Risk analysis lab 2016. 10. 15. (Chernoff bounds part 3)

1. A bank has two types of clients. In the first group clients withdraw $6 with probability 0.3. Clients in the other group withdraw $2.5 with probability 0.55. The bank has limited amount of cash, $100. Calculate the number of users the bank is able to serve. For each user combination $0 \leq n_1, n_2 \leq 50$ calculate the probability of bankruptcy. If the probability is less then $e^{-\gamma}$, the bank is able to serve:

$$P \left( \sum_{j=1}^{J} X_j > C \right) \leq e^{-\gamma}$$

where $\gamma = 3$ is the risk parameter.

For $0 \leq n_1, n_2 \leq 50$ estimate the probability of bankruptcy:

- First use the standard Chernoff bound:

  $$n_1 \mu_1 (s_{opt}) + n_2 \mu_2 (s_{opt}) \leq s_{opt} C - \gamma,$$

  where

  $$s_{opt} : \min \left\{ \sum_{j=1}^{J} n_j \mu_j (s) \right\} - sC,$$

  $$\mu_j (s) = \log \left( E \left\{ e^{s V_j} \right\} \right) = \log \left( e^{s v_j} p_j + 1 - p_j \right), n_1 \text{ and } n_2 \text{ are the number of clients in the groups and } J \text{ is the number of user groups.}$$

- Then, use universal $s$:

  $$s_{opt} : \max \left\{ \left( sC - \gamma \right)^{1/j} \right\} \prod_{j=1}^{J} \mu_j (s),$$

  $$n_1 \mu_1 (s_{opt}) + n_2 \mu_2 (s_{opt}) \leq s_{opt} C - \gamma \text{ and}$$

  $$\mu_j (s) = \log \left( E \left\{ e^{s V_j} \right\} \right) = \log \left( e^{s v_j} p_j + 1 - p_j \right).$$

For more help check: Risk analysis and management by Large Deviation Theory

(3 points)