Determination of organ pipes' acoustic parameters by means of numerical techniques

Orgonasípok akusztikai paramétereinek meghatározása numerikus technikákkal

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Kivonat – Az orgonasípok hangkeltési mechanizmusa rendkívül bonyolult fizikai folyamat, mivel akusztikai és áramlástani jelenségek párhuzamosan, csatolva jelennek meg. Ennek ellenére, pusztán akusztikai rezonátorként modellezve a sípot, megfelelő pontossággal becsülhetjük a hangzás több kulcsfontosságú paraméterét. A cikk célja az orgonasípok szimulációjára alkalmas numerikus technikák bemutatása. Munkám során többféle numerikus eljárást használtam a modellezéshez, melyeket kereskedelmi és saját fejlesztésű szoftverekkel valósítottam meg. A szimulációk eredményeit analitikus számításokkal és mérési adatokkal vetettem össze. Megmutattam, hogy a numerikus technikák jól alkalmazhatóak a síp főbb akusztikai paramétereinek meghatározására.

Abstract – The sound generation of an organ pipe is a very complex physical process, since the acoustical phenomena take place coupled with fluid flow effects. Even so, by modeling the organ pipe merely as an acoustic resonator, one can predict several key parameters of the sounding with sufficient accuracy. The aim of this article is to examine the simulation techniques that can be used for organ pipe modeling. In the course of the work reported herein, the author has modeled organ pipes by means of various numerical techniques. Commercial and self-developed software packages were used, and the obtained data were compared with analytical solutions and measurement results. It was shown that by using these techniques one can approximate key acoustic parameters of the pipe.

1. Introduction

Scaling of organ pipes is still performed according to the rules laid down in the 19th century. These rules prescribe pipe dimensions for the desired sounding, but in some cases changing the traditional geometry parameters is inevitable (for aesthetic and practical reasons). Then the organ builder can only rely on his experience, attempting to tune the sounding parameters of the pipe.

The aim of applying numerical techniques for organ pipe simulation is twofold. On the one hand to speed up the scaling and tuning processes, saving quite some time for organ manufacturers as an organ consists of thousands of pipes. On the other hand it will hopefully help developing new scaling methods. The purpose of the latter is to predict, how the traditional organ sounds can be preserved with changed geometrical parameters, and how new sounding characteristics can be achieved.

Firstly, the sound generation mechanism of organ pipes is examined and the acoustic parameters that typify the sounding are described. Section 3 presents the numerical techniques, that can be applied for modeling the pipe transfer function measurement. Section 4 focuses on mesh generation and simulation software. Impedance analysis and the results of pipe simulations are discussed in detail in section 5. Finally, conclusions are summarized and an outlook on further researches is given.

This work was performed at the Budapest University of Technology and Economics, Department of Telecommunications, Laboratory of Vibroacoustics.

2. Characteristics of the organ sound

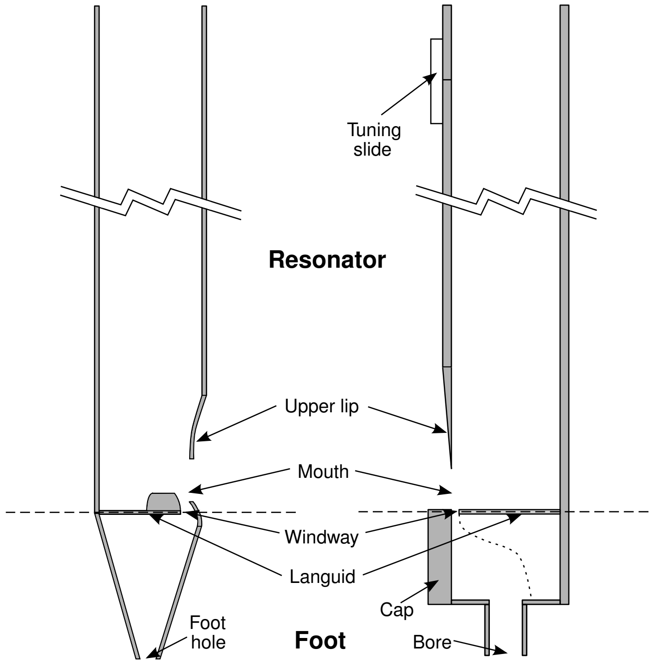
The sound quality of pipe ranks depends on the dimensioning (or scaling) of the pipes and on the voicing adjustment. Scaling concerns all about selecting geometrical parameters of the pipe, whereas voicing refers to tuning, the process of adjusting the parts of the pipe to produce the desired tone. The aim of scaling and voicing is to ensure the required quality of the perceived sound.

The quality of the produced sound can be characterized by various acoustical parameters that typify many aspects of the sounding. To understand these factors, the basics of the sound generation mechanism should be examined.

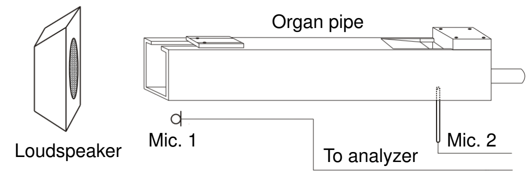
2.1. The sound generation mechanism

The organ sound is generated as the wind flows through the pipes. The wind, that is delivered from the blower to the pipes by complex mechanical structures of wind ducts, moving through metal or wooden pipes remains constant while a key is depressed. As the sound evolves when the wind excites the air column inside the pipe body, the pipe organ is classified into the aerophones group.

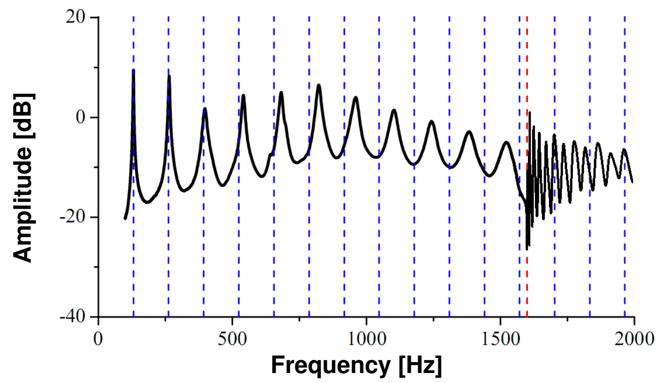
There are endless types of configuration and structure for organ pipes, whose main distinguishing features are: material (wood or metal), form of the resonator (cylindrical, conical or rectangular), sort of excitation (reed pipe or flue pipe) and ending of the resonator (open or closed – also known as gedackt or stopped). Longitudinal sections of a cylindrical metal and a rectangular wooden flue pipe can be seen in figure 1.



1. Figure: Longitudinal section of a cylindrical metal (left) and a rectangular wooden (right) flue organ pipe



2. Figure: Measurement of the pipe transfer function



3. Figure: Typical transfer function of an organ pipe
– – Exact harmonics of the fundamental – – Cut-off

Two main parts of the pipes are the foot and the body or resonator. The foot constitutes the bottom part of the pipe. At the foot base is the foot hole or the bore, through which wind gets into the pipe. The length of the pipe foot does not modify the pitch. Therefore, organ builders design the foot lengths of their pipes depending on several factors. The length and volume of the resonator and the voicing determine the fundamental pitch and timbre of the pipe.

The mouth of the pipe is the horizontal opening cut at the joint between the body and the foot and is delimited by the upper and the lower lips. At this joint a sheet of metal or wood called languid is attached horizontally inside the pipe. The languid divides the resonator and the foot completely, except for a small groove parallel to the mouth named windway. This separation creates a cavity inside the pipe foot, which allows air to flow into the resonator from the foot, but only as a thin jet of wind directed towards the mouth.

The air jet that evolves in the windway starts to oscillate around the upper lip, and this vibrating movement of air provides the excitation of the air column resonating inside the pipe body. This air column can resonate at different characteristic resonant frequencies.

As seen, the sound generation process of an organ pipe is a very complex physical procedure as the acoustic phenomena show up coupled with fluid flow effects. The examination of this problem in full detail would require the analysis of a coupled non-linear acoustic and fluid flow model.

At the same time, some key parameters of the sounding can be obtained if the pipe is regarded merely as an acoustic resonator. By this simplification, transient attacks can not be taken into consideration, our experiments are limited to the examination of stationary spectra. The stationary sound spectra of an organ pipe is mostly determined by the transfer function of the pipe resonator.

The aim of this brief summary was only to show the complexity of mechanism. A more detailed review on the sound generation process can be found in [1] or [2].

2.2. The pipe transfer function

The transfer function of the pipe determines how the resonator will react to the excitation respect to the frequency. In figure 2 the measurement setup of the pipe transfer function is shown. A loudspeaker is placed in the longitudinal axis of the pipe, creating a sound field, which excites the air column resonating in the pipe body. The excitation signal is a broadband signal, e.g. a logarithmic sweep sine function. Microphones are usually placed near the mouth of the pipe and at the open end. The signals of these microphones are analyzed by FFT analyzers. The whole setup is placed into an anechoic room, which provides the characteristics of a free sound field.

The geometry of the resonator determines the frequencies of eigenresonances, i.e. the frequencies at which the air column inside the body can resonate at. At these eigenfrequencies the transfer function shows peaks of significant amplification. A typical organ pipe transfer function can be seen in figure 3.

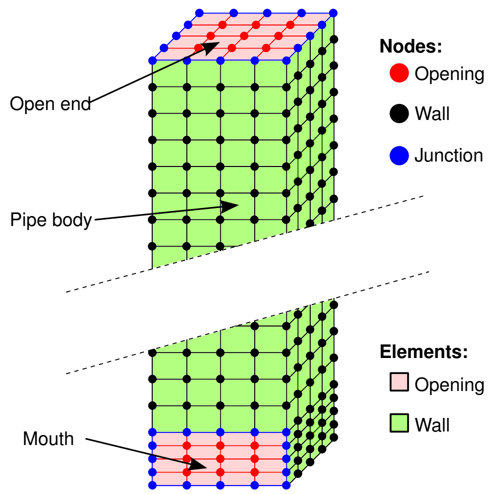
The transfer function of the resonator also determines the characteristics of the steady sound spectrum. Therefore, key information on the sounding can be obtained by the analysis of the pipe transfer function. This data can be summarized by the following acoustical parameters.

- **Fundamental frequency**

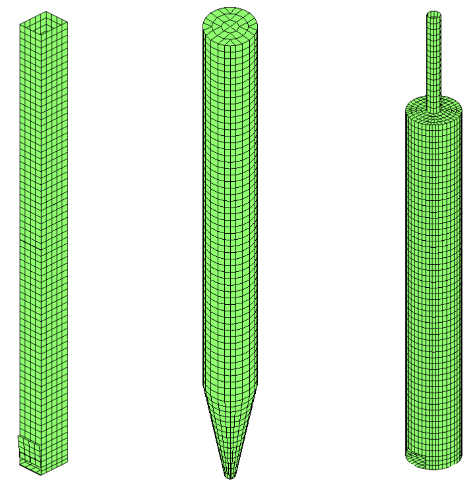
  This is the first resonant frequency of the pipe. Even though, other harmonics can be more dominant during transient attacks (see [3, 2]), it is the fundamental frequency that determines the tone of the pipe. This frequency has the highest amplitude in the stationary sound spectra.

- **Frequencies of harmonic partials**

  As can be seen in figure 3, in case of an organ pipe, the eigenresonances are not exact harmonics of the first resonance. The frequencies of these modes are slightly stretched. This effect is called stretching and it is an important attribute of the organ sound. The stretching effect is especially sensitive to the geometry parameters of the



4. Figure: Splitting the boundary into walls and openings



(a) Wooden (b) Cylindrical (c) Chimney

5. Figure: Meshes of different organ pipe types

pipe (see [2]). Generally, stretching values are higher of pipes with larger diameter.

- **Q-factors of eigenresonances**
  The peaks of harmonic partials are not equally sharp. Q-factors are higher in case of the first few harmonics and lower for the further harmonics. This means that amplification peaks become wider for the successive harmonics. Q-factors are also dependent of the resonator geometry. Larger diameter results in lower Q-factor in general.
- **Cut-off frequency**
  Since the diameter (or depth, e.g. in case of wooden pipes) of an organ pipe is much smaller than its length, pure longitudinal eigenmodes appear at lower frequencies. The frequency, where transversal resonances start to appear, is called cut-off frequency as the sound spectrum above this frequency shows irregularities compared to the slightly stretched harmonic peaks at lower frequencies. These irregularities are caused by the combined excitation of longitudinal and transversal modes.

There are many other attributes of the organ sound, which are not examined here. Detailed description of transient attacks and other characteristics of the organ sound can be found in [1], [3] or [2].

3. Numerical techniques

Since the computational capacity of computers has augmented exponentially in the last few decades, more and more accurate models can be examined by computer simulations. While analytical computations are limited to the simplest cases, problems that can be solved by numerical techniques can be much more complicated. The other reason why simulations get a wider and wider scope, is that they are more cost and time effective than prototyping for example.

There are two numerical methods that are applied in a very wide range in linear acoustics: the finite element method (FEM) and the boundary element method (BEM). These two are able to solve the Helmholtz equation with specified boundary conditions, i.e. they give a solution to a boundary value problem.

Both the FEM and the BEM are based on discretization of the geometry and – making use of the linearity – transforming the integral equations into a large number of linear equations, that can be expressed in matrix form. The matrices of these equations are called system matrices.

As the aim of our simulations is to simulate stationary spectra, it is obvious to solve the problems in the frequency domain. Therefore, effects such as transient attacks cannot be taken into consideration, but the transfer function of the pipe can easily be obtained.

In the following, two numerical methods that can be applied for organ pipe modeling will be presented.

3.1. The indirect boundary element method

The indirect boundary element method is a modification of the classical BEM, that is able to solve the internal and external acoustic radiation and scattering problem simultaneously. The indirect representation uses layer potentials that are the differences between the outside and inside values of pressure and its normal derivative respectively.

The acoustic variables at any point in the entire volume are computed as a function of these two layer potentials. The boundary conditions can also be formulated in terms of the layer potentials.

The system matrices in case of the indirect BEM are frequency dependent full matrices. This means that the system of equations must be solved one-by-one for each testing frequency and because of the denseness of the matrices fast matrix inversion techniques can not be used. On the other hand, the geometry is given as a surface mesh, which requires less nodes than a volume geometry and therefore the size of system matrices is reduced.

3.2. The coupled FE/BE method

The coupled FE/BE method is a combination of the finite element and the direct boundary element method and is able to solve the interior and exterior acoustic radiation and scattering problem, like the indirect BEM. The main difference is that the problem is solved here by means of FEM in the interior domain, while direct BEM is applied to set up boundary conditions.

The key of this method is the theory of superposition. As the model is linear, the evolving sound field is a superimposed

Perfectly Matched Layers

Many problems in acoustics, as well as in other fields of application like geophysics, oceanography and electro magnetics, involve waves in an unbounded medium. The solution of such problems using the finite element method or other domain-type methods usually requires the use of a finite computational domain, in which the entire calculation is to be done.

Thus, one has to introduce an artificial boundary that encloses the region of interest. To describe a well-posed mathematical problem in the finite computational domain, some boundary conditions must be imposed on the artificial boundary.

There are various methods that can be applied for this problem, like classical infinite elements (see [4, 5, 6]), low and high order boundary conditions (see [7], [8]) and absorbing layers (see [9, 10, 11]).

An absorbing layer is an artificial boundary layer, which is designed to damp or eliminate reflecting waves from the boundary of domain of interest. The perfectly matched layer (PML) is a special absorbing boundary, that was invented by Bérenger in the mid 90's for electro magnetic problems (see [12]). It is equipped with two basic properties:

1. It is designed to have *zero reflection* at the interface of the layer and the interior domain for *any* plane wave;
2. It is designed to make the solution *decay exponentially* inside the layer.

These two properties ensure excellent wave absorbance, at least on the continuous level. A wave outgoing from the interior domain enters the layer without any reflection, and then decays exponentially. By the time it arrives at the outer boundary of the PML it is very weak. Then it maybe reflected back into the PML, it decays exponentially again, and by the time it reaches the interface of the layer and the domain of interest it is too weak to cause any damage.

The damping is introduced as a modification of the wave equation, where the derivative operator is substituted by a damping operator. The PML can be formulated in many ways, as described in [10]. The most important of these are the split (Bérenger) and the unsplit (Zhao-Cangellaris) evaluations.

The PML has the distinct advantage that on the continuous level it is 'perfect' by construction. Indeed it performs extremely well in many circumstances, especially for high-frequency waves. However there are still a number of PML-related issues that remain open and are a subject to current research. These include:

- While the PML is perfect on the continuous level, it is not perfect on the discrete level. In some cases the PML performs poorly when incorporated in a discrete model, especially in low frequencies.
- The performance of the layer is sensitive to the choice of the PML parameters, i.e. the PML thickness and the PML damping function.

As there is no commercial software available that handles the acoustic PML, the author has implemented the perfectly matched layer for a simple, one-dimensional case by the split formulation, under `Matlab` environment.

First results showed, that the PML can perform well in a finite element implementation if the parameters are set correctly. However, the basic formulation should be improved in order to be able to extend the model efficiently for three-dimensional problems.

Since the PML is able to model the properties of a free sound field, a 3-D implementation will be able to be applied for setting up pipe simulation environments. Pipe simulations using perfectly matched layers are planned for further researches.

field of the incident and the reflected fields. The former is produced by the acoustical source placed in the vicinity of the resonator, while the latter is a scattered field that is determined by the shape and dimensions of the examined object.

For the inner sound field (inside the pipe) the Helmholtz equation is solved by the finite element method. The interaction between the exterior and interior fields is prescribed as admittance boundary conditions. These conditions can be obtained by expressing the load of the exterior field at the openings, by means of the direct boundary element method.

This way, the solution is carried out in the following steps.

1. Computation of the incident sound field.
2. Calculation of BE system matrices to determine the relation of sound pressure and particle velocity for the reflected field on the boundary. Admittance boundary conditions can be set up, expressed from these matrices.
3. Solving the interior problem with boundary conditions by means of FEM and evaluating the pressure field at any point of the exterior domain by the BEM.
4. Steps 1–3 have to be completed for each testing frequency. There are some simplification options that can be applied to speed up the process.

Since the geometry consists of openings and perfectly rigid walls in the simplest case, the boundary can be split up into two sub-domains (as it is seen in figure 4):

1. **Openings:** Admittance boundary conditions. ($v_n = Hp$)
2. **Walls:** The normal particle velocity is zero. ($v_n = 0$)

As the normal particle velocity is known at the walls, the BE system matrices can be expressed in their Schur's complement form. Hence the storage size of the admittance condition matrix can significantly be reduced.

To complete a solution, the pressure field of the region of interest must be computed for each testing frequency. This requires a large number of computational steps as the BE and FE equations have to be evaluated, which may take quite some time if the resolution of the model is fine. To speed up the process there are some options that can be applied.

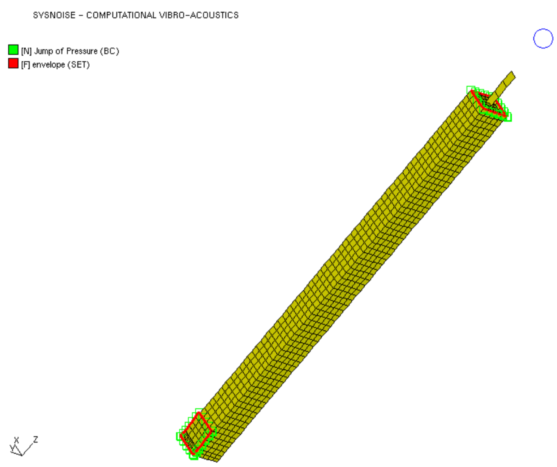
Firstly, the acoustic stiffness and mass matrices (FE system matrices) are independent of frequency, so they have to be calculated only once. These matrices are sparse, which means that their storage size can be reduced and fast matrix inversion algorithms can be used on them.

The BE system matrices are frequency dependent dense (or full) matrices, but their values are varying slowly with respect to the frequency. The same holds for their Schur's complement forms. Making use of this, the computational process can be sped up by using interpolation formulae to approximate their values. Taking this into consideration it is sufficient to evaluate BEM matrices only for a fraction of the whole number of testing frequencies.

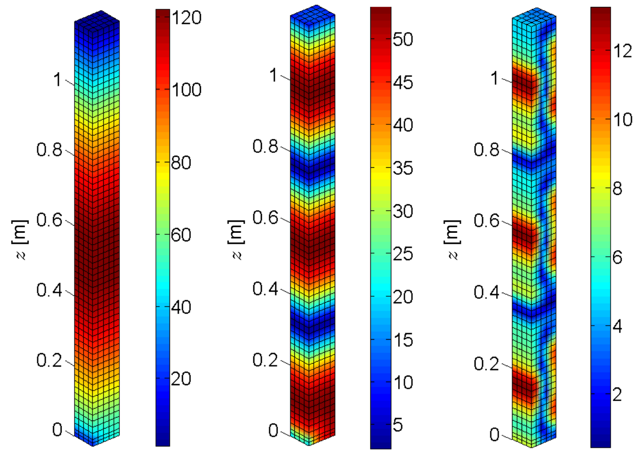
By these simplifications, the coupled method can be efficiently applied for the solution of a combined interior and exterior problem.

4. Simulation setup and software

In case of the simulation model, the organ pipe is given as a geometry mesh. The simplest case implies that the mesh consist of perfectly rigid, infinitely thin walls and openings. The rigidness of the walls means that the resonances of the mechanical structure of the pipe body are negligible, i.e. the resonating air



6. Figure: Simulation setup in LMS Sysnoise
○ Point source – Free edges – Boundary conditions



(a) 1st mode (b) 3rd mode (c) Combined mode

7. Figure: Pipe simulation in Matlab: Pressure distribution of different modes (pressure values given in Pa)

does not produce vibrations in the walls. This yields that the normal component of the particle velocity is equivalent to zero on the walls. Note, that this is just an approximation, in the real case the walls and the air inside are in active interaction, which would require the analysis of a coupled structure and air vibration model. For the following, the walls are considered to be perfectly rigid.

Simulation setups model the measurement of the pipe transfer function. As it was mentioned before, the excitation is produced by a loudspeaker and the response is recorded by microphones. The loudspeaker is simulated as a point source with given amplitude for each testing frequency, which is an acceptable approximation for the simulations. The microphones are substituted by simple measurement points, that can be placed anywhere in the domain of interest.

4.1. Mesh generation

The acoustic parameters of the organ sound are highly affected by the pipe's dimensions. Even small changes of the geometry can have major influence on the sounding and therefore, the discretization should be adequately fine. At the same time, by increasing the resolution of the model, storage size of the geometry and the computational effort will raise enormously.

The storage size of a mesh is proportional to the number of its nodes and elements. Hence in case of a surface geometry the storage size is $O(n^2)$, while for a volume geometry this value is $O(n^3)$, n denoting the average number of nodes in a unit length. As the system matrices are the size of $N \times N$ (where N represents the number of nodes), the storage sizes of the system matrices are $O(n^4)$ and $O(n^6)$, respectively. This means that a compromise must be made between the accuracy and the computational effort.

The validity range of the simulation is also dependent of the mesh resolution, the relation between the maximal element sizes (l_x, l_y and l_z) and the upper frequency limit ($f_{\max}$) is given as:

$$f_{\max} = \frac{c}{8 \max\{l_x, l_y, l_z\}}. \qquad (1)$$

It is useful to create a mesh that is discretized symmetrically and with uniform resolution. This way the values of l_x, l_y and l_z will be approximately equal, and effects of numerical instabilities caused by irregular meshing is minimized. The pipe meshes that were used for simulations were created by an algorithm that provides these attributes. Example meshes of different types of organ pipes can be seen in figure 5.

4.2. Simulations by the indirect BE method

In the case of the indirect BEM the region of our interest and thus, the sound field is split into an interior (inside the pipe) and an exterior (outside the pipe) domain. The connection and continuity between these two fields are described with boundary conditions, namely that the jump of pressure (the double layer potential) is zero at the boundaries (i.e. at the free edges of the mesh).

The resonator geometry is given here as a surface geometry. The acoustic parameters at the measurement locations are computed for each testing frequency, one at a time.

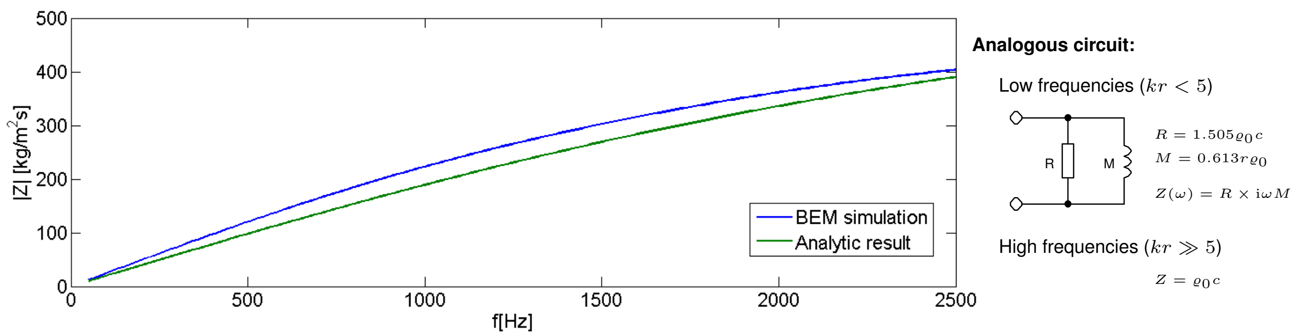
The `LMS Sysnoise` software package was used for simulation tests involving the indirect BE method. The simulation setup screen is shown in figure 6. A field point mesh can be seen in the longitudinal axis of the pipe, its nodes are the measurement points.

4.3. Simulations by the coupled FE/BE method

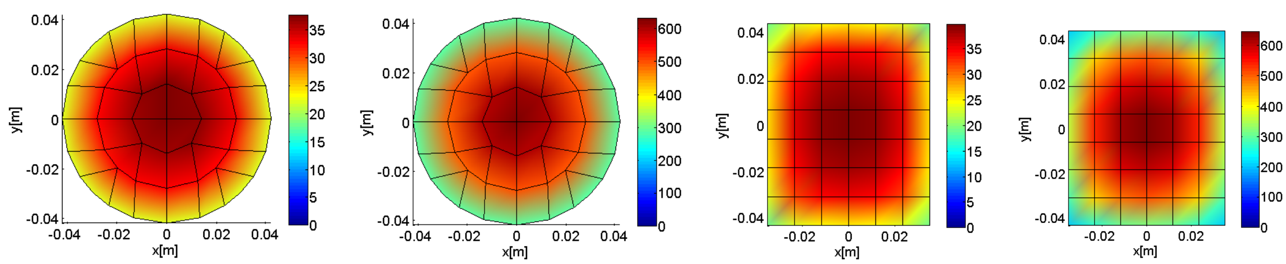
The application of this method involves the usage of the FEM for the interior domain, and utilization of the BEM to calculate the boundary conditions. The mesh is given here as a volume geometry, but the surface geometry is also needed, to be able to apply the BEM. This way the storage size required by the mesh is somewhat bigger than in case of the indirect BEM.

As the openings of the pipe consist of far less nodes than the walls, the Schur's complement technique can efficiently be used to set up the admittance boundary conditions for openings. Therefore, the frequency dependent part of the system matrices is relatively small compared to the size of the whole matrices. To speed up the computational process a spline interpolation formula were used on the admittance matrices, and thus, the BEM was only invoked for only a small fraction of the testing frequencies in case of pipe simulations. This resulted in a remarkable reduction of the computational time.

Simulations involving the coupled FE/BE method were run using a solution program, developed by the author in `Matlab`



8. Figure: Comparison of analytic and simulation results of the terminating impedance of a cylindric pipe. $L = 857.5$ mm; $r = 42$ mm.



(a) 100 Hz (b) 2 kHz (c) 100 Hz (d) 2 kHz

9. Figure: Distribution of terminating impedances of cylindrical (left) and rectangular (right) organ pipes. Values given in Rayl.

language. The AcouFEM toolbox (see [13]) and the AcouBEM software were also used under the Matlab environment. All geometry meshes that were used for simulations (both indirect BEM and coupled FEM/BEM) were created by self-written scripts. Simulation plots can be seen in figure 7.

5. Results

5.1. Analytic calculations

The simplest analytic formula, which determines the eigenfrequencies of an air column resonating inside a tube with both ends open is the well known relation

$$f_n = 2n\frac{c}{4l}; \qquad n = 1; 2; 3 \ldots, \tag{2}$$

where f_n is the frequency of the n-th eigenmode, c is the speed of sound, and l denotes the length of the resonator. This means that an integer multiple of the half wavelengths of the longitudinal modes are equal to the length of the tube. For a tube with one end closed we get

$$f_n = (2n-1)\frac{c}{4l}; \qquad n = 1; 2; 3 \ldots. \tag{3}$$

Note, that these modes are the eigenfunctions of the wave equation with boundary conditions $p = 0$ at an open end and $v = 0$ at a closed end. As these conditions naturally does not apply in a real case, an end correction formula (see e.g. [14]) can be used to take the interaction between the interior and exterior field into consideration. This prescribes the effective length of the tube as

$$l_{\text{eff}} = l + 0.62r, \tag{4}$$

r denoting the radius of the tube. If both ends are open, the correction factor is doubled.

5.2. Impedance analysis

Characteristics of the pipe transfer function and the organ sound itself can be understood by taking into consideration that the air column that resonates inside the pipe body is interacting with the exterior sound field.

The interaction between the interior and exterior sound field can be expressed by introducing terminating acoustic impedances at the enclosures of the pipes, i.e. the open end and the mouth. These impedances describe the load represented by the exterior sound field.

The radiation impedance in case of a plane piston moving in a long cylindric tube can be calculated analytically. In this case, the termination impedance can be described as an analogy of an electrical parallel R-L circuit (see: [14]). The equivalent acoustic circuit of concentrated parameters consists of an acoustic resistance and an acoustic mass. At high frequencies ($kr \gg 5$) the effect acoustic mass is negligible and the equivalent circuit is simplified to a pure acoustic resistance.

The impedance analysis of an organ pipe can be done by numerical means, using the boundary element method. The comparison of the results of analytic computation and simulation by the BEM can be seen in figure 8.

Despite, that in case of the piston problem, the piston is considered to be perfectly rigid, which would be a very rough approximation of the resonating air column, the similarity of the two curves is remarkable. It is also worth to mention, that for high frequencies the curves converge to the value of $\varrho_0 c$.

The analysis of the terminating impedances gives an explanation of some attributes of the pipe transfer function.

- The end correction formula and thus, the shifting of the fundamental frequency can be understood taking into consideration that the finite values of the terminating impedance is not equivalent to the ideal $p = 0$ case, which prescribes zero impedance for the open end.

- The stretching effect is caused by the frequency dependence of the terminating impedances. Since the value of the terminating impedances are higher for higher frequencies, the frequencies of the longitudinal modes become more and more stretched.
- Decreasing Q-factors of eigenresonances are partly caused by the increasing values terminating impedances. The other effect that determines the Q-factor of a certain resonance is the damping factor of air, which is also frequency dependent.

Analytic solutions can not take into account that the radiation impedance is not only dependent of the frequency, but also varying spatially over the openings of the pipe. The distribution of the impedance values in case of cylindrical and rectangular pipes can be seen in figure 9.

The impedance values near the edges are approximately half of the values near the center of the cross section. This means that the analytic approximation, which regards these impedances independent of the location, can not be successfully applied as boundary conditions in simulations. As seen, the distribution of the terminating impedance values is not, or just negligibly dependent of the frequency.

Making use of the facts, that the values of terminating impedances are varying slowly with respect to the frequency, and that their distribution over the cross section is nearly independent of the frequency, interpolation formulae can efficiently be applied in the coupled FE/BE method.

5.3. Pipe simulations

Pipe simulations were run on a series of wooden flue organ pipes, which already have been built and measured at the Fraunhofer Institut für Bauphysik, Stuttgart. These pipes were designed as a part of an experiment that examined how the dimensioning affects the sounding of wooden pipes. Therefore these pipes had different geometrical parameters, but similar steady sound characteristics, and were appropriate subjects for test simulations. The experiment is described in [3] in detail. The series consisted of five pipes of C tone, three from these were chosen and made simulation models of. Table 1 shows the exact dimensions of these pipes (4/16, 4/18 and 4/20 mean mouth width to circumference ratio).

The meshes were generated using a self developed, parametric mesh generator script. According to measurement data, cut-off frequencies of these pipes were in between 1.5 and 2 kHz. Hence, maximal element size was chosen not to be greater than 17.5 mm, which resulted a maximal validity frequency of approximately 2.5 kHz, making use of equation (1). At the mouth part, the model resolution was set higher to be able to follow the steep changes of acoustical variables near the free edges of the geometry.

Simulations were run by using both the indirect BEM and the coupled FE/BE method. Testing frequencies were chosen to start from 50 Hz and end at 2500 Hz with a 1 Hz resolution. In case of the coupled method, the Schur complement and interpolation technique were applied. The interpolation was carried out using a spline formula with 30 base points. The surface meshes consisted of approximately 1500 nodes, while the volume meshes had 2500 nodes.

Computational times were around 6 and $6\frac{1}{2}$ hours using the indirect BEM, and between 6 and 8 hours using the coupled method on the same computer. It is worth mentioning, that the coupled method under a self developed program performed nearly as fast as the indirect method under the commercial software package. This means that a more optimized implementation of the coupled method would perform very well in simulations.

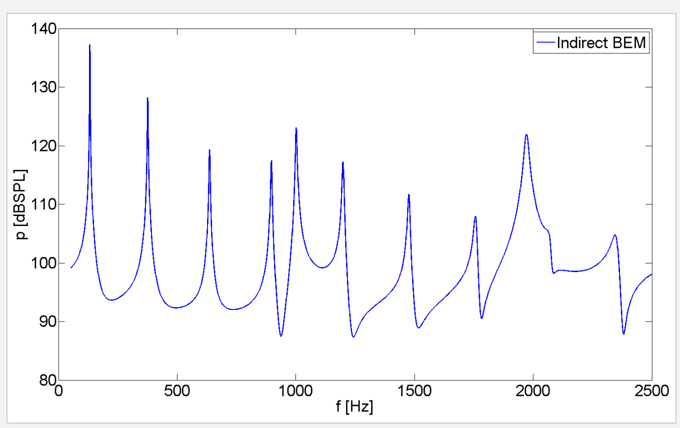
In the following tables and figures simulation results are compared to each other and measurement data. Frequencies of the first five harmonics and stretching factors were examined.

A chimney pipe experiment

Beside the simulations of wooden pipes a chimney pipe experiment was performed by using the indirect BEM. The chimney pipe is named after the small 'chimney' tube that is attached to the resonator body. The geometry is shown in figure 5.c). As seen, the resonator geometry is more complicated than in case of wooden pipes.
Only the fundamental frequency was given beside the geometry parameters. This had the value of 130.8 Hz according to measurement results. The indirect BEM simulation gave 131 Hz for the fundamental frequency, which is accurate, as the simulation was run with the resolution of 1 Hz.
As this experiment was not in the main line of the research, the results are presented as an example of application of the indirect BEM for a special mesh. It is worth mentioning that the same C tone is achieved by a completely different pipe geometry and dimensioning. This experiment was demonstrated here only as an outlook on further simulations that can be carried out by using the presented numerical techniques.

Parameter	Value
Resonator length	586.0
Resonator diameter	81.1
Chimney length	162.2
Chimney diameter	20.3
Mouth height	22.0
Mouth width	59.9

The table shows the exact dimensions of the pipe in mm. The simulated spectrum at the pipe mouth can be seen in the figure on the right hand side.



1. Table: Pipe dimensions given in mm

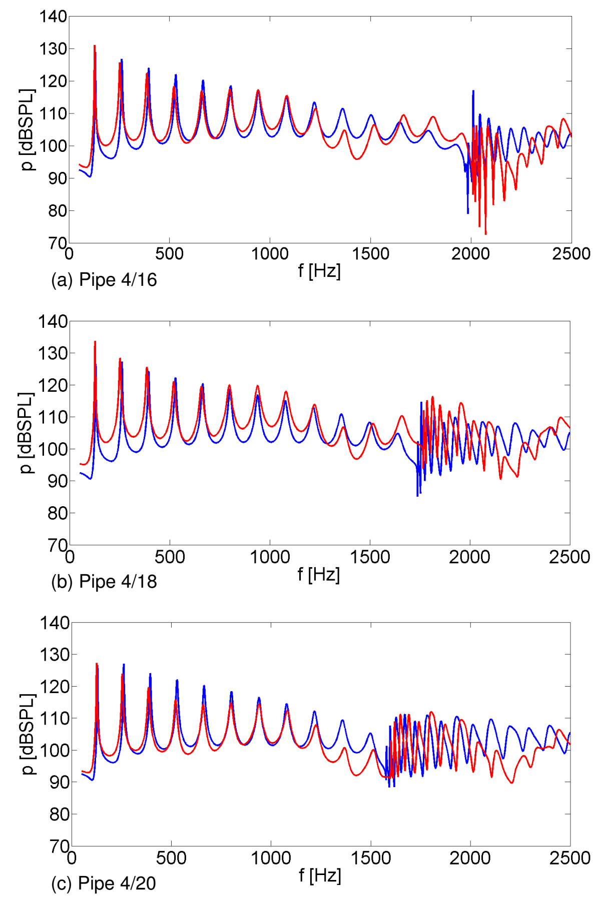
Pipe	Length	Width	Depth	Mouth height	Mouth width
4/16	1180	69.80	86.87	19.87	68.64
4/18	1181	61.20	98.32	21.53	60.76
4/20	1179	55.34	108.40	25.34	53.93

2. Table: Simulation results compared to measurement data

Pipe: 4/16	**Measurement**		**Indirect BEM**		**Coupled FE/BE**	
Harmonic	F [Hz]	Stretch	F [Hz]	Stretch	F [Hz]	Stretch
1. (Fund.)	129.87	1.000	131	1.000	128	1.000
2. (Octave)	261.76	2.016	263	2.008	253	1.977
3.	396.45	3.053	397	3.031	388	3.031
4.	536.98	4.135	531	4.053	522	4.078
5.	677.62	5.218	667	5.092	660	5.156
Cut-off [Hz]	1987		1987		2008	

Pipe: 4/18	**Measurement**		**Indirect BEM**		**Coupled FE/BE**	
Harmonic	F [Hz]	Stretch	F [Hz]	Stretch	F [Hz]	Stretch
1. (Fund.)	131.22	1.000	130	1.000	128	1.000
2. (Octave)	262.44	2.000	262	2.008	252	1.969
3.	400.38	3.051	394	3.025	387	3.023
4.	547.08	4.169	529	4.056	521	4.070
5.	680.99	5.190	664	5.095	660	5.156
Cut-off [Hz]	1740		1741		1768	

Pipe: 4/20	**Measurement**		**Indirect BEM**		**Coupled FE/BE**	
Harmonic	F [Hz]	Stretch	F [Hz]	Stretch	F [Hz]	Stretch
1. (Fund.)	131.22	1.000	130	1.000	126	1.000
2. (Octave)	265.12	2.020	262	2.007	255	2.024
3.	401.73	3.061	395	3.024	388	3.079
4.	543.71	4.143	529	4.053	524	4.159
5.	679.64	5.190	665	5.095	662	5.254
Cut-off [Hz]	1582		1579		1599	



10. Figure: Comparison of simulated spectra of wooden pipes at the pipe mouth – Indirect BEM – Coupled FEM/BEM

The stretching effect is represented by the stretching factor

$$\text{Stretch} = \frac{f_n}{f_1}. \tag{5}$$

Q-factors of these modes were also given among measurement data, but to be able to determine real Q-factors a damping model of air should be applied, which was not implemented herein. Thus, Q-factors determined by simulations can only be examined qualitatively, as without a damping model, simulated Q-factors are much higher than the real ones.

Table 2 shows comparison of acoustical parameters of the pipes, while in figure 10 diagrams of simulated spectra at the pipe mouth are displayed.

The fundamental frequencies are approximated within 1% range by the indirect boundary method, this means an absolute deviation that is less than 1.5 Hz. The coupled method predicts the fundamental frequencies with the average error 2-3% below the measured value. The maximal deviation is experienced in case of the 4/20 pipe, where the error is 5 Hz. This error is acceptable considering the simplicity of the model. The 1% deviance in case of the indirect BEM method is satisfactory and would alse be acceptable for an industrial application.

In case of the 4/16 and the 4/18 pipe the coupled method showed some irregularities for the octave and determined the stretching factor with significant error. The deviation of the measured and simulated frequencies is around 4-5% for these two pipes. For the further harmonics the coupled method estimates the stretching factors more accurately than the indirect method. However, the frequencies of these partials are generally determined more accurately by the indirect method, with a maximal error of 4%.

The cut-off frequencies are determined accurately by the indirect BEM and within a 1.5% error range by the coupled technique. This is a very accurate result taking into consideration that the resonater model implies remarkable simplification and neglects. The resulting cut-off frequencies are lower for the deeper pipes, as it is expected. Above the cut-off frequencies, the spectra become irregular as expected. As the irregularities are very sensitive to the model parameters, the simulated spectra are not expected to match up above the cut-off. In this frequency range the spectrum is not examined in details, only the cut-off effect is important.

Comparing the diagrams to figure 3, it can be seen, that the simulated transfer functions qualitatively correspond to a typical pipe transfer function. The amplification peaks are wider for the successive harmonics, as expected. The detailed analysis of Q-factors is not done herein, because of the reasons mentioned above.

As it can be seen on the comparison diagrams, minor irregularities are experienced in simulation results involving the coupled FE/BE method around 1.5 kHz. Except for these irregularities, as it can be seen, the simulated spectra approximately match up for the two methods. Therefore, both methods can efficiently be applied for pipe simulations. The accuracy analysis of the two methods is summarized in table 3.

3. Table: Comparison of realtive errors

Parameter	Indirect BEM	Coupled FEM/BEM
Fundamental frequency	<1%	2-4%
Octave frequency	<1%	3-5%
Further partials	2-4%	2-5%
Stretching factors	2-3%	<2%
Cut-off frequency	<1%	<1.5%

6. Conclusions

It was shown in this paper, that the indirect boundary element method and the coupled FE/BE method can be applied for the calculation of the steady sound field of an organ pipe resonator. These methods are unable to model the sound generation process in its whole complexity. Despite of the fact, that the acoustic model contains significant neglects and simplifications, some key parameters on the sounding can be determined using these methods.

Frequencies of the fundamental and other harmonics, stretching factors and cut-off frequencies were compared to each other and measurement data. Generally, the indirect BEM method gave a more accurate result for the frequencies of the partials, while stretching factors were approximated more accurately by the coupled method. The cut-off frequencies were predicted with sufficient accuracy by both methods.

It was also shown, that for a more detailed examination of the sounding characteristics, e.g. the analysis of Q-factors the acoustical model should be extended.

7. Future research

The author's plans concerning future research are the following. The PML method should be implemented for a three dimensional case, to be able to set up pipe simulations using this method. It would also be useful to implement other numerical techniques such as infinite elements or other types of artificial boundaries. In order to be able to increase the resolution of the model, optimization and further speed up techniques should be applied. The coupled method should also be further optimized and tested for different pipe geometries with various resolutions.

To enhance the accuracy of the simulations, the pure acoustical model should be extended with physical parts, by which resonances of the mechanical structure of the pipe could be examined by means of a coupled vibroacoustic model. A long term plan is to examine the sound generation mechanism by taking into consideration the fluid flow effects. To be able to do this the analysis of a non-linear coupled model needs to be done. By the simulation of these effects attack transients could be calculated, for example.

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