Risk analysis lab 8 2017. 10. 30. (Portfolio risk)

1. Load the supplied data from the binary (s.mat), the variable s will be a TxN matrix containing daily returns for N asset and T days. Calculate its mean vector as \( \mathbf{m} = \{E(s_1), E(s_2), ..., E(s_N)\} \), and its covariance matrix \( \mathbf{K} \).

2. Generate a random portfolio vector \( \mathbf{w} = \{w_1, w_2, ..., w_N\} \), where \( \|\mathbf{w}\| = \sum_{i=1}^{N} |w_i| = 1 \) and \(-1 < w_i < 1\) (short selling is allowed).
   If we define \( x(t) = \sum_{i=1}^{N} w_i s_i(t) \), then \( x \sim N(\mu, \sigma) \) (CLT), where \( \mu = \mathbf{w}^T \mathbf{m} \) and \( \sigma^2 = \mathbf{w}^T \mathbf{K} \mathbf{w} \).

   Calculate \( u \) to fulfill \( P(x < u) = 0.01 \).

   Display
   - the expected daily return
   - and the minimal daily return (with 1% uncertainty)

   of the given portfolio.

3. Determine the optimal portfolio to minimize the risk:

   \[
   \mathbf{w}_{\text{opt}} = \min_{\mathbf{w}} \frac{\mathbf{w}^T \mathbf{K} \mathbf{w}}{\mathbf{w}^T \mathbf{w}}
   \]

   The optimal portfolio can be calculated as that eigenvector of matrix \( \mathbf{K} \) which belongs to the smallest non-zero (>\( 10^{-6} \)) eigenvalue. Print the expected daily return and the minimal daily return (with 1% uncertainty) of the optimal portfolio.

4. It is known, that a single neuron model can be used to identify the largest or the smallest eigenvector of the correlation matrix of the observed vectors. Calculate the largest eigenvector with the following iterative weight update rule:

   \[
   \mathbf{w}(k + 1) = \mathbf{w}(k) + \eta y(k) (\mathbf{s}(k) - y(k) \mathbf{w}(k))
   \]

   where \( y(k) = \mathbf{w}^T(k) \mathbf{s}(k) \).

   Display
   - the expected daily return
   - and the minimal daily return (with 1% uncertainty)

   of the given portfolio.