Risk analysis lab 2019. 10. 22. (Chernoff bounds)

1. Let v_j denote the amount of deposit belongs to each client $j = 1, \dots, J$ with $p_j = P\left(X_j = v_j\right)$ and $X_j \in \{0; v_j\}$ probability of withdrawal. In a new notebook generate these vectors randomly as $v_j \sim U(1,4)$ and $p_j \sim U(0.25, 0.45)$. $J = 65$
   (0.5 points)

2. Let $y_i \in \{0;1\}$ stands for the event when the *j*-th customer withdraws their deposit, while $\psi \in \{0,1\}$ denotes the event that the bank exceeds its cash *C*. As a function of C calculate $P\left(\sum_{j=1}^{J} X_j > C\right)$ using Chernoff's bound:

   $$P\left(\sum_{j=1}^{J} X_j > C\right) \le \min\left\{ e^{\left[\sum_{j=1}^{J} \mu_j\left(s_{opt}\right)\right] - s_{opt} C}; 1 \right\}$$, where

   $$s_{opt} : \arg\min_{s} \left(\left(\sum_{j=1}^{J} \mu_j(s) \right) - sC \right)$$ and

   $$\mu_j(s) = \log\left(E\left\{e^{sV_j}\right\}\right) = \log\left(e^{sv_j} p_j + 1 - p_j\right).$$

   (4 points)

3. Plot and compare the results on one figure for *C = 1, …, 200* .
   (0.5 points)