

## Risk analysis lab 9 2019. 11. 26. (Modern Portfolio Theory)

1. Load the supplied data from `s.csv`, containing daily returns for  $N$  asset and  $T$  days. Calculate its mean vector as  $\mathbf{m} = \{E(s_1), E(s_2), \dots, E(s_N)\}$ , and its covariance matrix  $\mathbf{K}$ .
2. Generate a random portfolio vector  $\mathbf{w} = \{w_1, w_2, \dots, w_N\}$ , where  $\|\mathbf{w}\|_1 = \sum_{i=1}^N |w_i| = 1$  and  $-1 < w_i < 1$  (short selling is allowed).

If we define  $x(t) = \sum_{i=1}^N w_i s_i(t)$ , then  $x \sim N(\mu, \sigma)$  (CLT), where  $\mu = \mathbf{w}^T \mathbf{m}$  and  $\sigma^2 = \mathbf{w}^T \mathbf{K} \mathbf{w}$ .

Display the expected daily return and the variance of the given portfolio.

3. Determine the optimal portfolio to minimize the risk:

$$\mathbf{w}_{opt} = \min_{\mathbf{w}} \frac{\mathbf{w}^T \mathbf{K} \mathbf{w}}{\mathbf{w}^T \mathbf{w}}$$

The optimal portfolio can be calculated as that eigenvector of matrix  $\mathbf{K}$  which belongs to the smallest non-zero ( $>10^{-6}$ ) eigenvalue. Don't forget to normalize this portfolio as well:

$$\|\mathbf{w}\|_1 = \sum_{i=1}^N |w_i| = 1.$$

Display the expected daily return and the variance of the given portfolio.