1. Let $v_i$ denote the amount of deposit belongs to each client $j = 1, \ldots, J$ with $p_j = P(X_j = v_j)$ and $X_j \in \{0; v_j\}$ probability of withdrawal. In a new script generate these vectors randomly as $v_j \sim U(1,4)$ and $p_j \sim U(0.2,0.4)$. $J = 60$

2. Let $y_j \in \{0;1\}$ stands for the event when the $j$-th customer withdraws their deposit, while $\psi \in \{0,1\}$ denotes the event that the bank exceeds its cash $C$.

As a function of $C$ calculate $P \left( \sum_{j=1}^{J} X_j > C \right)$ using:

a. the Central Limit Theorem

\[
P \left( \sum_{j=1}^{J} X_j > C \right) \approx 1 - \Phi \left( \frac{C - \mu}{\sigma} \right), \quad \text{where} \quad \Phi \quad \text{is the cumulative distribution function of Normal distribution,} \quad \mu = \sum_{j=1}^{J} p_j v_j \quad \text{and} \quad \sigma = \sqrt{\sum_{j=1}^{J} p_j v_j^2 (1 - p_j)};
\]

b. Markov’s inequality

\[
P \left( \sum_{j=1}^{J} X_j > C \right) \leq \frac{m}{C}, \quad \text{where} \quad m = \sum_{j=1}^{J} p_j v_j;
\]

c. Chernoff’s bound

\[
P \left( \sum_{j=1}^{J} X_j > C \right) \leq \min \left\{ e^{-s \mu_J(s) - s C} ; 1 \right\}, \quad \text{where} \quad s_{opt} : \inf_{s} \left( \sum_{j=1}^{J} \mu_j(s) - s C \right) \quad \text{and} \quad \mu_j(s) = \log \left( E \left\{ e^{s v_j} \right\} \right) = \log \left( e^{s p_j} p_j + 1 - p_j \right).
\]

3. Plot and compare the results on one figure for $C = 1, \ldots, 200$.

(5 points)