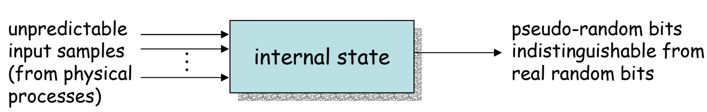
"Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin."
-- John von Neumann

Pseudo-random number generators

- motivation and definitions
- types of attacks
- analysis of ANSI X9.17, DSA PRNG
- guidelines for using vulnerable PRNGs
- design of Yarrow-160

Definitions

- a random number is a number that cannot be predicted by an observer before it is generated
  - if the number is generated within the range [0, N-1], then its value cannot be predicted with any better probability than 1/N
  - the above is true even if the observer is given all previously generated numbers
- a cryptographic pseudo-random number generator (PRNG) is a mechanism that processes somewhat unpredictable inputs and generates pseudo-random outputs
  - if designed, implemented, and used properly, then even an adversary with enormous computational power should not be able to distinguish the PRNG output from a real random sequence



Motivation

- sources of true randomness may be available …
  - keystroke timing
  - mouse movement
  - disc access time
  - network usage statistics
  - …
- … but the amount of random bits obtained per time unit or available at a given point in time may not be sufficient
- random number generators used for simulation purposes are not good for cryptographic purposes
  - example: $s_{i+1} = (a \cdot s_i + b) \bmod n$
    - has nice statistical properties
    - but it is predictable
- weakly designed PRNGs can easily destroy security even if very strong cryptographic primitives (ciphers, MACs, etc.) are used
  - example: early version of Netscape PRNG (to be used for SSL)

Early version of Netscape's PRNG

```
RNG_CreateContext()
    (seconds, microseconds) = time of day;
    pid = process ID; ppid = parent process ID;
    a = mklcpr(microseconds);
    b = mklcpr(pid + seconds + (ppid << 12) );
    seed = MD5(a, b);

mklcpr(x)
    return((0xDEECE66D*x + 0x2BBB62DC) >> 1)

RNG_GenerateRandomBytes()
    x = MD5(seed);
    seed = seed+1;
    return x;

create_key()
    RNG_CreateContext();
    RNG_CreateRandomBytes(); RNG_CreateRandomBytes();
    challenge = RNG_CreateRandomBytes();
    secret_key =  RNG_CreateRandomBytes();
```

Attacking the Netscape PRNG

- if an attacker has an account on the UNIX machine running the browser
  - *ps* command lists running processes → attacker learns pid, ppid
  - the attacker can guess the time of day with seconds precision
  - only unknown is the value of microseconds → ~2^{20} possibilities
  - each possibility can be tested easily against the challenge sent in clear within SSL
- if the attacker has no account on the machine running the browser
  - a has 20 bits of randomness, b has 27 bits of randomness → seed has 47 bits of randomness (compared to 128 bit advertised security)
  - ppid is often 1, or a bit smaller than pid
  - sendmail generates message IDs from its pid
    - send mail to an unknown user on the attacked machine
    - mail will bounce back with a message ID generated by sendmail
    - attacker learns the last process ID generated on the attacked machine
    - this may reduce possibilities for pid

Classification of attacks

- various ways to compromise the PRNG's state
  - cryptanalytic attacks
    - between receiving input samples the PRNG works as a stream cipher
    - a cryptographic weakness in this stream cipher might be exploited to recover its internal state
  - side-channel attacks
    - additional information about the actual implementation of the PRNG may be exploited
    - example: measuring the time needed to produce a new output may leak information about the current state of the PRNG (timing attacks)

```
x = MD5(seed);
seed = seed+1;  // increment needs m+1 byte additions if the last m bytes are all 0xFF
return x;       // long output time → last couple of bytes of seed are 0x00
```

  - input-based attacks
    - known-input attacks: an attacker is able to observe (some of) the PRNG inputs
    - chosen-input attacks: an attacker is able to control (some of) the PRNG inputs
      - typically applicable against smart cards
  - mishandling of seed files

Classification of attacks

- in practice, it is prudent to assume that occasional compromises of the state may happen
- various ways to exploit compromised states
  - permanent compromise attacks
    - given: state at time t_0
    - find: all future (or past) states
  - iterative guessing attacks
    - given: state at time t_0, outputs in $[t_0, t_1]$
    - find: state at time t_1
  - backtracking attacks
    - given: state at time t_0
    - find: outputs before t_0
  - meet-in-the-middle attacks
    - given: state at time t_0 and $t_2 > t_0$
    - find: state at time t_1, where $t_0 < t_1 < t_2$

ANSI X9.17

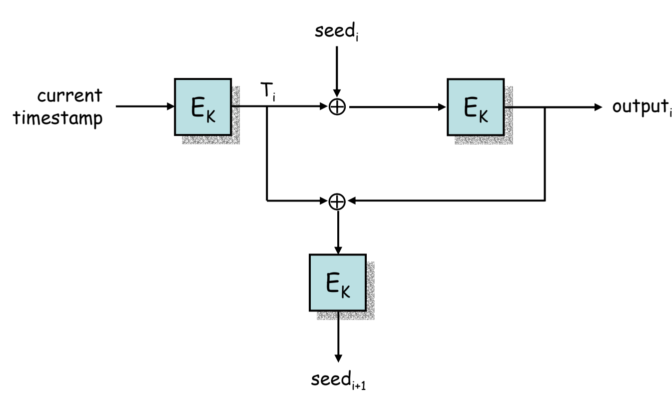
state: K, $seed_i$

output generation:

$T_i = E_K(\text{current timestamp})$

$output_i = E_K(T_i \oplus seed_i)$

$seed_{i+1} = E_K(T_i \oplus output_i)$



Attacks on X9.17

- cryptanalytic attacks
  - it seems that they require to break the block cipher E
  - however, this has never been proven formally

- input based attacks
  - assume that an attacker can freeze the clock (T_i = T for all i)
  - $\text{output}_{i+1} = E_K(T \oplus \text{seed}_{i+1}) = E_K(T \oplus E_K(T \oplus \text{output}_i)) = E'_K(\text{output}_i)$
  - for a good cipher E, we expect a repeating value in the above sequence after $\sim 2^{n-1}$ steps, where n is the block size of E
  - in a sequence of true n-bit random values, a collision is expected after $\sim 2^{n/2}$ steps (birthday paradox)
  - the attacker can distinguish the output of X9.17 from a sequence of true random numbers given that he can observe sufficiently many ($\sim 2^{n/2}$) outputs
    - not practically important
    - *certificational weakness*

ANSI X9.17

Attacks on X9.17

- weaknesses leading to state compromise extensions
  - part of the state (K) never changes
    - → if K is compromised, then the PRNG can never fully recover
  - seed_{i+1} depends on seed_i only via output_i
    - → if K is known from a previous state compromise and output_i is observable, then finding seed_{i+1} is not so difficult (timestamps can usually be assumed to have only 10-20 bits of entropy)

- deriving the seed from two consecutive outputs (and K)

  $\text{seed}_{i+1} = E_K(T_i \oplus \text{output}_i)$ (1)

  $\text{seed}_{i+1} = D_K(\text{output}_{i+1}) \oplus T_{i+1}$ (2)

  - assume that timestamps has 10 bits of entropy
  - try all values for T_i, and form a sorted list of possible values for seed_{i+1} using (1)
  - try all values for T_{i+1}, and form another sorted list of possible values for seed_{i+1} using (2)
  - the correct seed_{i+1} value is the one that appears on both lists (expected number of matching pairs is $\sim 1+2^{20-n}$)

ANSI X9.17

Attacks on X9.17

- iterative guessing attack
  - if an attacker knows K and $seed_i$ and sees (some public function f of) $output_i$, then he can determine $seed_{i+1}$ easily
    - let $f(output_i) = v$
    - try all possible values t for T_i, and form a list of values $v_t = f(E_K(t \oplus seed_i))$
    - select t* such that $v_{t^*} = v$
    - $seed_{i+1} = E_K(t^* \oplus E_K(t^* \oplus seed_i))$
- backtracking
  - if an attacker knows K and $seed_{i+1}$ and sees (some public function f of) $output_i$, then he can determine $output_i$ and $seed_i$ easily (EXERCISE)
- timer entropy issues
  - if larger amount of random bytes are needed (e.g., RSA key pair generation), then the PRNG is called repeatedly within a very short time

  → consecutive T_i values have much less entropy than 10-20 bits

Exercise

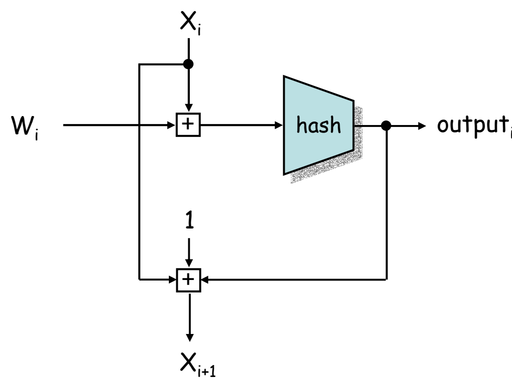
- Explain how the backtracking attack (given K and $seed_{i+1}$ and some public function f of $output_i$, find $output_i$ and $seed_i$) works on the X9.17 PRNG.

DSA PRNG

state: X_i
optional input: W_i (W_i = 0 if not supplied)
output generation:

$output_i = hash((W_i + X_i) \bmod 2^{160})$

$X_{i+1} = (X_i + output_i + 1) \bmod 2^{160}$



Attacks on the DSA PRNG

- cryptanalytic attacks
  - if the hash function is good, then the PRNG output seems to be hard to distinguish from a real random sequence
  - no formal proof
- input based attacks
  - assume the attacker can control W_i
  - setting $W_i = (W_{i-1} - output_{i-1} - 1) \bmod 2^{160}$ will force the PRNG to repeat its output

$$\begin{aligned} output_i &= hash((W_i + X_i) \bmod 2^{160}) = \\ &= hash(((W_{i-1} - output_{i-1} - 1) + (X_{i-1} + output_{i-1} + 1)) \bmod 2^{160}) = \\ &= hash((W_{i-1} + X_{i-1}) \bmod 2^{160}) = \\ &= output_{i-1} \end{aligned}$$

  - this works only if input samples are sent directly into the PRNG
    - in practice, they are often hashed before sent in

Attacks on the DSA PRNG

- a weakness that may make state compromise extensions easier
  - X_{i+1} depends on W_i only via $output_i$
    - → if an attacker compromised X_i and can observe $output_i$, then he knows X_{i+1} no matter how much entropy has been fed into the PRNG by W_i
- iterative guessing attack
  - if an attacker knows X_i and observes (a public function f of) $output_i$, then he can find X_{i+1}
    - let $f(output_i) = v$
    - assume that W_i has only 20 bits of entropy
    - the attacker can try all possible values w for W_i, and compute $v_w = f(hash((w + X_i) \bmod 2^{160}))$
    - let w* be the value such that $v = v_{w*}$
    - $X_{i+1} = (X_i + hash((w^* + X_i) \bmod 2^{160}) + 1) \bmod 2^{160}$
- filling the gaps
  - if an attacker knows X_i and X_{i+2}, and observes $output_{i+1}$, then he can compute $output_i$ as

$$output_i = (X_{i+2} - X_i - 2 - output_{i+1}) \bmod 2^{160}$$

Strengthening the DSA PRNG

- all inputs should be hashed together before feeding them into the PRNG (to make input based attacks harder)
- X_{i+1} should depend on W_i directly and not via the output
  - example: $X_{i+1} = X_i + hash(output_i + W_i)$

Guidelines for using vulnerable PRNGs

- use a hash function at the output to protect the PRNG from direct cryptanalytic attacks
- hash all inputs together with a counter or timestamp before feeding into the PRNG to make chosen-input attacks harder
- pay special attention to PRNG starting points and seed files to make it harder to compromise the PRNG state
- occasionally generate a new starting state and restart the PRNG to limit the scope of state compromise extensions

The Yarrow-160 PRNG

- design philosophy
  - accumulate entropy from as many different sources as possible
  - reseed the key (state) only when enough entropy has been collected (this puts the PRNG in an unguessable state at each reseed)
  - between reseeds, use strong crypto algorithms to generate outputs from the key (like a stream cipher)
- four major components
  - entropy accumulator
    - collects samples from entropy sources into two entropy pools (slow and fast pool)
  - reseed mechanism
    - periodically reseeds the key with new entropy from the pools
  - reseed control
    - determines when a reseed should be performed
  - generation mechanism
    - generates PRNG output from the key (state)

Entropy accumulator

- inputs from each source are fed alternately into two entropy pools
  - fast pool
    - provides frequent reseeds
    - ensures that state compromises has as short a duration as possible
  - slow pool
    - rare reseeds
    - entropy is estimated very conservatively
    - rationale: even if entropy estimation of the fast pool is inaccurate, the PRNG still eventually gets a secure reseed from the slow pool
- entropy estimation
  - entropy of each sample is measured in three ways:
    - a: programmer supplies an estimate for the entropy source
    - b: a statistical estimator is used to estimate the entropy of the sample
    - c: length of the sample multiplied by $\frac{1}{2}$
  - entropy estimate of the sample is min(a, b, c)
  - entropy contribution of a source is the sum of entropy estimates of all samples collected so far from that source
  - entropy contribution of each source is maintained separately

Reseed control

- periodic reseed
  - the fast pool is used to reseed when any of the sources reaches an estimated entropy contribution of 100 bits
  - the slow pool is used to reseed when at least two sources reaches an estimated entropy contribution of 160 bits
- explicit reseed
  - an application may explicitly ask for a reseed operation (from both pools)
  - should be used only when a high-valued random secret is to be generated

Reseed mechanism

- reseed from the fast pool (h is SHA1, E is 3DES):

  v_0 := h(fast pool)

  $v_i := h(v_{i-1} \mid v_0 \mid i)$ for i = 1, 2, …, P_t

  $K := h'(h(v_{Pt} \mid K), k)$

  $C := E_K(0)$

  where h' is a "size adaptor"

  $h'(m, k)$ = first k bit of $s_0 \mid s_1 \mid s_2 \mid \ldots$

  $s_0 = m$

  $s_i = h(s_0 \mid \ldots \mid s_{i-1})$ i = 1, 2, …

  reset all entropy estimates to 0

  wipe the memory of all intermediate values

- reseed from the slow pool:
  - feed h(slow pool) into fast pool
  - reseed from fast pool as described above

Reseed mechanism

- observations
  - new value of K directly depends on previous value of K and current pool content (pool → v_0 → v_{Pt})
    - if an attacker has some knowledge of the previous value of K, but does not know most of the pool content, then he cannot guess the new K
    - if an attacker does not know the previous value of K, but observed many inputs of the pool, then he still cannot guess the new K
  - execution time depends on security parameter P_t
    - this makes the time needed for iterative guessing attacks longer

Generation mechanism

- algorithm (E is 3DES):

  $C := (C+1) \bmod 2^n$ // n is the block size of E

  $R := E_K(C)$

  output: R

- generator gate
  - after P_g output has been generated, a new key is generated
    K := next k bits of PRNG output
  - P_g is a security parameter currently set to 10
  - rationale: if a key is compromised, then only 10 previous output can be computed by the attacker (prevention of backtracking attacks)

Yarrow-160

Protecting the entropy pool

- the pool can be swapped into swap files and stored on disk
  - several operating systems allow to lock pages into memory
    - mlock() (UNIX), VirtualLock() (Windows), HoldMemory() (Macintosh)
  - memory mapped files can be used as private swap files
    - the files should have the strictest possible access permissions
    - file buffering should be disabled to avoid that the buffer is swapped
- allocated memory blocks can be scanned through by other processes
  - entropy pool is often allocated at the beginning when the security subsystem is started → pool is often at the head of allocated memory blocks
  - the pool can be embedded in a larger allocated memory block
  - its location can be changed periodically (by allocating new space and moving the pool) in the background
  - this background process can also be used to prevent the pool from being swapped (touched pages are kept in memory with higher probability)

Summary

- PRNGs for cryptographic purposes needs special attention
  - simple congruential generators are predictable
  - naïve PRNG design will not do (cf. early Netscape PRNG)
- widely used cryptographic PRNGs may have weaknesses too
  - ANSI X9.17
  - DSA PRNG
  - RSAREF 2.0
  - …
- some guidelines for using vulnerable PRNGs
- design of Yarow-160
  - careful design that seems to resist various attacks
- protecting the entropy pools

Recommended readings

- Kelsey, Schneier, Wagner, Hall. Cryptographic attacks on PRNGs. Workshop on Fast Software Encryption, 1998.
- Kelsey, Schneier, Ferguson. Yarrow-160: Notes on the design and analysis of the Yarrow cryptographic PRNG.
- Gutmann. Software generation of random numbers for cryptographic purposes. USENIX Security Symposium, 1998.