Risk analysis lab 2019. 10. 15. (CLT, Markov)

1. Let v_j denote the amount of deposit belongs to each client $j = 1,\dots,J$ with $p_j = P(X_j = v_j)$ and $X_j \in \{0; v_j\}$ probability of withdrawal. In a new notebook generate these vectors randomly as $v_j \sim U(1.5, 3)$ and $p_j \sim U(0.35, 0.55)$. $J = 75$

   (0.5 points)

2. Let $y_i \in \{0;1\}$ stands for the event when the *j*-th customer withdraws their deposit, while $\psi \in \{0,1\}$ denotes the event that the bank exceeds its cash *C*. As a function of C calculate $P\left(\sum_{j=1}^{J} X_j > C\right)$ using:

   a. the Central Limit Theorem
   $P\left(\sum_{j=1}^{J} X_j > C\right) \approx 1 - \Phi\left(\frac{C-m}{\sigma}\right)$, where Φ is the cumulative distribution function of Normal distribution, $m = \sum_{j=1}^{J} p_j v_j$ and $\sigma = \sqrt{\sum_{j=1}^{J} p_j v_j^2 (1-p_j)}$;

   (2 points)

   b. Markov's inequality
   $P\left(\sum_{j=1}^{J} X_j > C\right) \le \frac{m}{C}$, where $m = \sum_{j=1}^{J} p_j v_j$;

   (2 points)

3. Plot and compare the results on one figure for *C = 20, …, 200* .

   (0.5 points)