Risk analysis lab 2018. 11. 20. (Optimizing mean-reverting portfolios)

1. Load the supplied data (s.csv). The data is a $T \times N$ matrix containing daily closing prices for $N$ asset and $T$ days. Calculate the covariance matrix $G$ for the given time series $s(t) = \{s_1(t), s_2(t), ..., s_N(t)\}$, $t = 1, ..., T$.

2. Assume that $s(t)$ is subject to a first order vector autoregressive process – VAR(1) –, defined as follows: $s(t) = As(t) + W(t)$, where $A$ is a matrix of size $N \times N$ and $W(t) \sim N(0, \sigma I)$ are i.i.d. random variables for some $\sigma > 0$.

Estimate $A$ using least squares estimation techniques, as $\hat{A} = \min_A \left| \sum_{t=2}^{T} s(t) - As(t - 1) \right|^2$ where $\| \cdot \|^2$ denotes the Euclidian norm. Solving the minimization problem above, by equating the partial derivatives to zero with respect to each element of the matrix $A$, we obtain a system of linear equations. Solving that for $A$ and switching back to vector notation for $s$, we obtain

$$\hat{A} = \left( s^T (t-1) s(t-1) \right)^{-1} \left( s^T (t-1) s(t) \right)$$

where $M^+$ denotes the Moore-Penrose pseudoinverse of a matrix $M$. (Note that the Moore-Penrose pseudoinverse is preferred to regular matrix inversion, in order to avoid problems which may arise because of the potential singularity.)

function $A = \text{est}_A(s)$

3. The traditional way to identify the optimal mean-reverting portfolio is to find a portfolio vector subject to maximizing its predictability. One may note that

$$w_{opt} = \max \lambda = \max_s \frac{w^T A G A^T w}{w^T G w}$$

is equivalent to finding the eigenvector corresponding to the maximum eigenvalue in the following generalized eigenvalue problem:

$$AGA^T w = \lambda G w$$

Calculate the optimal mean reverting portfolio and print its $\lambda$ parameter.