

Efficient Symmetric-key Private Authentication

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Private authentication – the problem

- authentication protocols often reveal the identity of the authenticating party (prover) to an eavesdropper
- when devices move around and authenticate themselves frequently, the location of them can be tracked
- typical examples are RFID tags and contactless smart card based systems

An example – ISO 9798-2

- the protocol:

- (1) $B \rightarrow A: r_B$
- (2) $A \rightarrow B: E(K, r_B | B^*)$

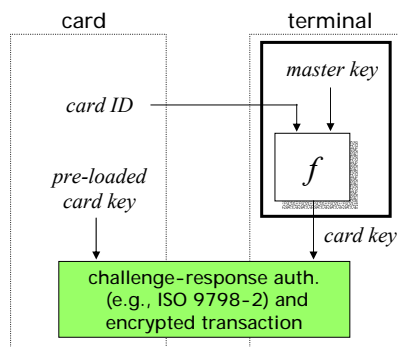
where K is a shared key between A and B , and $E(.)$ denotes encryption

- “it is assumed that the parties are aware of the claimed identity of the other either by context or *by additional cleartext data fields*”

- (0) $A \rightarrow B: A$

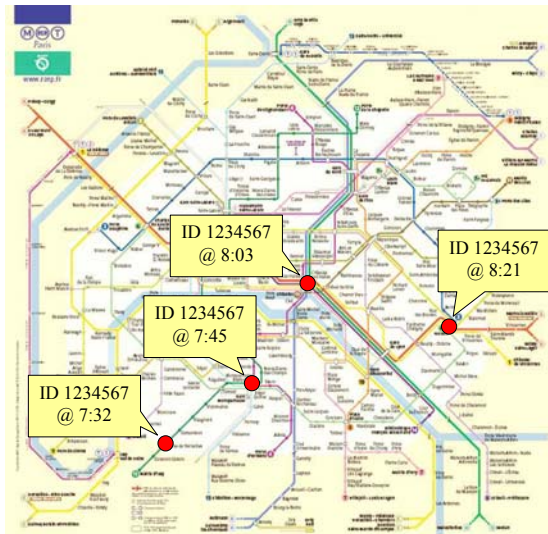
Authentication in AFC systems

- off-line terminals should be able to authenticate any transport card



- key diversification is used
 - each card has its own key
 - card key is generated from the card ID and a master key using a one-way function
 - terminals store only a few master keys, and compute card keys on-the-fly when they are needed
- this requires transmitting the ID of the card at the beginning of the transaction*

Private authentication – the problem (cont'd)

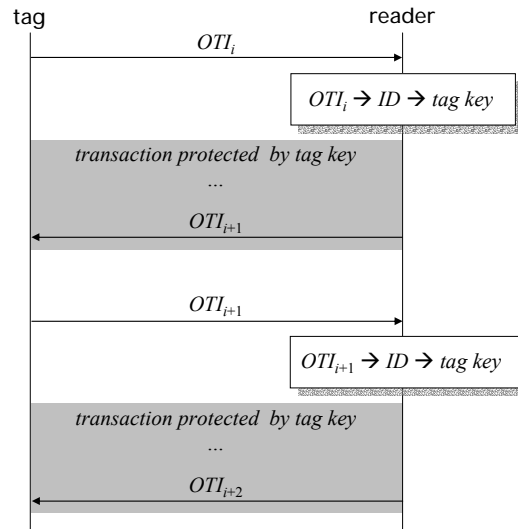


Solutions based on public-key cryptography

- encrypt identity information of the authenticating party with the public key of the verifier
- setup a confidential channel between the parties using the basic Diffie-Hellman protocol and send identity information through that channel
 - IKE in main mode works in this way
- common disadvantage: public key operations may not be affordable in devices with limited resources (e.g., public transport cards, RFID tags)

One-time identifiers – a solution for high-end tags

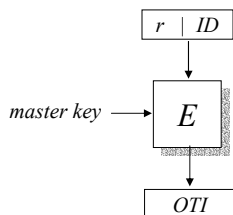
[Buttyan et al., 2006]



Assumptions and requirements for OTIs

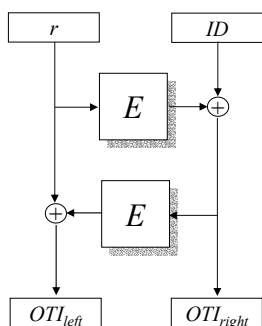
- tags must have some writable memory
- OTIs should be unlinkable
- mapping an OTI to the real ID should be fast
- OTI generated by one reader should be decodable by another
 - since readers may be off-line, OTIs must be generated from a master key known by every reader
- it should be difficult to break the master key even if many OTIs are observed

Possible implementations



- let's use the block cipher already implemented on many readers (e.g., DES)
- if IDs are not too long, then a random number and an ID may fit in a single block
- if r is n bit long, then first repeating OTI is expected after about $2^{n/2}$ transactions (birthday paradox)
- typical ID lengths
 - Mifare Classic: 32 bits
 $n = 32, 2^{32/2} = \sim 60000$
 - Mifare DESfire: 56 bits
 $n = 56, 2^{56/2} = 16$ (too small!)

Possible implementations (cont'd)



- if IDs are long, then we need two blocks
- one could use the block cipher in a 2-round Feistel structure to effectively double its block length
- $n = 64$ should be enough

Naïve solutions for low-cost tags

- encrypt (hash) identity information with a single common key
 - drawback:
 - compromise of a single member of the system has fatal consequences

- encrypt (hash) identity information with a unique key
 - drawback:
 - number of keys need to be tested by the verifier grows linearly with the number of potential provers
 - doesn't scale (potentially long authentication delay in large systems)

Better solutions for low cost tags

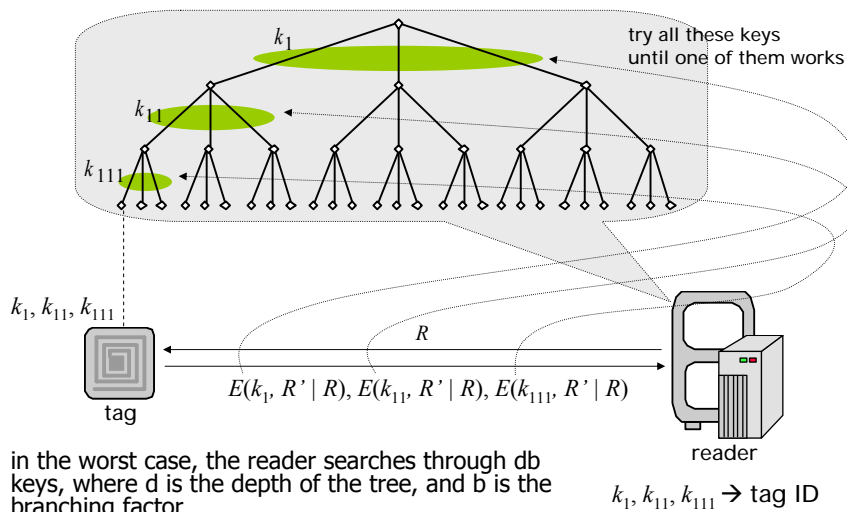
- tree-based approach
 - proposed by Molnar and Wagner in 2004
 - improved by Buttyan, Holczer, and Vajda in 2006
 - advantage:
 - authentication delay is logarithmic in the number of members
 - drawback:
 - increased overhead (at the prover's side)
 - level of privacy quickly decreasing as the number of compromised members increases

- group-based approach
 - proposed by Avoine, Buttyan, Holczer, Vajda in 2007
 - advantage:
 - higher level of privacy and smaller overhead than in the tree-based approach
 - drawback: ???

Outline

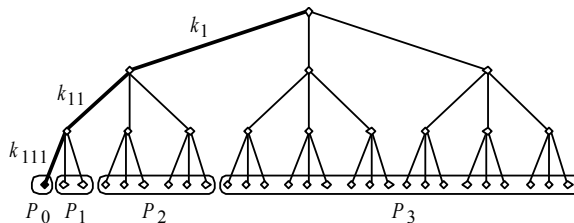
- introduction to private authentication
- overview of the tree-based approach
 - original proposal
 - optimization
- privacy metrics
 - normalized average anonymity set size [Buttyan et al., 2006]
 - probability of traceability [Avoine et al., 2005]
 - entropy based anonymity set size [Nohl and Evens, 2006]
- description and analysis of the group-based approach
- comparison of the tree-based and the group-based approaches
- conclusion and open problems

The tree-based approach



- in the worst case, the reader searches through db keys, where d is the depth of the tree, and b is the branching factor
- compare this to b^d , which is the total number of tags !

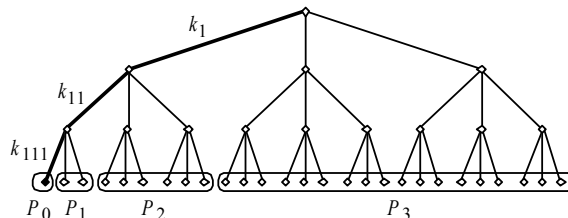
Tree-based private authentication – a problem



- if a member is compromised, its keys are learned by the adversary
- however, most of those keys are used by other members too
- the adversary can recognize the usage of those compromised keys
- consequently, the level of privacy provided by the system to *non-compromised* members is decreased

but:
this decrease can be minimized by careful design of the tree!

Resistance to single member compromise (RSMC)



- the level of privacy provided by the system to a randomly selected member is characterized by the *average anonymity set size when a single member is compromised*:

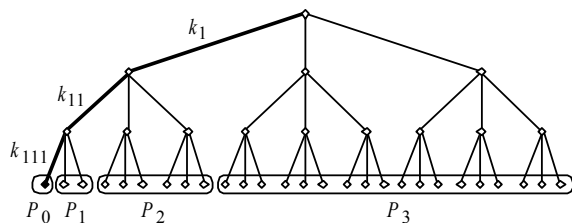
$$\bar{S} = \sum_{i=0}^{\ell} \frac{|P_i|}{N} |P_i| = \sum_{i=0}^{\ell} \frac{|P_i|^2}{N}$$

where N is the total number of members

- we normalize this to obtain the benchmark metric called *resistance to single member compromise*:

$$R = \frac{\bar{S}}{N} = \sum_{i=0}^{\ell} \frac{|P_i|^2}{N^2}$$

Computing RSMC



$$\begin{aligned}
 |P_0| &= 1 \\
 |P_1| &= b_\ell - 1 \\
 |P_2| &= (b_{\ell-1} - 1)b_\ell \\
 |P_3| &= (b_{\ell-2} - 1)b_{\ell-1}b_\ell \\
 &\dots \quad \dots \\
 |P_\ell| &= (b_1 - 1)b_2b_3 \dots b_\ell
 \end{aligned}$$

$$\begin{aligned}
 R &= \frac{S}{N} = \sum_{i=0}^{\ell} \frac{|P_i|^2}{N^2} \\
 &= \frac{1}{N^2} (1 + (b_\ell - 1)^2 + ((b_{\ell-1} - 1)b_\ell)^2 + \dots + ((b_1 - 1)b_2b_3 \dots b_\ell)^2) \\
 &= \frac{1}{N^2} \left(1 + (b_\ell - 1)^2 + \sum_{i=1}^{\ell-1} (b_i - 1)^2 \prod_{j=i+1}^{\ell} b_j^2 \right)
 \end{aligned}$$

A trade-off between privacy and efficiency

- efficiency of the system is characterized by the *maximum authentication delay*:

$$D = \sum_{i=1}^{\ell} b_i$$

- examples:
 - naïve linear key search ($l = 1$)
 - $R = 1 - 2(N+1)/N^2 \approx 1 - 2/N \approx 1$ (if N is large)
 - $D = N$
 - binary key-tree ($l = \log N$)
 - $R = 1/3 + 2/(2N^2) \approx 1/3$ (if N is large)
 - $D = 2 \log N$
- how to maximize R while keeping D below a threshold?

The optimization problem

Given the total number N of members and the upper bound D_{\max} on the maximum authentication delay, find a branching factor vector $B = (b_1, b_2, \dots, b_\ell)$ such that

$$R = \frac{1}{N^2} \left(1 + (b_\ell - 1)^2 + \sum_{i=1}^{\ell-1} (b_i - 1)^2 \prod_{j=i+1}^{\ell} b_j^2 \right)$$

is maximal, subject to the following constraints:

$$\prod_{i=1}^{\ell} b_i = N$$
$$\sum_{i=1}^{\ell} b_i \leq D_{\max}$$

Analysis of the optimization problem

Lemma 1: we can always improve a branching factor vector by ordering its elements in decreasing order

Lemma 2: lower and upper bounds on $R(B)$ (where B is ordered):

$$\left(1 - \frac{1}{b_1}\right)^2 \leq R(B) \leq \left(1 - \frac{1}{b_1}\right)^2 + \frac{4}{3b_1^2}$$

Lemma 3: given two branching factor vectors (that satisfy the constraints), the one with the larger first element is always at least as good as the other

Lemma 4: given two branching factor vectors the first j elements of which are equal, the vector with the larger $(j+1)$ -st element is always at least as good as the other

A solution

- let P be the ordered vector of prime factors of N
- if P doesn't satisfy the conditions, then no solution exists
- otherwise, let P' be a subset of P such that
 - if we multiply the prime factors in P' (let the product be Q), then the vector $(Q, P \setminus P')$ still satisfy the constraints, and
 - Q is maximal
- the first element of the optimal branching factor vector is Q
- if all prime factors are used ($P \setminus P' = \emptyset$), then stop
- else repeat the procedure recursively with the remaining primes

An example for the operation of the algorithm

- let $N = 27000$ and $D_{\max} = 90$

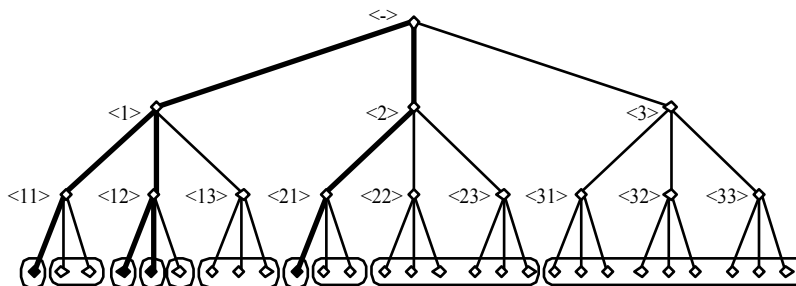
recursion level	P	d	P'	Q
1	(5, 5, 5, 3, 3, 3, 2, 2, 2)	90	(3, 3, 2, 2, 2)	72
2	(5, 5, 5, 3)	18	(5)	5
3	(5, 5, 3)	13	(5)	5
4	(5, 3)	8	(5)	5
5	(3)	3	(3)	3

- the optimal tree for these parameters is (72, 5, 5, 5, 3)
 - $R \approx 0.9725$
 - $D = 90$

Proof sketch of the algorithm

- let $B^* = (b^*_1, \dots, b^*_L)$ be the output of the algorithm
- assume that there's a $B' = (b'_1, \dots, b'_K) \neq B^*$ such that $R(B') > R(B^*)$
- B^* is obtained by maximizing $b^*_1 \rightarrow b^*_1 \geq b'_1$
- if $b^*_1 > b'_1$ then $R(B^*) \geq R(B')$ by Lemma 3 $\rightarrow b^*_1 = b'_1$ must hold
- B^* is obtained by maximizing b^*_2 (once b^*_1 is determined) $\rightarrow b^*_2 \geq b'_2$
- if $b^*_2 > b'_2$ then $R(B^*) \geq R(B')$ by Lemma 4 $\rightarrow b^*_2 = b'_2$ must hold
- ...
- $B^* = B'$ must hold, which is a contradiction

General case



- compromised tags partition the set of all tags
 - tags in a given partition are indistinguishable
 - tags in different partitions can be distinguished
- we measure the level of privacy again as the normalized average anonymity set size (NAASS)

Approximation of NAASS for key-trees

- let the branching factors of the tree be b_1, b_2, \dots, b_L
- select a tag T randomly (without loss of generality, we assume that the left most tag of the tree is selected)
- we want to compute the expected size of T 's anonymity set when *some* tags are compromised
- we assume that each tag is compromised with probability

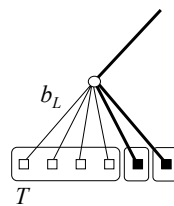
$$p = q/N$$

- the probability that a given edge (key) is compromised at level i is

$$q_i = 1 - (1 - p)^{N_i}$$

where $N_i = N/(b_1 b_2 \dots b_i)$ is the number of tags below that edge

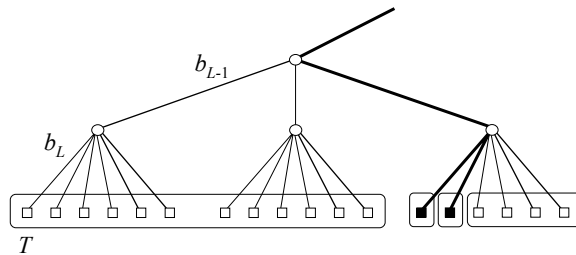
Approximation of NAASS for key-trees



- the probability that T 's anonymity set size is exactly k ($k = 1, 2, \dots, b_L - 1$) is:

$$(1 - q_L) \binom{b_L - 1}{k - 1} (1 - q_L)^{k-1} q_L^{b_L - k} = \binom{b_L - 1}{k - 1} (1 - q_L)^k q_L^{b_L - k}$$

Approximation of NAASS for key-trees



- the probability that T 's anonymity set size is kb_L ($k = 1, 2, \dots, b_{L-1}-1$) is:

$$\binom{b_{L-1} - 1}{k - 1} (1 - q_{L-1})^k q_{L-1}^{b_{L-1}-k}$$

Approximation of NAASS for key-trees

- in general, the probability that T 's anonymity set size is $kb_L b_{L-1} \dots b_{i+1} = kN_i$ ($i = 1, 2, \dots, L$ and $k = 1, 2, \dots, b_i-1$) is:

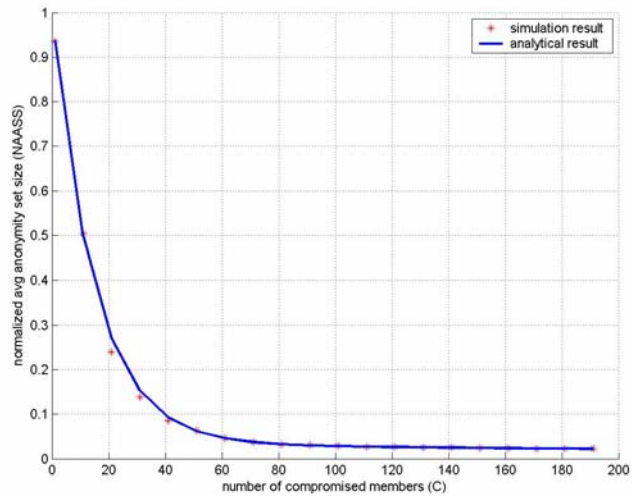
$$\binom{b_i - 1}{k - 1} (1 - q_i)^k q_i^{b_i-k}$$

- from this, the expected size of T 's anonymity set is:

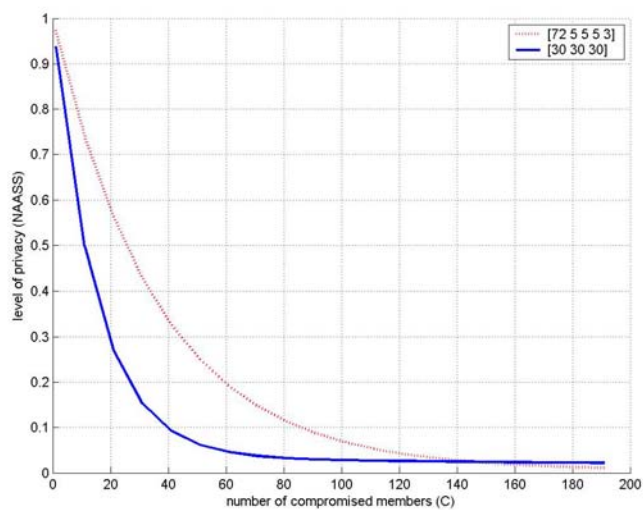
$$\bar{S} = \sum_{i=1}^L \sum_{k=1}^{b_i-1} kN_i \binom{b_i - 1}{k - 1} (1 - q_i)^k q_i^{b_i-k} + 1 \cdot p + N \cdot (1 - p)^N$$

Verification of the approximation

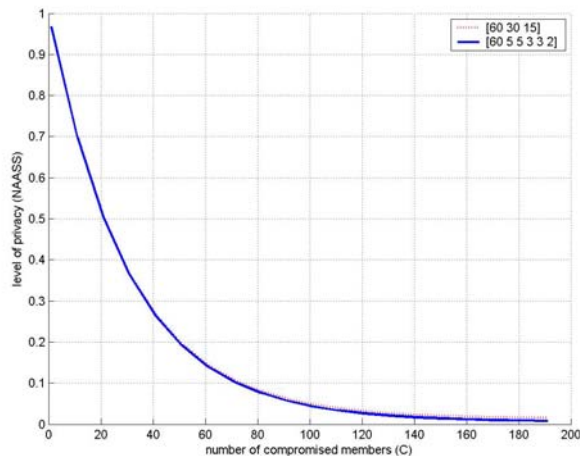
$$B = [30 \ 30 \ 30]$$



Comparison of key-trees in the general case



Comparison of key-trees in the general case



conclusion:

first element of the branching factor vector determines the level of privacy in the general case too

Probability of traceability as a privacy metric

- proposed by Avoine et al. at SAC 2005
- traceability game:
 1. The adversary can tamper with a certain number C of tags (Compromised tags are put back in circulation)
 2. The adversary chooses a tag T and queries it as much as she wants (but she cannot compromise T)
 3. The adversary is presented two tags T_1 and T_2 such that T is in $\{T_1, T_2\}$. The adversary can query both T_1 and T_2 as much as she wants, and she has to decide which one is T .
- the success probability of the adversary in the traceability game is a measure of privacy

Relation to NAASS

- if T_1 and T_2 are in the same partition, then the adversary cannot distinguish them \rightarrow she cannot tell which one is T
- otherwise, she can distinguish T_1 and $T_2 \rightarrow$ she can decide which one is T
- prob. of success = $1 - \Pr\{T_1 \text{ and } T_2 \text{ are in the same partition}\}$

$$\sum_{\forall |P|} \Pr\{\text{size of } T\text{'s partition is } |P|\} \cdot \frac{|P|}{N} =$$

$$\sum_{i=1}^L \sum_{k=1}^{b_i-1} \binom{b_i-1}{k-1} (1-q_i)^k q_i^{b_i-k} \cdot \frac{kN_i}{N} + p \cdot \frac{1}{N} + (1-p)^N \cdot \frac{N}{N} = \bar{S}$$

Normalized entropy based anonymity set size (NEASS)

- proposed by Nohl and Evans in 2006 (tech report)
- main idea:
 - assume that a tag is compromised and this results in two equal size ($N/2$) partitions
 - the adversary can tell each tag in either one of the partitions \rightarrow 1 bit of information has been disclosed
 - in general, the amount of information that is disclosed due to tag compromise is

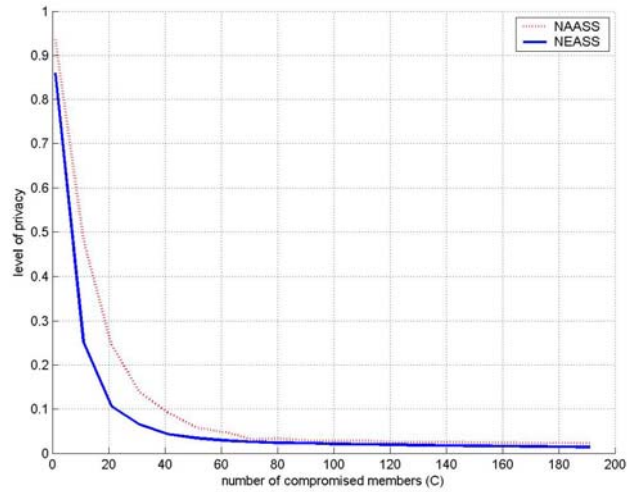
$$I = \sum_{\forall P} \frac{|P|}{N} \log_2 \frac{N}{|P|}$$

- (normalized) entropy based anonymity set size:

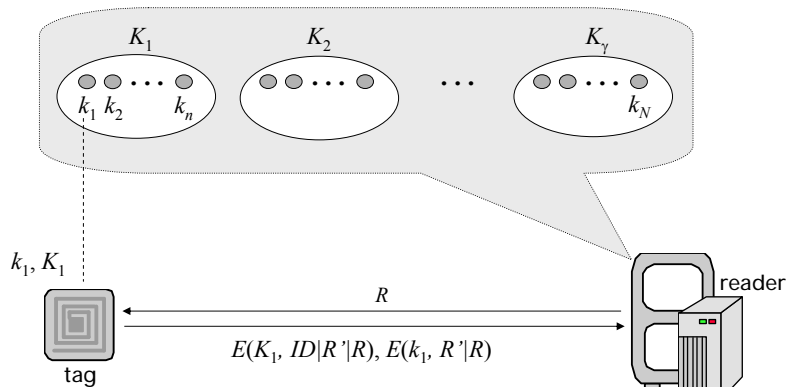
$$\bar{S}_{entropy} = \frac{N}{2I} \qquad \frac{\bar{S}_{entropy}}{N} = \frac{1}{2I}$$

Comparison of NAASS and NEASS (simulation)

$B = [30 \ 30 \ 30]$



The group-based approach



immediate advantage:
each tag stores and uses only two keys

- 1.) try all group keys until one of them works
- 2.) authenticate the tag by using its individual key

Computing NAASS for groups



- partitioning depends on the number C of compromised *groups*
- NAASS can be computed as:

$$\frac{\bar{S}}{N} = \sum_{\forall i} \frac{|P_i|^2}{N^2} = \frac{nC + (n(\gamma - C))^2}{N^2}$$

- if tags are compromised randomly, then C is a random variable
 - we are interested in the expected value of S/N
 - for this we need to compute $E[C]$ and $E[C^2]$

Computing NAASS for the group-based approach

- let A_i be the event that at least one tag is compromised from the i -th group
- let I_{A_i} be the indicator function of A_i
- the probability of A_i can be computed as:

$$P(A_i) = 1 - \frac{\binom{N-n}{c}}{\binom{N}{c}} = 1 - \prod_{j=0}^{c-1} \left(1 - \frac{n}{N-j}\right)$$

Computing NAASS for the group-based approach

$$E[C] = E\left[\sum_{i=1}^{\gamma} I_{A_i}\right] = \sum_{i=1}^{\gamma} P(A_i) = \gamma \left(1 - \prod_{j=0}^{c-1} \left(1 - \frac{n}{N-j}\right)\right)$$

$$\begin{aligned} E[C^2] &= E\left[\sum_{i=1}^{\gamma} I_{A_i}\right]^2 = E\left[\sum_{i=1}^{\gamma} I_{A_i}\right] + E\left[\sum_{i \neq j} I_{A_i \cap A_j}\right] = \\ &= E[C] + (\gamma^2 - \gamma) P(A_i \cap A_j) \end{aligned}$$

Computing NAASS for the group-based approach

$$P(A_i \cap A_j) = 1 - P(\overline{A_i} \cap \overline{A_j}) - 2P(A_i \cap \overline{A_j})$$

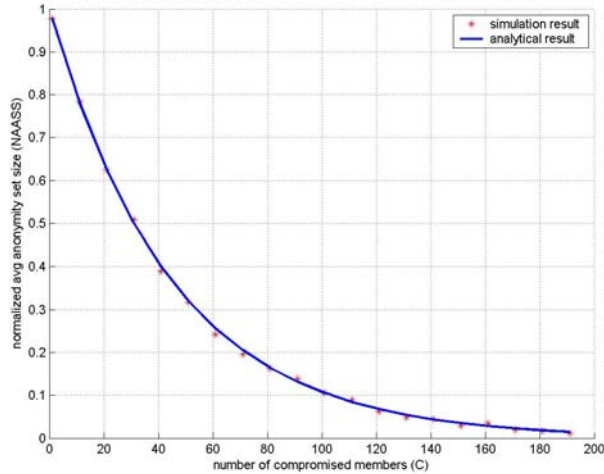
$$P(\overline{A_i} \cap \overline{A_j}) = \frac{\binom{N-2n}{c}}{\binom{N}{c}} = \prod_{j=0}^{c-1} \left(1 - \frac{2n}{N-j}\right)$$

$$\begin{aligned} P(A_i \cap \overline{A_j}) &= P(A_i | \overline{A_j}) P(\overline{A_j}) = \\ &= \left[1 - \prod_{j=0}^{c-1} \left(1 - \frac{n}{N-n-j}\right)\right] \cdot \prod_{j=0}^{c-1} \left(1 - \frac{n}{N-j}\right) \end{aligned}$$

Verification of the approximation

$N = 27000$

$\gamma = 90$



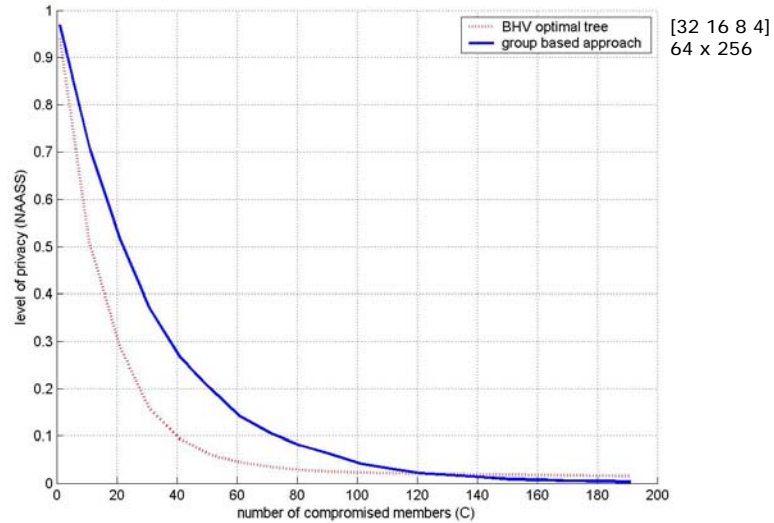
Comparison of trees and groups

- select a privacy metric (e.g., NAASS)
- for a given set of parameters (number N of tags, max authentication delay D), determine the optimal key-tree
- compute the privacy metric for the optimal tree (as a function of the number c of compromised tags)
- determine the corresponding parameters for the group based approach ($\gamma = D-1$)
- compute the privacy metric for the groups (as function of c)

Comparison in NAASS for a specific N and D pair

$$N = 2^{14}$$

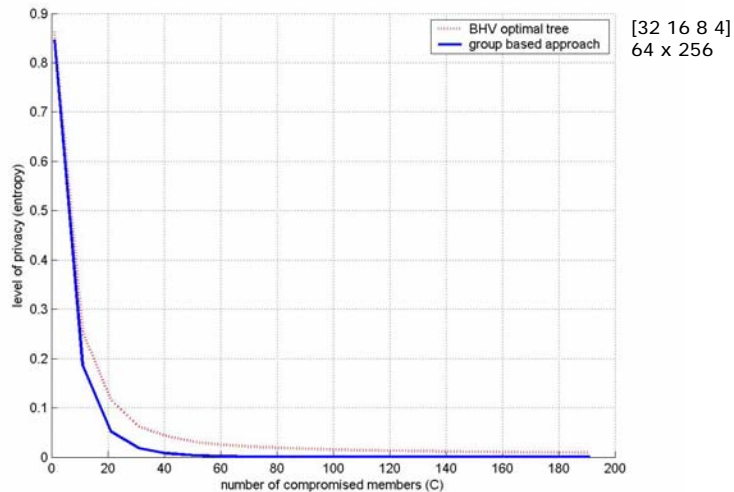
$$D = 65$$



Comparison in NEASS

$$N = 2^{14}$$

$$D = 65$$



Summary

- we studied the problem of (efficient) symmetric-key private authentication
- we gave an overview of the tree-based and the group-based approaches
- we gave an overview of proposed privacy metrics
 - NAASS, NEASS, prob. of traceability
- we showed some relationships between the metrics
 - prob. of traceability \sim NAASS, NEASS $<$ NAASS
- we gave precise approximations of the NAASS for trees and for groups
- we compared the tree and the group based approaches using NAASS and NEASS

Conclusions

- we obtained controversial results
 - group-based approach achieves better privacy if we use NAASS
 - tree-based approach achieves better privacy if we use NEASS
- be cautious which metric you use!
- yet, the difference between trees and groups does not seem to be large in terms of privacy
 - groups may be a better trade-off, due to the smaller overhead
- the group-based approach could be a serious alternative to the tree-based approach

Open problems

1. Closed form approximation of the NEASS (both for trees and groups) ?
2. How to find the optimal tree when the metric is the NEASS ?
3. How to preserve the efficiency of the tree and the group-based approaches *and* eliminate the exponential decrease of the level of privacy at the same time ???

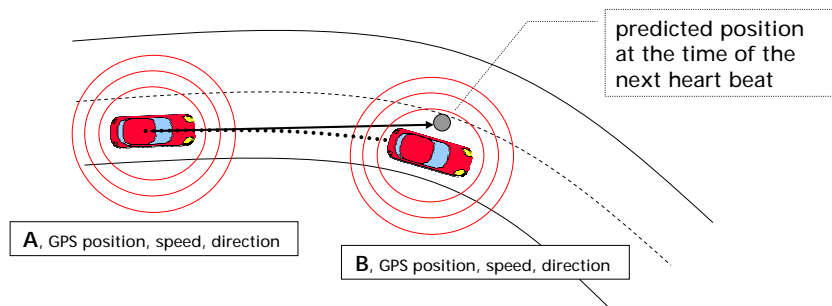
Location privacy in vehicular communication systems

The location privacy problem and a solution

- vehicles continuously broadcast *heart beat* messages, containing their ID, position, speed, etc.
- tracking the physical location of vehicles is easy just by eavesdropping on the wireless channel
- one possible solution is to change the vehicle identifier, or in other words, to use *pseudonyms*

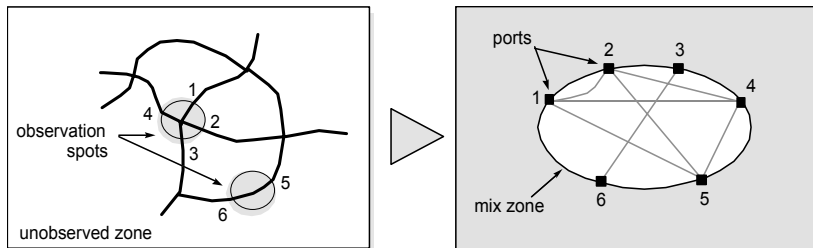
Adversary model

- changing pseudonyms is ineffective against a global eavesdropper



- hence, the adversary is assumed to be able to monitor the communications only at a limited number of places and in a limited range

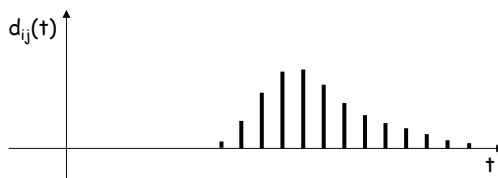
The mix zone concept



- the unobserved zone functions as a *mix zone* where the vehicles change pseudonym and mix with each other
- note that the vehicles do not know where the mix zone is (this depends on where the adversary installs observation spots)
- we assume that the vehicles change pseudonyms frequently so that each vehicle changes pseudonym while in the mix zone

Model of the mix zone

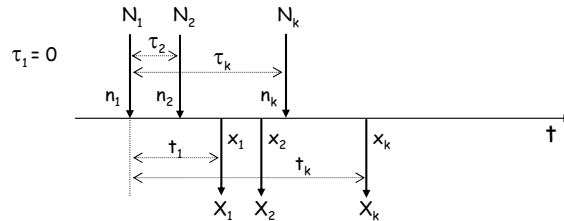
- time is divided into discrete steps
- $p_{ij} = \Pr\{ \text{exiting at } j \mid \text{entering at } i \}$
- D_{ij} is a random variable (delay) that represents the time that elapses between entering at i and exiting at j
- $d_{ij}(t) = \Pr\{ D_{ij} = t \}$



- $\Pr\{ \text{exiting at } j \text{ at } t \mid \text{entering at } i \text{ at } \tau \} = p_{ij} d_{ij}(t-\tau)$

Observations

- the adversary can observe the points (n_i, x_i) and the times (τ_i, t_i) of enter and exit events (N_i, X_i)



- by assumption, the nodes change pseudonyms inside the mix zone \rightarrow there's no easy way to determine which exit event corresponds to which enter event
- each possible mapping between exit and enter events is represented by a permutation π of $\{1, 2, \dots, k\}$:

$$m_\pi = (N_1 \sim X_{\pi[1]}, N_2 \sim X_{\pi[2]}, \dots, N_k \sim X_{\pi[k]})$$

where $\pi[i]$ is the i -th element of the permutation

- we want to determine $\Pr\{m_\pi \mid \underline{N}, \underline{X}\}$

Computing the level of privacy

$$\Pr\{m_\pi \mid \bar{N}, \bar{X}\} = \frac{\Pr\{m_\pi, \bar{X} \mid \bar{N}\}}{\Pr\{\bar{X} \mid \bar{N}\}}$$

$$\Pr\{m_\pi, \bar{X} \mid \bar{N}\} = \prod_{i=1}^k p_{n_i x_{\pi(i)}} d_{n_i x_{\pi(i)}}(t_{\pi(i)} - \tau_i) = q_\pi$$

$$\Pr\{\bar{X} \mid \bar{N}\} = \sum_{\pi'} \Pr\{m_{\pi'}, \bar{X} \mid \bar{N}\} = \sum_{\pi'} q_{\pi'}$$

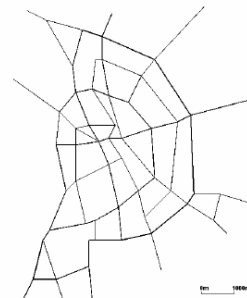
$$H(\bar{N}, \bar{X}) = - \sum_{\pi} \frac{q_\pi}{\sum_{\pi'} q_{\pi'}} \log \left(\frac{q_\pi}{\sum_{\pi'} q_{\pi'}} \right)$$

Another privacy metric

- tracking game:
 - the adversary picks a vehicle v in the observed zone
 - she tracks v until it enters the mix zone at port s
 - then, she observes the exiting events until time T (where the probability that v leaves the mix zone until T is close to one)
 - for each exiting vehicle at port j and time t , the adversary computes $q_{jt} = p_{sj}d_{sj}(t)$
 - the adversary decides to the exiting vehicle v' for which q_{jt} is maximal
 - this realizes a Bayesian decision (minimizes the error probability of the decision)
 - the adversary wins if $v' = v$
- the level of privacy achieved is characterized by the success probability of the adversary
 - if success probability is high, then level of privacy is low

Simulation settings

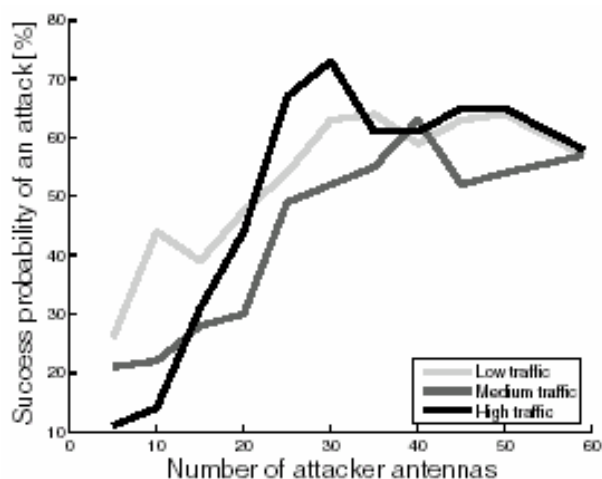
- we generated a simplified map of Budapest with MOVE
- we generated movement of the vehicles on the map with SUMO
 - low traffic: 250 new vehicles / time step
 - medium traffic: 500 new vehicles / time step
 - high traffic: 750 new vehicles / time step
- we selected the adversary's observation spots in intersections of roads
 - number of observation spots were varied from 5 to 59 with a step size of 5



Simulation settings

- we let the adversary build her model of the mix zone by letting her fully track vehicles for some time
- after that, we let the adversary pick a vehicle, track it until it enters the mix zone, observe exiting vehicles, and make a decision
- we run 100 simulations for each simulation setting
- we look at the percentage of the simulation runs where the adversary is successful

Simulation results



Summary

- changing pseudonyms has been proposed as a mechanism to provide location privacy in vehicular networks
- we studied the effectiveness of this approach
 - a model based on the concept of the mix zone
 - characterization of the adversary's tracking strategy
 - privacy metric
 - simulation results using realistic settings
- how about the frequency of the pseudonym change?