Morgan Stanley

Financial Derivatives and Risk

Norbert Fogarasi
Managing Director, Morgan Stanley

Financial Information Systems
BME-VIK

Session Outline

- Financial Risks - Summary
- Derivatives Market
- Forward and Futures Contracts
- Options
- Option Pricing
- Option Risk

Financial Risks - Summary

- Market Risk
  - Change in value due to change in market variables.
  - Mathematical derivative
- Credit Risk
- Operational Risk
- Liquidity Risk

Derivatives - Overview

- Derivatives:
  - Financial instruments whose value depends on (derived from) other, underlying variables.
  - Underlying variables in most cases are prices of other financial product (**underlying instruments**).
- Huge market
  - Estimated volume over $1.2 quadrillion ($10^{15}$)
  - 10 times the total world GDP!

NYSE Trading Floor; late '80s
(http://www.nyse.com/images/about/NYSETradingFloorCrowds.jpg)

Types of Derivatives

- Trading methods
  - Over-The-Counter (OTC)
  - Exchange-Traded (ETD)

- Underlying products
  - Commodities (e.g. oil, gold)
  - Stocks
  - Bonds
  - Credits
  - Equity indices
  - etc.

New York Stock Exchange; NYSE
(http://wallstreetbusinessfunding.com/images/nyse.jpg)

Basic Contract Types

- Overview of the following types
  - Forwards and Futures
  - Simple Options
  - Complex Options

NYSE Today
(http://www.borev.net/nyse.jpg)

Forward Contracts – I.

- *Forward contracts* are contracts of sales, in which two parties agree on trading (buying or selling) an asset at a future time at a certain price. These contracts also include other regulations, such as quality and quantity of the asset, rules of delivery, location, rights and liabilities, etc.

- Short position – the party who sells the underlying asset

- Long position – the party who buys the underlying asset

- Common contract details
  - type of the underlying asset
  - asset quality
  - asset quantity
  - delivery date (maturity date)
  - delivery price (forward price)
  - delivery method
  - other details…

- No cash or asset of any kind actually changes hands until the maturity date, except for collateralized transactions, where margins may apply according to an agreed schedule

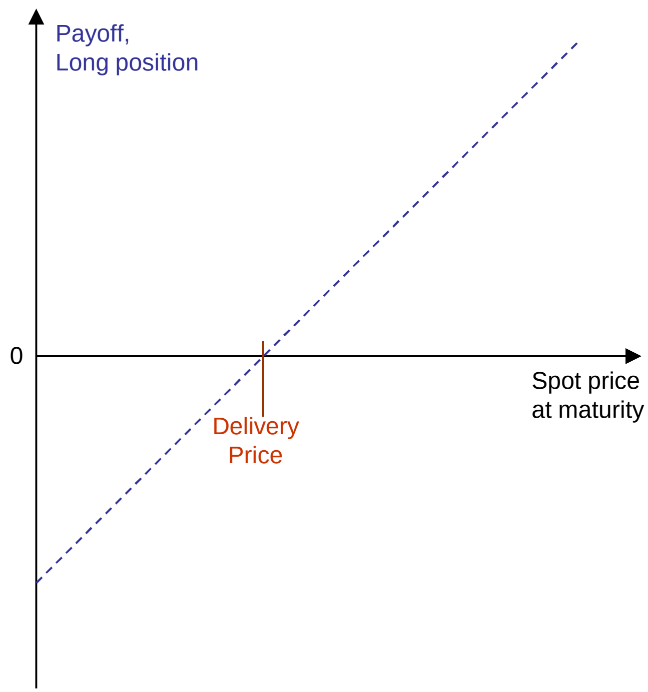
Forward Contracts – II.

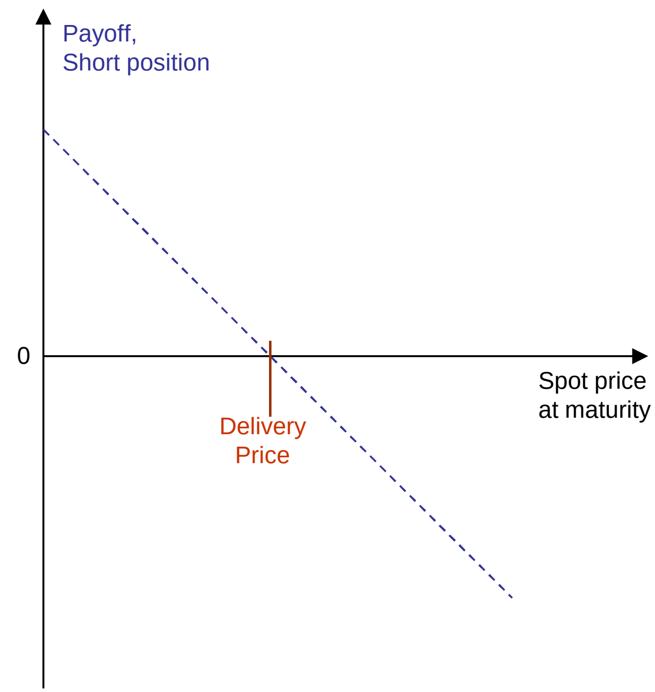
- Companies often use derivatives (eg. forward contracts) for hedging market risks

- FX Forward Contract example:
  - Hungarian company X will have to pay GBP 1M in 6 months time
  - Holds all of its assets in HUF, no excess assets available today to convert
  - Now has significant RISK / exposure to the HUF/GBP FX rate
  - Company X can enter into an FX forward agreement with a financial institution
  - Forward rate (price) is 375 HUF/GBP for a notional of GBP 1M. Fixed 375M HUF cashflow in 6m.
  - Current (spot) FX may be 370 HUF/GBP, forward rate is computed so that there is no cost/fee upfront.
  - Can be profitable to company X as it “Hedges the FX risk”:
    - If in 6 months time the rate is 400 HUF/GBP, company X will have paid only 375M instead of 400M, gain of 25M
    - If in 6 months time the rate is 350 HUF/GBP, company X will have paid 375M instead of 350M, loss of 25M
  - Financial institution may apply a margin rule: if loss is over 10M then margin payments may be asked for from company X ahead of the maturity date

Forward Contracts – III.

- Forward premium – difference between the *spot* and the *forward* price
- Payoff functions at maturity for both positions:





- Cash settlement: no physical delivery, only the forward premium is paid

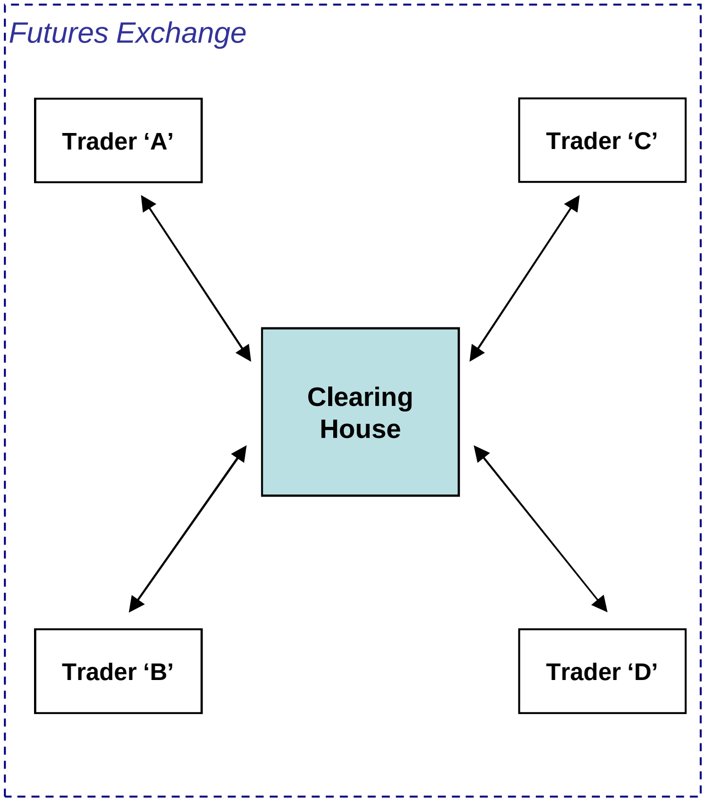
Futures Contracts

- Questions about forward contracts:
  - Finding the counterparty
  - Problems with warranties
  - etc.

- *Futures exchanges* are specific financial exchanges created for trading *futures contracts*. In order to regulate trading activities, exchanges determine contract details and provide a standardized structure for each contract type. Summing up the key points, prescriptions usually defined for the followings:
  - Types of assets
  - Asset quality (different categories, substitutions, etc)
  - Traded quantities (sets/packages)
  - Maturity dates
  - Delivery methods (cash settle or physical delivery)
  - Other delivery details (how, when, where, etc.)
  - Method to determine forward prices
  - Other payment details, etc

Futures Exchange

- The Clearing House – act on almost all futures exchanges as centralized counterparties for each trade
  - *Novation* – Clearing Houses present themselves as counterparties to every trade on the exchange, and guarantee that trades will be settled as defined in the contracts
- Margins
  - *Initial margins*: margin paid by exchange players (trading firms) to cover possible future losses
  - *Mark-To-Market margins* (often referred as M2M-margins): difference between costs of positions held by a trading firm and actual market value of these positions
  - Margin account and maintenance margin level
  - Daily Cash Settlement
- Futures contracts are not issued but are created as *open interests* increase ("somebody wants to go long or short")



Futures Exchange Example

Types of Traders

- *Speculators*
  - use derivatives to bet on future market movements
- *Arbitrageurs*
  - take offsetting positions to lock in a profit (look for arbitrages)
- *Hedgers*
  - use derivatives to reduce the risk that they know they will face in the future (see a forward contract example below)

Pricing Forwards and Futures – I.

- Consider a portfolio of:
  - different assets
  - forward contracts in which these assets are the underlying instruments
- Make the value of this portfolio independent of the changes in the asset's price

- The value of this portfolio is known at maturity
  - N: time to maturity (e.g. number of years)
  - A_0: spot price of the asset 'today'
  - A_N: spot price of the asset at maturity
  - F_0: forward price 'today'
  - F_N: forward price at maturity

- *Convergence of prices*: $F_N = A_N$

- The yield of the portfolio is equal to the risk-free interest rate (*r*), which can be used to calculate the forward price:

$$\frac{F_0}{A_0} = (1+r)^N \Rightarrow F_0 = A_0 (1+r)^N$$

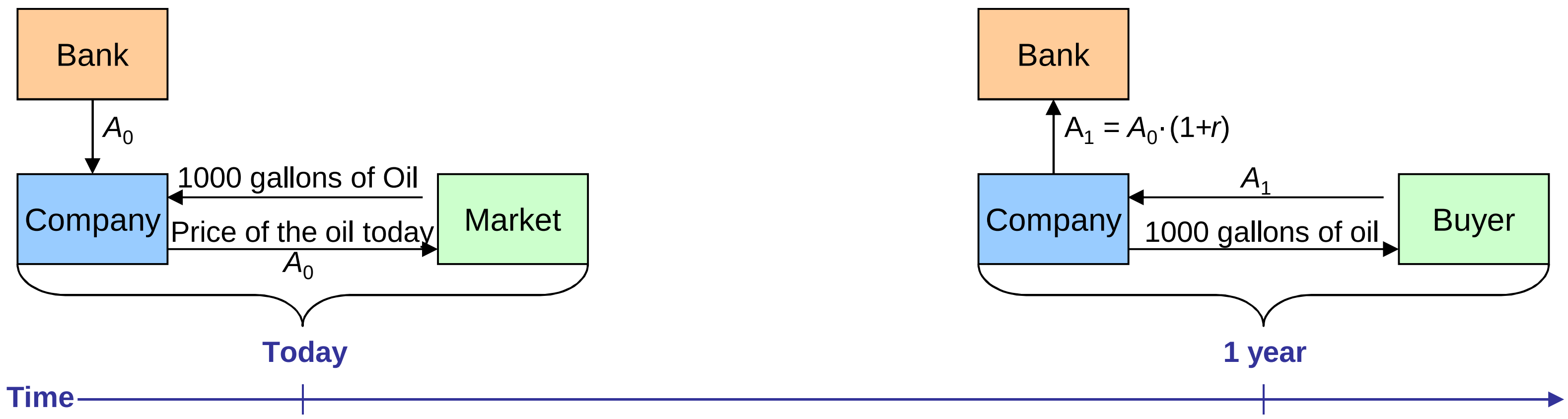
- Potential dividends and storage costs would make the pricing more complicated.

Pricing Forwards and Futures – II.

- Differences between forward and futures trades (mainly)
  - standardization
  - daily cash settlement

- This makes futures pricing a bit more complicated

- However, it can be shown that forward and futures price of an asset are usually very close if the maturity date of the contracts are the same



Options – Definitions I.

- *Option* agreements give the right to one of the parties to buy or sell an asset, while the other is obligated to take the opposite position
- Traded both over-the-counter markets and exchanges
- Call – Put options
  - *call options* give the holder the right to buy
  - *put options* give the holder the right to sell
- Positions
  - the party who has the right to buy or sell the asset takes the *long position*
  - the other party who is obligated to sell or buy the asset takes the *short position*

put option

- Long Put (LP) – right to sell the asset
- Short Put (SP) – obligated to buy the asset

call option

- Long Call (LC) – right to buy the asset
- Short Call (SC) – obligated to sell the asset

Option types

Options – Definitions II.

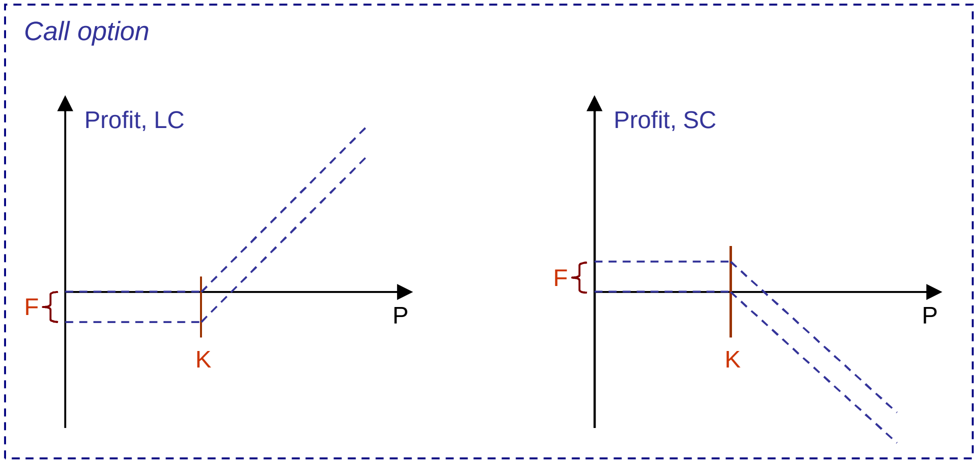
- Fees
  - – The party who buys an option (takes the long position) has to pay a *fee* for the right to *exercise* the option in the future.

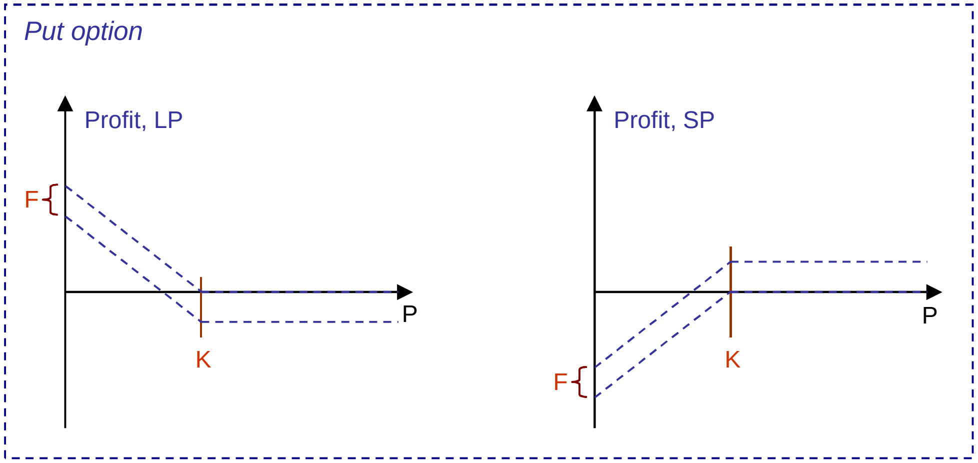
- Ways to end an option contract
  - – *Exercise:* the buyer of the option exercises it
  - – *Expire*: after the expiration date, the contract is terminated
  - – *Buy/sell* the contract on the market

- Option Types
  - – Vanilla options
    - *American options*: can be exercised any time up to the expiration (or maturity) date
    - *European options*: can be exercised on the expiration date only
  - – Exotic (or non-vanilla) options
    - All other contracts for which the payoff is calculated differently; e.g. Bermudan options, in which the holder has the right to exercise the option at specific, previously agreed times, path dependent options with knock-in/knock-out features
- Terminology
  - – Similar to other derivatives (strike price, maturity date, etc.)

Profit at Maturity

- Calculating the profit of an option contract at maturity
- Based on the *strike price*, *spot price* and the *future value* of the *fee* paid on the trade date





Options - Example

- Companies often use derivatives (eg. forward contracts) for hedging market risks

- Revisit Company X example from earlier:
  - Hungarian company X will have to pay GBP 1M in 6 months time
  - Holds all of its assets in HUF, no excess assets available today to convert
  - Now has significant RISK / exposure to the HUF/GBP FX rate
  - Company X can enter into a long FX call option position with a financial institution
  - Strike price is 375 HUF/GBP for notional of GBP 1M.
  - Current (spot) FX may be 370 HUF/GBP, there is a fee/price to pay for this option upfront!!
  - Can be profitable to company X as it "Hedges the FX risk":
    - If in 6 months time the rate is 400 HUF/GBP, company X will exercise the option and pay only 375M instead of the market rate of 400M for gain of 25M
    - If in 6 months time the rate is 350 HUF/GBP, company X will NOT have the loss of 25M, as it will just let the option expire and pay 350M HUF
  - Company X has to pay the option price, but is guaranteed to pay maximum of 375M, may be less if the market moves favorably!

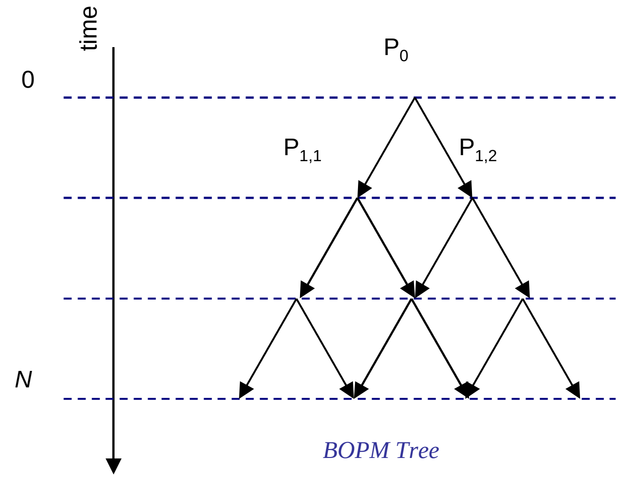
Pricing an Option Contract Before Maturity

- Concerns
  - how to calculate expected cash flows
  - how to calculate changes in the risk of the option
  - …

- Mathematical Models
  - Binomial Options Pricing Model (BOPM)
  - Black–Scholes Options Pricing Model (BS)

Pricing Options Using the Binomial Model

- Assumptions
  - price of the underlying asset changes according to the binomial distribution
  - discrete time scale is used, and the price of the underlying asset may have two different values for each point (so it is assumed that the price move up or down by a specific factor at each step)
  - *Risk neutrality assumption** for the entire lifetime of the option
    - determine the probability of each node and use this probability to calculate the prices
- Build a binomial tree of the possible prices
- Nodes represent possible prices of the underlying asset at a specified times
- Algorithm
  - starting at final nodes, determine the prices at maturity based on possible spot prices of the asset
  - backward calculation of prices for internal nodes
  - Calculate P0, the price of the option

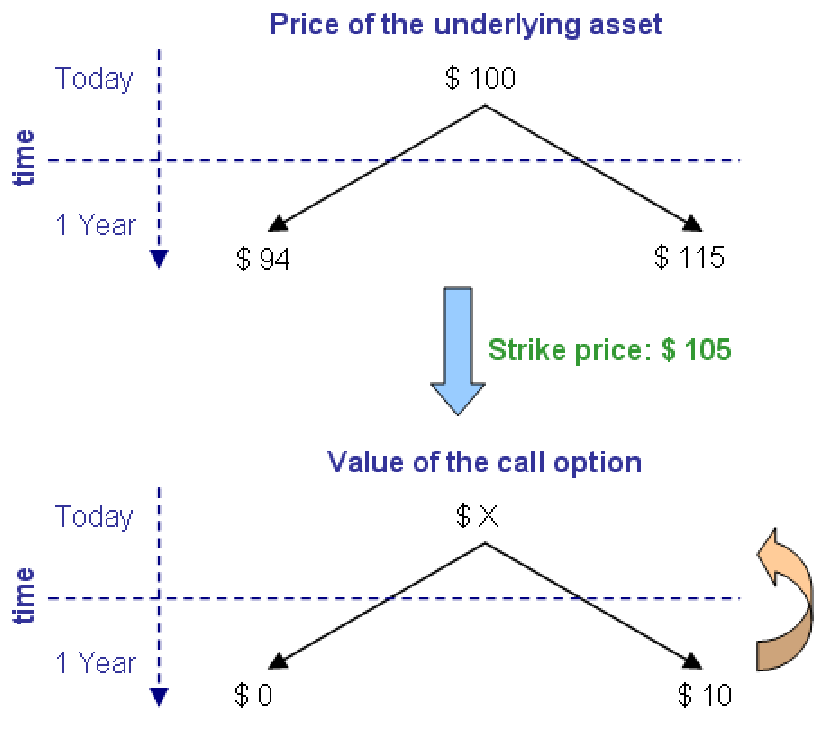


BOPM Tree

**In general, risk neutrality assumption means that when we price an asset, we calculate the probabilities of the future cash flows first, and discount them at the risk free rate*

BOPM Example – I.

- Construct a portfolio
  - contains an asset and a short call option on the same asset
  - the value of this portfolio at maturity is independent of the changes in the asset's price



BOPM Example

- *A*=$100
  - Case 1: price moves up to $115; value of the portfolio at maturity = 115y - 10
  - Case 2: price moves down to $94; portfolio's value = 94y

- To create the portfolio, we'd need

$$115y\text{-}10 = 94y \Rightarrow y = \frac{10}{21}$$

BOPM Example – II.

- Value of the portfolio at maturity:

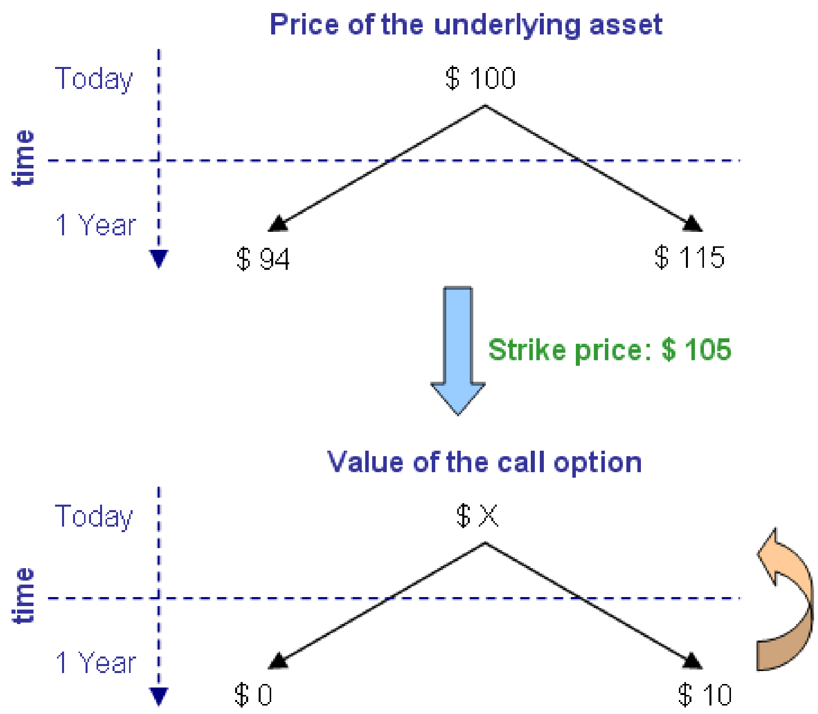
$$P = 115 \times \frac{10}{21} - 10 = 94 \times \frac{10}{21} \approx 44.7$$

- The yield of the portfolio is equal to the risk free rate, e.g. *r* = 4%.

$$\left(1 + \frac{r}{100}\right) \times (y \times A - X) = P$$

$$1.04 \times \left(\frac{1000}{21} - X\right) = 44.7$$

$$X = \$\, 0.9025$$



BOPM Example

Black-Scholes Option Pricing Model (BS) - I

- Continuous time models for the underlying price (continuous-time stochastic processes, eg. geometric Brownian motion)
- 1973: Black, Scholes and Merton showed that under the below assumptions there is a closed-form formula for the price of European call and put options. They were awarded a Nobel Prize for this finding in 1997.
  - Underlying instrument is non-dividend paying. No transaction costs or taxes. Instrument is perfectly divisible and short-sellable in continuous trading.
  - The risk free interest rate is constant and the same for all maturities
  - The underlying process follows a geometric Brownian motion with constant drift and volatility
  - There are no riskless arbitrage opportunities (efficient markets)

Black-Scholes Option Pricing Model (BS) - II

$$C = SN(d_1) - N(d_2)Ke^{-rt}$$

C = Call premium
S = Current stock price
t = Time until option exercise
K = Option striking price
r = Risk-free interest rate
N = Cumulative standard normal distribution
e = Exponential term

s = St. Deviation
ln = Natural Log

$$d_1 = \frac{\ln\left(S/K\right) + \left(r + s^2/2\right)t}{s \cdot \sqrt{t}}$$

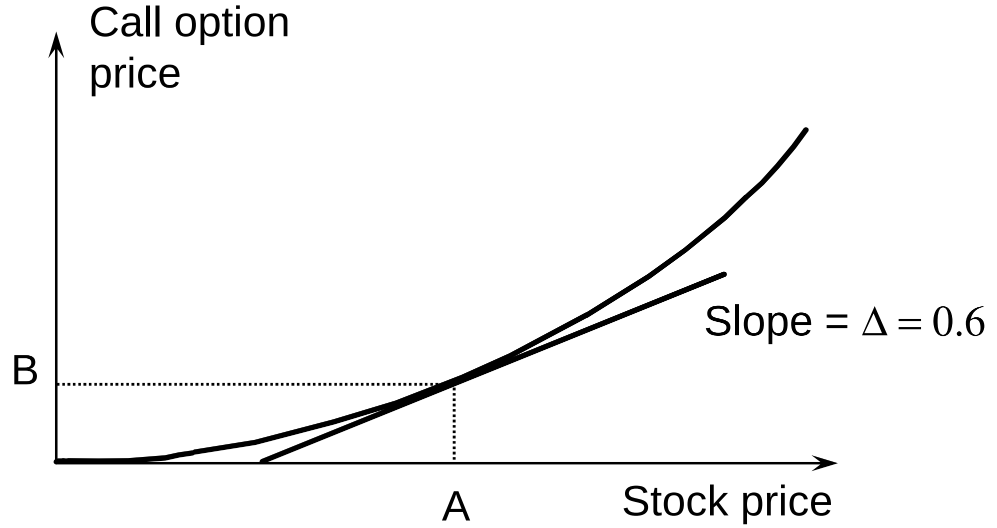
$$d_2 = d_1 - s \cdot \sqrt{t}$$

Greek Letters

- Greek letters are the partial derivatives with respect to the model parameters that are liable to change
- Usually traders use the Black-Scholes-Merton model when calculating partial derivatives
- The volatility parameter in BSM is set equal to the implied volatility when Greek letters are calculated. This is referred to as using the "practitioner Black-Scholes" model

Delta

- Delta (Δ) is the rate of change of the option price with respect to the underlying asset price



Hedge

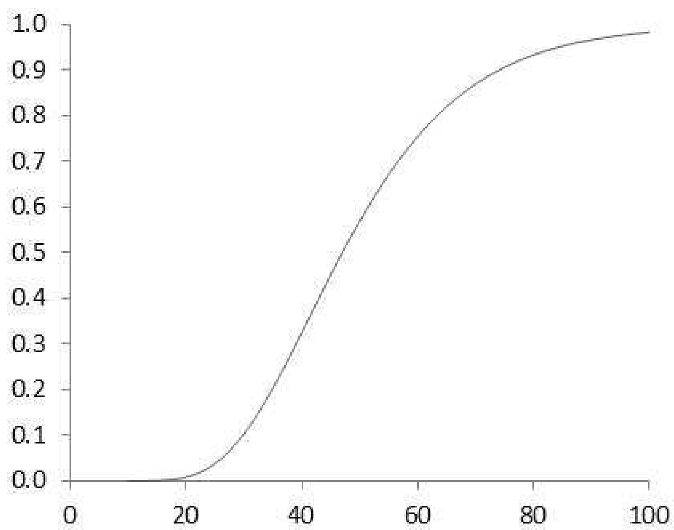
- Trader would be hedged with the position:
  - short 1000 options
  - buy 600 shares
- Gain/loss on the option position is offset by loss/gain on stock position
- Delta changes as stock price changes and time passes
- Hedge position must therefore be rebalanced

Delta Hedging

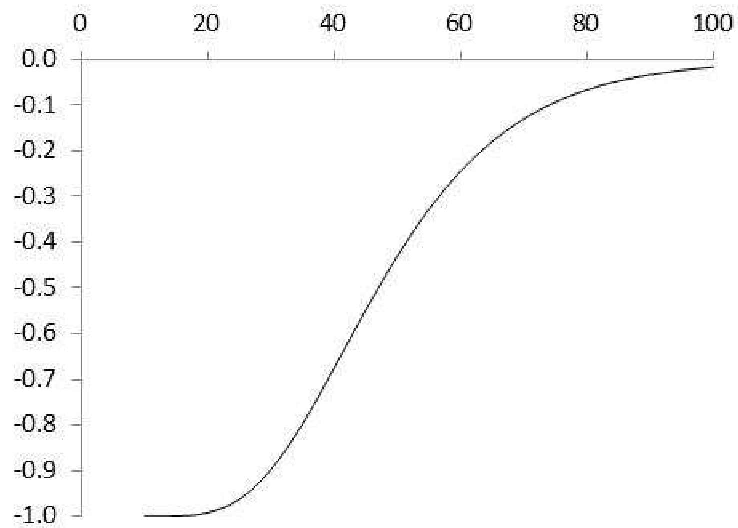
- This involves maintaining a delta neutral portfolio
- The delta of a European call on a non-dividend paying stock is $N(d_1)$
- The delta of a European put on the stock is

$$N(d_1) - 1$$

Delta of a Stock Option (K=50, r=0, σ = 25%, T=2)



Call

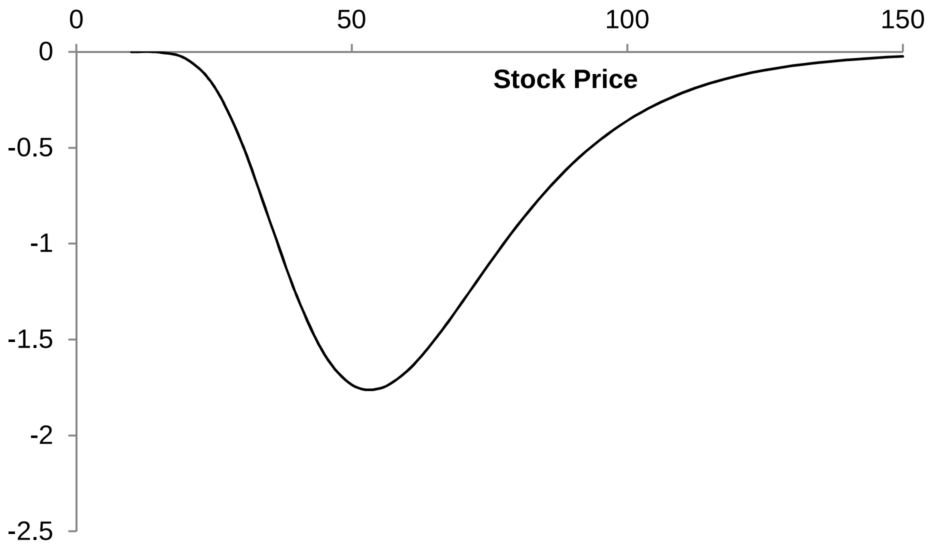


Put

Theta

- Theta (Θ) of a derivative (or portfolio of derivatives) is the rate of change of the value with respect to the passage of time
- The theta of a call or put is usually negative. This means that, if time passes with the price of the underlying asset and its volatility remaining the same, the value of a long call or put option declines

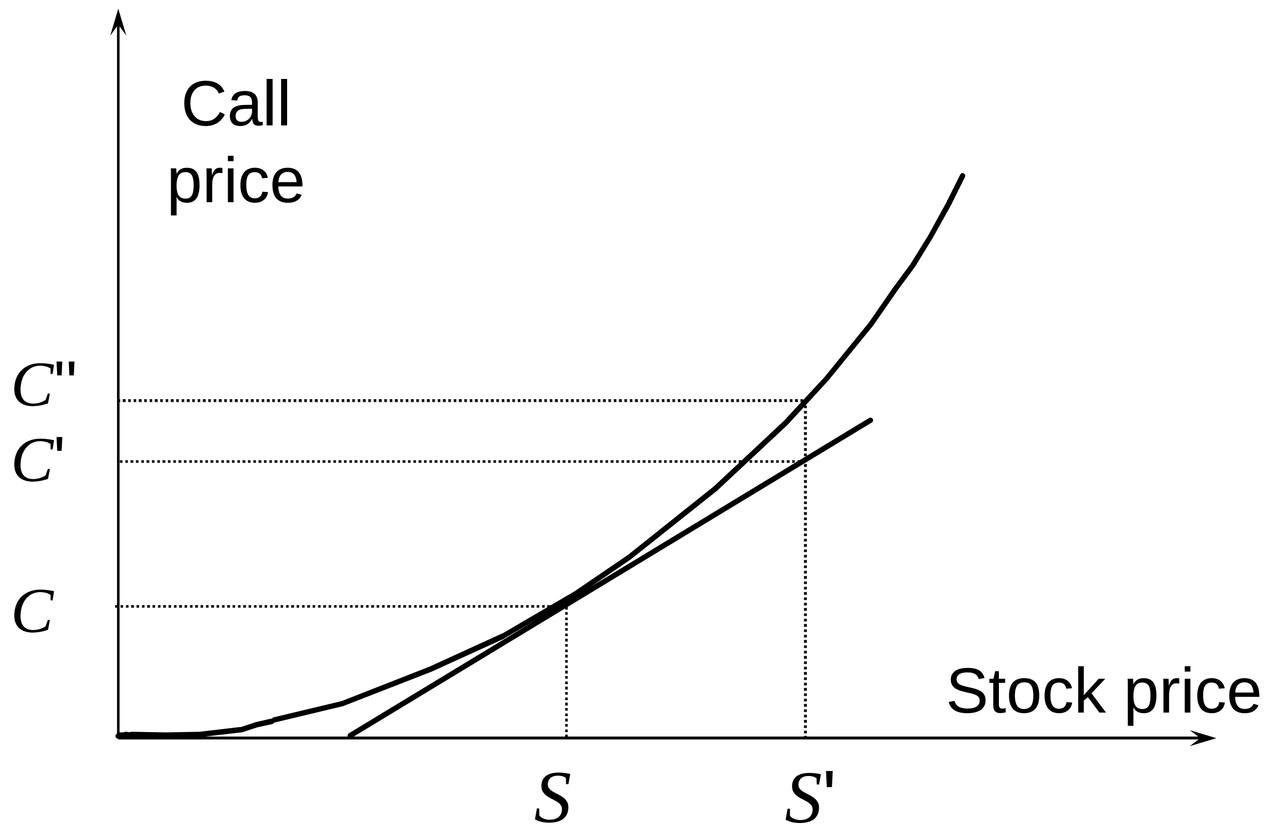
Theta for Call Option (K=50, σ = 25%, r = 0, T = 2)



Gamma

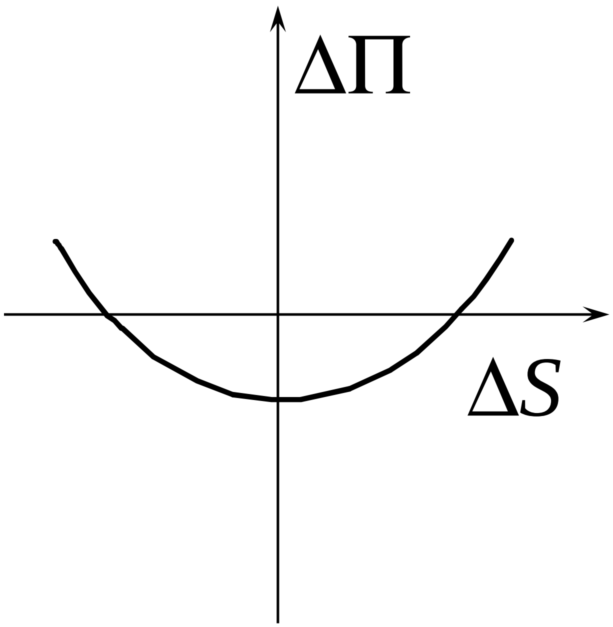
- Gamma (Γ) is the rate of change of delta (Δ) with respect to the price of the underlying asset
- Gamma is greatest for options that are close to the money

Gamma Addresses Delta Hedging Errors Caused By Curvature

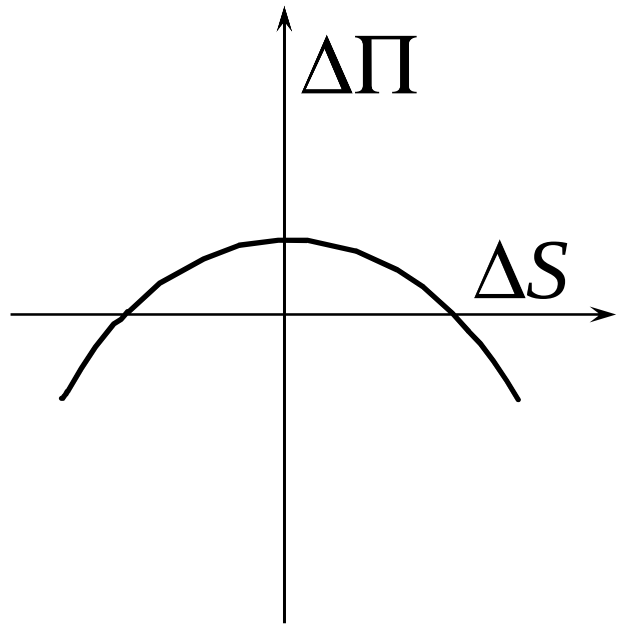


Interpretation of Gamma

For a delta neutral portfolio, $\Delta\Pi \approx \Theta\,\Delta t + \frac{1}{2}\Gamma\Delta S^2$

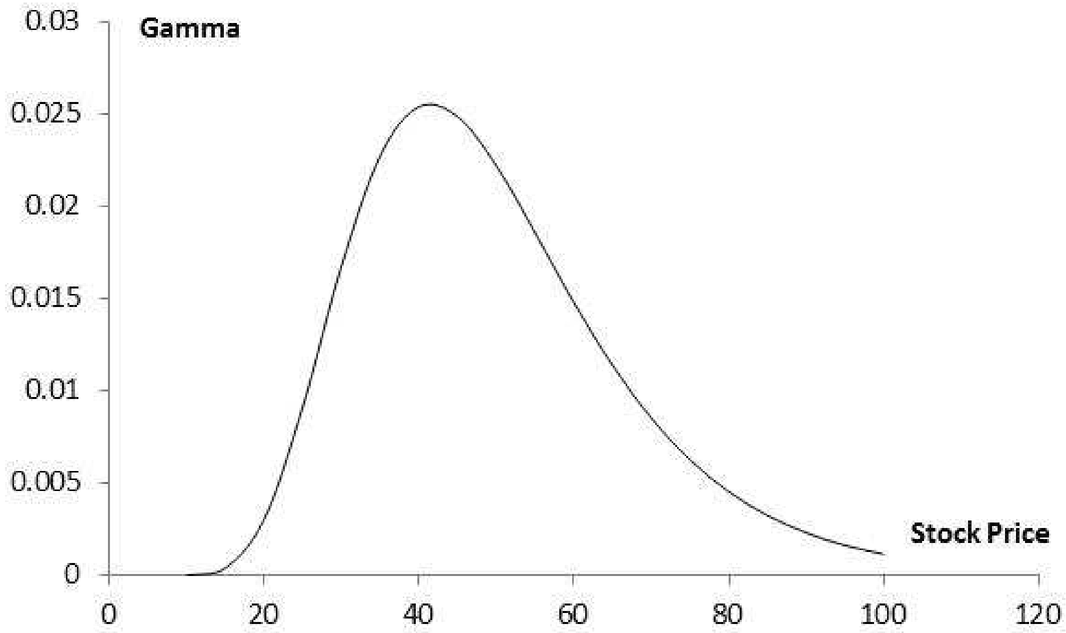


Positive Gamma



Negative Gamma

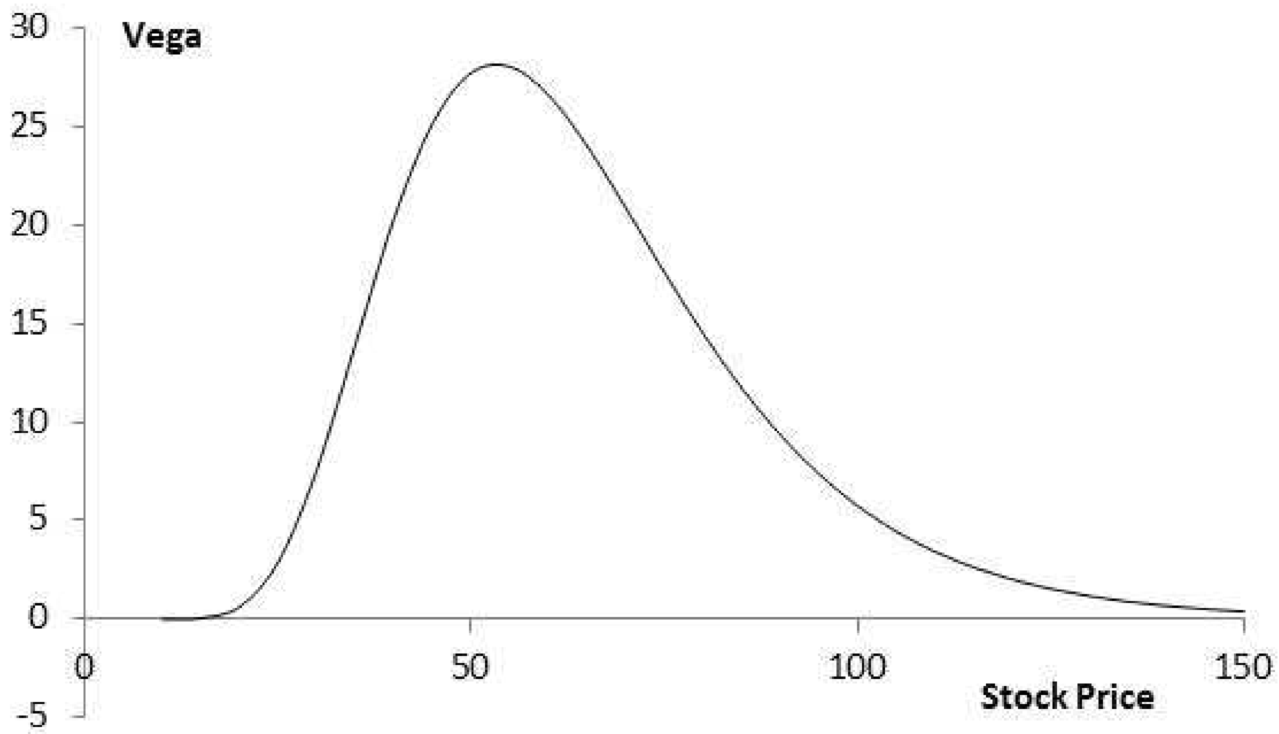
Gamma for Call or Put Option: (K=50, σ = 25%, r = 0%, T = 2)



Vega

- Vega (ν) is the rate of change of the value of a derivatives portfolio with respect to volatility
- If vega is calculated for a portfolio as a weighted average of the vegas for the individual transactions comprising the portfolio, the result shows the effect of all implied volatilities changing by the same small amount

Vega for Call or Put Option (*K*=50, σ = 25%, *r* = 0, *T* = 2)



Taylor Series Expansion

- The value of a portfolio of derivatives dependent on an asset is a function of the asset price *S*, its volatility σ, and time *t*

$$\Delta\Pi = \frac{\partial \Pi}{\partial S}\Delta S + \frac{\partial \Pi}{\partial \sigma}\Delta\sigma + \frac{\partial \Pi}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 \Pi}{\partial S^2}(\Delta S)^2 + \text{K}$$

$$= \text{Delta}\times\Delta S + \text{Vega}\times\Delta\sigma + \text{Theta}\times\Delta t + \frac{1}{2}\text{Gamma}\times(\Delta S)^2 + \text{K}$$

Managing Delta, Gamma, & Vega

- Delta can be changed by taking a position in the underlying asset
- To adjust gamma and vega it is necessary to take a position in an option or other derivative