Risk analysis lab 2019. 12. 03. (Optimizing mean-reverting portfolios)

1. Load the supplied data (`s.csv`). The data is a TxN matrix containing daily closing prices for N asset and T days. Calculate the covariance matrix $\mathbf{G}$ for the given time series $\mathbf{s}(t)=\{s_1(t), s_2(t),..., s_N(t)\}$, $t=1,...,T$.

2. Assume that $\mathbf{s}(t)$ is subject to a first order vector autoregressive process – VAR(1) –, defined as follows: $\mathbf{s}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{W}(t)$, where $\mathbf{A}$ is a matrix of size NxN and $\mathbf{W}(t) \sim N(\mathbf{0}, \sigma\mathbf{I})$ are i.i.d. random variables for some $\sigma > 0$.

   Estimate $\mathbf{A}$ using least squares estimation techniques, as $\hat{\mathbf{A}}=\min_{\mathbf{A}}\left\|\sum_{t=2}^{T}\mathbf{s}(t)-\mathbf{A}\mathbf{s}(t-1)\right\|^2$ where $\|.\|^2$denotes the Euclidian norm. Solving the minimization problem above, by equating the partial derivatives to zero with respect to each element of the matrix $\mathbf{A}$, we obtain a system of linear equations. Solving that for $\mathbf{A}$ and switching back to vector notation for $\mathbf{s}$, we obtain

$$\hat{\mathbf{A}}=\left(\mathbf{s}^T(t-1)\mathbf{s}(t-1)\right)^+\left(\mathbf{s}^T(t-1)\mathbf{s}(t)\right)$$

   where M^+denotes the Moore-Penrose pseudoinverse of a matrix $\mathbf{M}$. Use only the last sample for the estimation. (Note that the Moore-Penrose pseudoinverse is preferred to regular matrix inversion, in order to avoid problems which may arise because of the potential singularity.)

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def est_A(s)
    return A
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3. The traditional way to identify the optimal mean-reverting portfolio is to find a portfolio vector subject to maximizing its predictability. One may note that

$$\mathbf{w}_{opt}=\max\lambda=\max_{x}\frac{\mathbf{w}^T\mathbf{A}\mathbf{G}\mathbf{A}^T\mathbf{w}}{\mathbf{w}^T\mathbf{G}\mathbf{w}}$$

   is equivalent to finding the eigenvector corresponding to the maximum eigenvalue in the following generalized eigenvalue problem:

$$\mathbf{A}\mathbf{G}\mathbf{A}^T\mathbf{w}=\lambda\mathbf{G}\mathbf{w}\,.$$

   Calculate the optimal mean reverting portfolio and print its λ parameter.