = \{I(t), J(t)\}$ on state space $S = \{(i, j) : 0 \leq i \leq 3, \;\; j \geq 0\}$. We denote the steady state probabilities by

$$\pi_{i,j} = \lim_{t\to\infty} Prob(I(t) = i, J(t) = j),$$

and let $\mathbf{v}_j = [\pi_{0,j}, \pi_{1,j}, \pi_{2,j}, \pi_{3,j}]$. Note that $\pi_{2,0} = 0$. That is, when no customer in the orbit, the probability that the server is not on a working vacation and does not serve a customer is zero.

The following types of possible transitions between the states of the Markov chain Y are identified:

(a) a purely phase transition rate from state (i,j) to state (k,j) ($\forall$ $(i,j) \in S$ and $(k,j) \in S$) is denoted by $A_j(i,k)$;

(b) an one–step upward transition rate from state (i,j) to state $(k,j+1)$ ($\forall$ $(i,j) \in S$ and $(k,j+1) \in S$) is represented by $B_j(i,k)$;

(c) an one−step downward transition rate from state (i,j) to state $(k,j-1)$ ($\forall$ $(i,j) \in S$ and $(k,j-1) \in S$) is $C_j(i,k)$.

Let A_j, B_j and C_j be matrices of size 4×4 with elements $A_j(i,k)$, $B_j(i,k)$ and $C_j(i,k)$, respectively.

For $J(t) = j \geq 1$ the following events can happen in the system:

(1A) upon the arrival of a new customer

(1A.1) if the server is free, then the server changes to the busy state (i.e.: $I(t)$ changes either from 0 to 1, or 2 to 3). The transition belongs to transition type (a) since $J(t)$ is unchanged.

(1A.2) if the server is occupied ($I(t) = 1$ or $I(t) = 3$), then the customer goes into the orbit. Therefore, the transition is of type (b).

(2A) the departure of a request after the finish of its service, then the server becomes free ($I(t)$ changes either from 1 to 0 or from 3 to 2) and $J(t)$ remains unchanged. This is the transition of type (a).

(3A) the status change of the server (i.e.: the end of the vacation), then $I(t)$ changes either from 0 to 2 or from 1 to 3. It is the transition of type (a).

(4A) the successful service request of a customer from the orbit, then $I(t)$ changes either from 0 to 1 or from 2 to 3.

Therefore, we can write

$$A_j = A = \begin{bmatrix} 0 & \lambda & \theta & 0 \\ \mu_v & 0 & 0 & \theta \\ 0 & 0 & 0 & \lambda \\ 0 & 0 & \mu_b & 0 \end{bmatrix}; \; B_j = B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}; \; C_j = C = \begin{bmatrix} 0 & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 \end{bmatrix} \; \forall j \geq 1.$$

If no customer is in the orbit ($J(t) = 0$) the following events are possible in the system:

(1B) upon the arrival of a new customer

(1B.1) if the server is free, then the server changes to the busy state (i.e.: $I(t)$ changes either from 0 to 1). The transition belongs to transition type (a) since $J(t)$ is unchanged.

(1B.2) if the server is occupied ($I(t) = 1$ or $I(t) = 3$), then the customer goes into the orbit. Therefore, the transition is of type (b).

(2B) the departure of a request after the finish of its service, then the server becomes free ($I(t)$ changes either from 1 to 0 or from 3 to 0) and $J(t)$ remains unchanged. This is the transition of type (a).

(3B) the status change of the server (i.e.: the end of the vacation), then $I(t)$ changes from 1 to 3. It is the transition of type (a).

Therefore, we obtain

$$A_0 = \begin{bmatrix} 0 & \lambda & 0 & 0 \\ \mu_v & 0 & 0 & \theta \\ 0 & 0 & 0 & 0 \\ \mu_b & 0 & 0 & 0 \end{bmatrix}; \; B_0 = B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}.$$

Let D^A and D^C be introduced as the diagonal matrices of size 4×4 with their i^{th} diagonal elements as

$$D^A(i,i) = \sum_{k=0}^{3} A(i,k); \quad D^C(i,i) = \sum_{k=0}^{3} C(i,k).$$

Markov process Y is on a two-dimensional lattice, finite in the phase $I(t)$ and infinite in the level $J(t)$ of the process. Transitions in process Y are possible within the same level or between adjacent levels. Therefore, Y is a Quasi-Birth-and-Death (QBD) process (c.f. [6, 14]).

Theorem 1 *The necessary and sufficient condition for the existence of the steady state probabilities of continuous time Quasi-Birth-and-Death (QBD) process Y is*

$$\alpha > \frac{\lambda^2}{\mu_b - \lambda}.$$

Proof. We can write the balance equations for $j \geq 1$ as follows

$$\begin{aligned} \mathbf{v}_{j-1}B + \mathbf{v}_j(A - D^A - B - D^C) + \mathbf{v}_{j+1}C = 0, \\ \mathbf{v}_{j-1}Q_0 + \mathbf{v}_jQ_1 + \mathbf{v}_{j+1}Q_2 = 0, \end{aligned} \tag{1}$$

where $Q_0 = B,\ Q_1 = A - D^A - B - D^C$ and $Q_2 = C$.

The characteristic matrix polynomial associated with equations (1) is given by $Q(x) = Q_0 + Q_1x + Q_2x^2$. It is proved in [14] that the solution of equations (1) is closely related to the eigenvalues and left-eigenvectors of $Q(x)$. If $(x, \boldsymbol{\psi})$ is an eigenvalue-eigenvector pair of $Q(x)$, then $\boldsymbol{\psi}Q(x) = 0,\ \ Det[Q(x)] = 0$. Note that

$$\begin{aligned} Det[Q(x)] &= Det \begin{bmatrix} Q_{00}(x) & \lambda x + \alpha x^2 & \theta x & 0 \\ \mu_v x & Q_{11}(x) & 0 & \theta x \\ 0 & 0 & Q_{22}(x) & \lambda x + \alpha x^2 \\ 0 & 0 & \mu_b x & Q_{33}(x) \end{bmatrix} \\ &= (Q_{00}(x)Q_{11}(x) - (\lambda + \alpha x)\mu_v x^2)(Q_{22}(x)Q_{33}(x) - (\lambda + \alpha x)\mu_b x^2) \end{aligned}$$

holds, where

$$\begin{aligned} Q_{00}(x) &= -(\alpha + \lambda + \theta)x, \\ Q_{11}(x) &= \lambda - (\lambda + \theta + \mu_v)x, \\ Q_{22}(x) &= -(\lambda + \alpha)x, \\ Q_{33}(x) &= \lambda - (\lambda + \mu_b)x. \end{aligned}$$

As a consequence the roots of $Det[Q(x)]$ can be determined from $Q_{00}(x)Q_{11}(x) - (\lambda + \alpha x)\mu_v x^2 = 0$ and $Q_{22}(x)Q_{33}(x) - (\lambda + \alpha x)\mu_b x^2 = 0$. This means that $Q(x)$ has six eigenvalues:

$$
\begin{aligned}
x_0 &= \frac{G+\alpha\mu_v+\sqrt{(G+\alpha\mu_v)^2-4\alpha\mu_v\ \ (\lambda^2+\alpha\lambda+\theta\lambda)}}{2\alpha\ \mu_v},\\
x_1 &= \frac{G+\alpha\mu_v-\sqrt{(G+\alpha\mu_v)^2-4\alpha\mu_v\ \ (\lambda^2+\alpha\lambda+\theta\lambda)}}{2\alpha\ \mu_v},\\
x_2 &= 0,\\
x_3 &= 0,\\
x_4 &= \frac{\alpha\lambda+\lambda^2}{\alpha\mu_b},\\
x_5 &= 1,
\end{aligned}
$$

where $G = \lambda^2 + \alpha\lambda + 2\theta\ \lambda + \theta^2 + \alpha\theta + \mu_v\theta$,

- x_0, x_1 and x_2 are the roots of equation $Q_{00}(x)Q_{11}(x) - (\lambda + \alpha x)\mu_v x^2 = 0$, and
- x_3, x_4 and x_5 are the roots of equation $Q_{22}(x)Q_{33}(x) - (\lambda + \alpha x)\mu_b x^2 = 0$.

We have

$$
\begin{aligned}
G &> \lambda^2+\alpha\lambda+\theta\lambda\\
4\alpha\mu_v G &> 4\alpha\mu_v(\lambda^2+\alpha\lambda+\theta\lambda)\\
(G+\alpha\mu_v)^2-4\alpha\mu_v G &< (G+\alpha\mu_v)^2-4\alpha\mu_v(\lambda^2+\alpha\lambda+\theta\lambda)\\
\sqrt{(G+\alpha\mu_v)^2-4\alpha\mu_v G} &< \sqrt{(G+\alpha\mu_v)^2-4\alpha\mu_v(\lambda^2+\alpha\lambda+\theta\lambda)}.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
x_1 &< \frac{G+\alpha\mu_v-\sqrt{(G+\alpha\mu_v)^2-4\alpha\mu_v G}}{2\alpha\mu_v} = \frac{G+\alpha\mu_v-|G-\alpha\mu_v|}{2\alpha\mu_v} \le 1\\
x_0 &> \frac{G+\alpha\mu_v+\sqrt{(G+\alpha\mu_v)^2-4\alpha\mu_v G}}{2\alpha\mu_v} = \frac{G+\alpha\mu_v+|G-\alpha\mu_v|}{2\alpha\mu_v} \ge 1.
\end{aligned}
$$

Based on [14], the necessary and sufficient condition for the ergodicity of the Markov process Y is that the number of eigenvalues of $Q(x)$ inside the unit disk is 4, which follows $|x_4| < 1$. Therefore, $\alpha > \frac{\lambda^2}{\mu_b - \lambda}$ holds. □

Remark 1 We have the following observations concerning the left-hand-side (LHS) eigenvectors of $Q(x)$:

- $\mathbf{\Psi}_i = [\mu_v, \alpha + \lambda + \theta, y_{1,i}, y_{2,i}]$ is the LHS eigenvector of $Q(x)$ for eigenvalue x_i ($i = 0..1$), where $y_{1,i}$ and $y_{2,i}$ can be determined as the solution of the following linear equations:

$$
\begin{aligned}
\theta x_i\mu_v + Q_{22}(x_i)y_{1,i} + \mu_b x_i y_{2,i} &= 0,\\
\theta x_i(\alpha+\lambda+\theta) + (\lambda x_1 + \alpha x_1^2)y_{1,i} + Q_{33}(x_i)y_{2,i} &= 0.
\end{aligned}
$$

Therefore, we get

$$
\begin{aligned}
y_{1,i} &= -\frac{\mu_b Q_{00}(x_i)\theta x_i + \mu_v Q_{33}(x_i)\theta x_i}{Q_{22}(x_i)Q_{33}(x_i) - \lambda\mu_b x_i^2 - \alpha\mu_b x_i^3},\\
y_{2,i} &= -\frac{Q_{00}(x_i)Q_{22}(x_i)\theta - \mu_v\lambda\theta x_i^2 - \mu_v\alpha\theta x_i^3}{Q_{22}(x_i)Q_{33}(x_i) - \lambda\mu_b x_i^2 - \alpha\mu_b x_i^3}.
\end{aligned}
$$

- $\mathbf{\Psi}_2 = [1, 0, 0, 0]$ is the corresponding LHS eigenvector of zero eigenvalue x_2 of $Q(x)$.
- $\mathbf{\Psi}_3 = [0, 0, 1, 0]$ is the corresponding LHS eigenvector of zero eigenvalue x_3 of $Q(x)$.
- $\mathbf{\Psi}_i = [0, 0, \mu_b, \alpha + \lambda]$ is the LHS eigenvector of $Q(x)$ for eigenvalue x_i $(i = 4..5)$.

The steady state probabilities can be expressed as a linear sum of factors $x_i^j \mathbf{\Psi}_i$ (where $|x_i| < 1$):

$$\mathbf{v}_j = \sum_{i=1}^{4} a_i x_i^j \mathbf{\Psi}_i \ \ (j \geq 0). \tag{2}$$

where a_i $(i = 1, \ldots, 4)$ are the coefficients to be determined (see Appendix).

3 Conditional stochastic decomposition

Theorem 2 *If the ergodicity condition for the retrial M/M/1 queue with working vacations holds, the conditional queue length* $J_b = lim_{t\to\infty}\{J(t)|I(t) = 1 \ or \ I(t) = 3\}$ *given that the server is busy can be decomposed into the sum of two independent random variables*

$$J_b = J_0 + J_c,$$

where J_0 *is the conditional queue length of the retrial M/M/1 queue given that the server is busy and* J_c *is the additional queue length due to vacations. The queue length* J_c *has the probability generating function*

$$G_{J_c}(z) = \frac{1}{a_4^*(\lambda + \alpha)} \left(\frac{m + n - (mx_4 + nx_1)z}{1 - zx_1} \right),$$

where

$$m = \frac{a_1(\alpha + \lambda + \theta + y_{2,1})}{p_b}; \ \ n = \frac{a_4(\alpha + \lambda)}{p_b}.$$

Proof. The probability that the server busy is

$$p_b = \sum_{j=0}^{\infty} (\pi_{1,j} + \pi_{3,j}) = \frac{a_1(\alpha + \lambda + \theta + y_{2,1})}{1 - x_1} + \frac{a_4(\alpha + \lambda)}{1 - x_4}.$$

Thus, the probability generating function of J_b is as follows:

$$\begin{aligned}
G_{J_b}(z) &= \sum_{j=0}^{\infty} \frac{z^j (a_1(\alpha + \lambda + \theta + y_{2,1})x_1^j + a_4(\alpha + \lambda)x_4^j)}{p_b} \\
&= \frac{a_1(\alpha + \lambda + \theta + y_{2,1})}{p_b} \frac{1}{1 - zx_1} + \frac{a_4(\alpha + \lambda)}{p_b} \frac{1}{1 - zx_4} \\
&= \frac{m}{1 - zx_1} + \frac{n}{1 - zx_4} = \frac{m + n - (mx_4 + nx_1)z}{(1 - zx_1)(1 - zx_4)} \\
&= G_{J_0}(z) G_{J_c}(z) \quad \text{(substituting (8))}.\square
\end{aligned}$$

4 Appendix: The derivation of the coefficients a_i

We have the steady state balance equations for $J(t) = 0$ as

$$\mathbf{v}_0 \left(A_0 - D^{A_0} - B_0 \right) + \mathbf{v}_1 C_1 = 0,$$

where D^{A_0} is a diagonal matrix whose diagonal elements are the sum of the elements in the corresponding row of matrix A_0. After some algebra a_2, a_3 and a_4 can be expressed in terms of of a_1 as

$$a_2 = -\frac{-a_1\lambda^2 - a_1\alpha\lambda - 2a_1\theta\lambda - a_1\theta^2 - a_1\alpha\mu_v - a_1\alpha\theta - a_1\mu_v\theta + a_1\alpha\mu_v x_1}{\lambda}, \tag{3}$$

$$a_3 = -\frac{a_1\lambda^2 + a_1\alpha\lambda + 2a_1\theta\lambda + a_1 y_{1,1}\lambda + a_1\theta^2 + a_1\alpha\theta - a_1\alpha\mu_v x_1 + a_1\alpha y_{1,1} - a_1\mu_b y_{2,1}}{\alpha + \lambda}, \tag{4}$$

$$a_4 = -\frac{-a_1\lambda^2 - a_1\alpha\lambda - 2a_1\theta\lambda - a_1\theta^2 - a_1\alpha\theta + a_1\alpha\mu_v x_1 + a_1\mu_b y_{2,1}}{(\alpha + \lambda)\mu_b}. \tag{5}$$

The normalization equation $\sum_{j=0}^{\infty} \mathbf{v}_j \boldsymbol{e} = 1$ can be rewritten as

$$\sum_{j=0}^{\infty} \mathbf{v}_j \boldsymbol{e} = \sum_{j=0}^{\infty} \sum_{i=1}^{4} a_i x_i^j \boldsymbol{\Psi}_i \boldsymbol{e} = \sum_{i=1}^{4} a_i \frac{1}{1 - x_i} \boldsymbol{\Psi}_i \boldsymbol{e} = 1, \tag{6}$$

where $\boldsymbol{e}$ is the column vector of size 4 with each element equal to unity. Substituting (3), (4) and (5) into equation (6) we obtain coefficient a_1. Finally, a_2, a_3 and a_4 are determined from equations (3), (4) and (5).

5 Appendix: The retrial M/M/1 queue

The evolution of the retrial M/M/1 queue at any time t is described by two integer-valued random variables:

- $I^*(t) = \begin{cases} 0 \text{ the server is free at time } t \\ 1 \text{ the server is busy at time } t \end{cases}$,
- $J^*(t)$ represents the number of customers in the orbit at time t.

The retrial M/M/1 queue is a continuous time discrete state Markov process, $\{I^*(t), J^*(t)\}$, on the state space $\{(i, j) : i = 0..1, j \geq 0\}$. The matrices, which contain the transition rates, are written as

$$A_j^{(*)} = A^{(*)} = \begin{bmatrix} 0 & \lambda \\ \mu_b & 0 \end{bmatrix} \ \forall j \geq 1; \ B_j^{(*)} = B^{(*)} = \begin{bmatrix} 0 & 0 \\ 0 & \lambda \end{bmatrix} \ \forall j \geq 0;$$
$$C_j^{(*)} = C^{(*)} = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} \ \forall j \geq 1.$$

The characteristic matrix polynomial of the retrial M/M/1 queue is obtained as

$$Q^{(*)}(x) = \begin{bmatrix} Q_{22}(x) & \lambda x + \alpha x^2 \\ \mu_b x & Q_{33}(x) \end{bmatrix}.$$

Thus, the characteristic matrix polynomial has 3 eigenvalues x_3, x_4 and x_5, which form the subset of the eigenvalues of the characteristic matrix polynomial associated with the retrial M/M/1 queue with working vacations.

The corresponding LHS eigenvector of zero eigenvalue x_3 of $Q^*(x)$ is $\mathbf{\Psi}_3^* = [1, 0]$, while $\mathbf{\Psi}_4^* = [\mu_b, \alpha + \lambda]$ is the corresponding LHS eigenvectors of eigenvalue x_4. Thus we can write

$$\mathbf{v}_j = \sum_{i=3}^{4} a_i^* x_i^j \mathbf{\Psi}_i^* \quad (j \geq 0), \tag{7}$$

where a_i^* is the coefficients, which can be determined from the balance equation for $J^*(t) = 0$ and the normalization equation

$$a_3^* = \frac{\alpha\mu_b - \alpha\lambda - \lambda^2}{(\alpha + \lambda)\mu_b},$$
$$a_4^* = \frac{\alpha\lambda\mu_b - \alpha\lambda^2 - \lambda^3}{\alpha(\alpha + \lambda)\mu_b^2}.$$

The probability generating function of the number of customers in the orbit, given the server is busy, is

$$G_{J_0}(z) = \sum_{j=0}^{\infty} z^j a_4^* x_4^j (\alpha + \lambda) = \frac{a_4^*(\lambda + \alpha)}{1 - zx_4}. \tag{8}$$

Acknowledgements The author thanks the anonymous reviewers for constructive comments which greatly help the author to improve the presentation of this paper.

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