# Solution for A Retrial Queueing Problem in Cellular Networks with the Fractional Guard Channel Policy

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**Abstract**

In this paper, we introduce a retrial queueing problem for wireless cellular networks applying the Fractional Guard Channel (FGC) admission control policy and propose a new algorithm to solve the retrial queue problem involving the FGC policy.

Comparison between analytical and simulation results shows that our algorithm is accurate and fast to evaluate the performance of the system.

## 1 Introduction

In wireless networks it is of paramount importance to handle handover calls in an appropriate way. The limited number of channels in a specific cell and the competition between calls may cause great annoyances for traveling subscribers because of the ongoing call disruption problem. The guard channel policy [8] is the well-known technique to cope with the problem by giving a priority for handover calls over fresh calls. The Fractional Guard Channel (FGC) concept has recently been introduced by Ramjee et al [13]. They showed that FGC can be used to optimally control call admission policy in cellular networks. Furthermore, the Guard Channel policy is a special case of FGC. The efficient recursive algorithm to evaluate the performance of the Fractional Guard Channel Policy was presented in [16].

However the retrial phenomenon of calls in wireless networks with FGC was not discussed in the literature until now. In this paper we deal with a new retrial queueing model for cellular networks with the Fractional Guard Channel policy. The

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*Preprint submitted to Elsevier* *29 August 2010*

consideration of the retrial phenomenon allows the investigation of important performance measures related to the quality of services experienced by subscribers such as the average number of calls in the orbit, the probability of fresh and handover calls leaving the system due to the control policy, and the probability that fresh and handover calls are forced to the orbit. Furthermore, we propose a new efficient algorithm to solve the retrial queue problem involving the FGC policy.

The rest of the paper is organized as follows. In Section 2 we provide the description of the retrial queueing problem. In Section 3 an efficient computational procedure is proposed. In Section 4 some representative numerical results are presented. Finally, Section 5 concludes the paper.

## 2 System description

We consider a particular cell in a cellular mobile system with infinite user population, where there are $c$ channels to serve incoming calls. The interarrival times of new and handover calls are exponentially distributed with rate $\lambda_F$ and $\lambda_H$, respectively. Call durations (of new and handover calls) in the cell follow an exponential distribution with mean $1/\mu$. A blocked call due to the lack of capacity or the resource allocation policy (e.g., the guard channel concept) will retry with probability $\theta$ and rate $\alpha$. Note that $\theta$ is used to represent the degree of impatience of users. Throughout this paper, the orbit is defined as a collection of blocked handover and fresh calls which will repeat a request for service (see Figure 1). Let $I(t)$ $(0 \leq I(t) \leq c)$ denote the number of occupied channels at time $t$.

The Fractional Guard channel [13] policy is defined as follows. When $I(t) = i$ $(0 \leq i < c)$, a new call or retried call is accepted with probability $\beta_i$ and handover calls are accepted with probability 1.

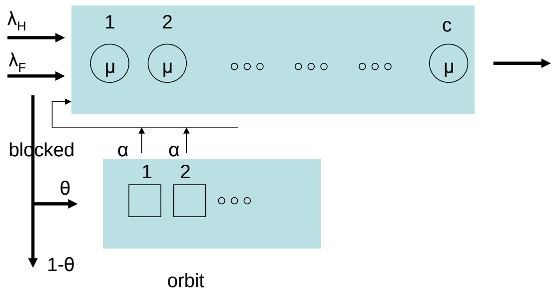


Fig. 1. A retrial queueing model

The system is described by the two-dimensional continuous time Markov process $(I(t), J(t))$, where $I(t)$ $(0 \leq I(t) \leq c)$ represents the number of occupied servers (channels in the context of this paper) and $J(t)$ $(0 \leq J(t))$ is the number of customers waiting for reattempt at time $t$. The steady state probabilities are denoted by $\pi_{i,j} = \lim_{t \to \infty} Prob(I(t) = i, J(t) = j)$. We introduce vector $\mathbf{v}_j$ as follows $\mathbf{v}_j = (\pi_{0,j}, \ldots, \pi_{c,j})$.

Retrial queues have been used to model the queueing problem of new and handover calls in cellular mobile networks [4,5,9,14,15] and telecommunication systems [1–3]. The fact that the retrial rate depends on the number of retried calls waiting in the system leads to an analytically intractable model [5,7]. Therefore, approximation procedures should be used to compute the performability of the system. The well-known technique is based on the truncation of the state space. That is, instead of the computation of the probabilities of the whole state space $\{0, \ldots, c\} \times \{0, 1, \ldots\}$ we calculate $\pi_{i,j}$ ($(i, j) \in \{0, \ldots, c\} \times \{0, 1, \ldots, q\}$), where $q$ is an appropriate large integer number. Another approach applies the assumption that the retrial rate of calls/requests waiting in the orbit has a fixed value after a certain value of $J(t)$. The development of a proposed algorithm in this paper is based on the latter approach.

We assume that the rate $\alpha_0$ of repeated calls is independent of the number of calls in the orbit. Note that due to the efficient estimation of the average number of calls in the orbit, we can apply the iterative procedure (see Algorithm 3) to estimate $\alpha_0 = E(J)\alpha$, where $E(J)$ is the average number of waiting calls in the orbit. The result of the fixed point iteration (Algorithm 3) is used to approximate the performance of wireless cellular networks with the FGC policy, where the rate of repeated calls depends on the number of calls in the orbit.

The following notations are introduced.

- $A_j(i, k)$ denotes a transition rate from state $(i, j)$ to state $(k, j)$ ($0 \leq i, k \leq c; j = 0, 1, \ldots$), which is caused by either the departure of a call after service or the arrival of a call. For $j \geq 0$, matrix $A_j$ is defined as the matrix with elements $A_j(i, k)$. Since $A_j$ is $j$-independent, it can be written as

$$A_j = A = \begin{bmatrix} 0 & \lambda_0 & 0 & \ldots & 0 & 0 & 0 \\ \mu & 0 & \lambda_1 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & & \ldots & (c-1)\mu & 0 & \lambda_{c-1} \\ 0 & 0 & & \ldots & 0 & c\mu & 0 \end{bmatrix},$$

  where $\lambda_i = \lambda_F \beta_i + \lambda_H$.
- $B_j(i, k)$ represents one step upward transition from state $(i, j)$ to state $(k, j + 1)$ ($0 \leq i, k \leq c; j = 0, 1, \ldots$), which is due to a call joining the orbit. In the similar way, matrix $B_j$ ($B$) with elements $B_j(i, k)$ is defined as

$$B_j = B = \begin{bmatrix} \sigma_0 & 0 & 0 \ldots 0 & 0 & 0 \\ 0 & \sigma_1 & 0 \ldots 0 & 0 & 0 \\ \vdots & \vdots & \vdots \; \vdots & \vdots \; \vdots & \vdots \\ 0 & 0 & \ldots 0 & \sigma_{c-1} & 0 \\ 0 & 0 & \ldots 0 & 0 & \sigma_c \end{bmatrix} \quad \forall j \geq 0,$$

where $\sigma_i = \lambda_F(1 - \beta_i)\theta$ $(0 \leq i < c)$ and $\sigma_c = (\lambda_F + \lambda_H)\theta$

- $C_j(i, k)$ is the transition rate from state $(i, j)$ to state $(k, j-1)$ $(0 \leq i, k \leq c; j = 1, \ldots)$, which is due to a call which leaves the orbit. Matrix $C_j$ $(\forall j \geq 1)$ with elements $C_j(i, k)$ is written as

$$C_j = C = \begin{bmatrix} \omega_0 & \alpha_0\beta_0 & 0 & \ldots 0 & 0 \\ 0 & \omega_1 & \alpha_0\beta_1 & \ldots 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \; \vdots & \vdots \\ 0 & 0 & & \ldots \omega_{c-1} & \alpha_0\beta_{c-1} \\ 0 & 0 & & \ldots 0 & (1-\theta)\alpha_0 \end{bmatrix},$$

where $\omega_i = \alpha_0(1 - \beta_i)(1 - \theta)$.

$D^A$ and $D^C$ are diagonal matrices whose diagonal elements are the sum of the elements in the corresponding row of $A$ and $C$. The infinitesimal generator matrix of Markovian process $(I(t), J(t))$ is given by

$$\begin{bmatrix} A_{00} & \text{B} & 0 & \ldots & \ldots & \ldots & \ldots \\ \text{C} & Q_1 & \text{B} & 0 & \ldots & \ldots & \ldots \\ 0 & \text{C} & Q_1 & \text{B} & 0 & \ldots & \ldots \\ 0 & 0 & \text{C} & Q_1 & \text{B} & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix}, \quad (1)$$

where $A_{00} = A - D^A - B$ and $Q_1 = A - D^A - B - D^C$.

It is obvious that the system is described by a Quasi-Birth and Death (QBD) process. For $j \geq 1$, the balance equations is written as follows

$$\mathbf{v}_{j-1}B + \mathbf{v}_j Q_1 + \mathbf{v}_{j+1}C = 0 \ \ (j \geq 1). \tag{2}$$

We have the balance equation for $j = 0$

$$\mathbf{v}_0 A_{00} + \mathbf{v}_1 C = 0. \tag{3}$$

In addition, the normalization equation is

$$\sum_{i=0}^{c} \sum_{j=0}^{\infty} \pi_{i,j} = 1. \tag{4}$$

Since the system can be approximately modeled by a QBD process, some existing methods can be used such as the improved version [11] of the matrix-geometric method [12], the spectral expansion method [10]. We will show that we can develop a faster algorithm than the matrix-geometric method.

Following [10], the characteristic matrix polynomial associated with equations (2) is $Q(x) = B + Q_1 x + Cx^2$. Assume that $Q(x)$ has $d+1$ pairs of eigenvalue-eigenvectors $(x_i, \boldsymbol{\psi}_i)$, thus satisfying the equation:

$$\boldsymbol{\psi}_i Q(x_i) = 0; \ \ det[Q(x_i)] = 0 \ \text{ for } i = 0, \ldots, d. \tag{5}$$

For the $k^{th}$ $(k = 0, \ldots, d)$ eigenvalue-eigenvector pair, $(x_k, \boldsymbol{\psi}_k)$, by substituting $\mathbf{v}_j = \boldsymbol{\psi}_k x_k^j$ in the equations (2), this set of equations is satisfied. This means $\mathbf{v}_j = \boldsymbol{\psi}_k x_k^j$ is a particular solution. As a consequence, the general solution for $\mathbf{v}_j$ is a linear sum of all the factors $(\boldsymbol{\psi}_k x_k^j)$. That is, we can write

$$\mathbf{v}_j = \sum_{k=0}^{d} b_k x_k^j \boldsymbol{\psi}_k \ (j \geq 0), \tag{6}$$

where $(x_0, \boldsymbol{\psi}_0)$, ..., $(x_d, \boldsymbol{\psi}_d)$ are left-hand-side eigenvalue-eigenvector pairs of $Q(x)$ and $b_0, \ldots, b_d$ are coefficients to be determined. In order to satisfy the normalization equation 4, only $|x_i| < 1$ should be considered. Without the loss of generality, let $d+1$ be the number of these eigenvalues $x_i$. The number of coefficients $b_0, \ldots, b_d$ to be determined is $d+1$. We can utilize equation (3) and the normalization equation to compute the coefficients. Thus, the number of linear equations is $c+2$. However, only $c+1$ equations are linearly independent. Therefore, we obtain a unique solution if and only if $c = d$. This means, the exact number of eigenvalues $(|x_i| < 1)$ is $c+1$.

## 3 A Computational Procedure

From equation (6) the steady state probabilities are given by

$$\mathbf{v}_j = \sum_{k=0}^{c} b_k x_k^j \boldsymbol{\psi}_k \ (j \geq 0), \tag{7}$$

where $(x_0, \boldsymbol{\psi}_0)$, ..., $(x_c, \boldsymbol{\psi}_c)$ are left-hand-side eigenvalue-eigenvector pairs of $Q(x)$ and $b_0, \ldots, b_c$ are coefficients to be determined.

In the present paper, $Q(x)$ is a tridiagonal matrix

$$Q(x) = \begin{bmatrix} q_{0,0}(x) & q_{0,1}(x) & 0 & \ldots & 0 & 0 \\ \mu x & q_{1,1}(x) & q_{1,2}(x) & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & & \ldots & c\mu x & q_{c,c}(x) \end{bmatrix},$$

where

$$\begin{aligned} q_{i,i+1}(x) &= (\lambda_F \beta_i + \lambda_H)x + x^2 \alpha_0 \beta_i \quad (i = 0, \ldots, c-1), \\ q_{0,0}(x) &= \lambda_F(1-\beta_0)\theta - (\lambda_F \beta_0 + \lambda_H)x - \\ &\quad (\lambda_F(1-\beta_0)\theta + \alpha_0(1-\beta_0)(1-\theta) + \alpha_0\beta_0)x \\ q_{i,i}(x) &= \lambda_F(1-\beta_i)\theta - (i\mu + \lambda_F\beta_i + \lambda_H)x - \\ &\quad (\lambda_F(1-\beta_i)\theta + \alpha_0(1-\beta_i)(1-\theta) + \alpha_0\beta_i)x + \\ &\quad x^2\alpha_0(1-\beta_i)(1-\theta) \ \ (i = 1, \ldots, c-1), \\ q_{c,c}(x) &= \lambda\theta - x(\lambda\theta + c\mu + (1-\theta)\alpha_0) + (1-\theta)\alpha_0 x^2, \\ \lambda &= \lambda_F + \lambda_H. \end{aligned}$$

If $\boldsymbol{\xi}(x) = \{\xi_0(x), \xi_1(x), \ldots, \xi_c(x)\}$ is the corresponding eigenvector of eigenvalue $x$ of $Q(x)$ (i.e: $\boldsymbol{\xi}(x)Q(x) = 0$), we can write

$$\begin{aligned} 0 &= \xi_0(x)q_{0,0}(x) + \xi_1(x)q_{1,0}(x), \\ 0 &= \xi_{i-1}(x)q_{i-1,i}(x) + \xi_i(x)q_{i,i}(x) + \xi_{i+1}(x)q_{i+1,i}(x), \\ &\quad i = 1, \ldots, c-1, \\ 0 &= \xi_{c-1}(x)q_{c-1,c}(x) + \xi_c(x)q_{c,c}(x). \end{aligned}$$

We set $\xi_0(x) = 1$ and $q_{c+1,c}(x) = 1 + x$, therefore

$$\begin{aligned} \xi_1(x) &= -q_{0,0}(x)/q_{1,0}(x), \\ \xi_{i+1}(x) &= -\frac{\xi_i(x)q_{i,i}(x) + \xi_{i-1}(x)q_{i-1,i}(x)}{q_{i+1,i}(x)}, \\ &\quad i = 1, \ldots, c. \end{aligned} \tag{8}$$

The sequence $\{\xi_i(x), i = 0, \ldots, c+1\}$ associated with the characteristic matrix polynomial of tridiagonal form is a Sturm sequence within a given interval if for any fixed $x$ within this interval $\xi_0(x) = 1$ and $\xi_i(x) = 0$ ($i = 1, \ldots, n$) imply $\xi_{i-1}(x)\xi_{i+1}(x) < 0$. In order to determine the eigenvalues of $Q(x)$, the number of sign variations is defined as

$$mnsv(x) = \#\{\xi_i(x)\xi_{i+1}(x) < 0, 0 \le i \le c\}. \tag{9}$$

As a consequence, a divide-and-conquer procedure [6] (called the `getx` in Algorithm 1) can be applied to find eigenvalues (see Algorithm 2). The eigenvector $\boldsymbol{\psi}_i = [\psi_{i,0}, \ldots, \psi_{i,c}]$ can be computed by the use of equation $\boldsymbol{\psi}_i Q(x_i) = 0$ and $\psi_{i,0} = 1$.

**Algorithm 1** `getx` procedure

{$Xeg$ is the vector of eigenvalues}
{$\epsilon$ is the required accuracy}
PROCEDURE getx($xx_1, nx_1, xx_2, nx_2$)
**if** $nx_1 == nx_2$ **then**
  Return
**end if**
**if** $xx_2 - xx_1 < \epsilon$ **then**
  **if** $nx_1 == nx_2 + 1$ **then**
    $Xeg_{nx_2} = xx_1$
  **end if**
  Return
**end if**
$x = \frac{xx_1 + xx_2}{2}$
$nx = nsv(x)$
Call getx($xx_1, nx_1, xx, nx$)
Call getx($x, nx, xx_2, nx_2$)
END OF PROCEDURE getx

Note that for each eigenvalue, the corresponding eigenvector can be determined by equation (8), then the steady state probabilities can be computed after the calculation of coefficients $b_i$'s. To compute $b_i$'s, we have to use equation (3) and the normalization equation. Based on the efficient computation of the approximated

**Algorithm 2** The proposed computation algorithm

{ $\epsilon$ is the required accuracy}

$xx_1 := \epsilon$
$xx_2 := 1 - \epsilon$
$nx_1 := mnsv(xx_1)$
$nx_2 := mnsv(xx_2)$
Call getx($xx_1, nx_1, xx_2, nx_2$)
{nonzero-eigenvalues are returned in $Xeg$}

model, we propose a procedure to evaluate the performance of the system (see Algorithm 3). The result of the fixed point iteration is used to approximate the performance of wireless cellular networks with the FGC policy, where the rate of repeated calls depends on the number of calls in the orbit.

**Algorithm 3** The proposed computation algorithm

{$\alpha$ is the retrial rate of each call in the orbit}
{$\alpha_0$ is the retrial rate of the approximated model}
{$E_n(J)$ is the average number of calls in the orbit in the approximated model}

$\alpha_0 = \alpha$
**repeat**
  Compute $x_i$'s based on Algorithm 2
  Compute $\boldsymbol{\psi}_i$'s
  Compute $b_i$'s
  Compute $E_n(J)$
  $\alpha_0 = \alpha E_n(J)$
**until** $E_n(J)$ converges

### *3.1 Influence of $\beta_i = 1$*

**Theorem 1** *The number of zero-eigenvalues of $Q(x)$ is equal to the number of $\beta_i$'s whose value is 1.*

**Proof.** If $b_i = 1$, then all the elements ($q_{i-1,i}(x), q_{i,i}(x), q_{i,i+1}(x)$) in row $i$ of $Q(x)$ are divideable by $x$, but not by $x^2$. Therefore, it follows that $Q(x)$ should have a zero-eigenvalue because of the computation of $det[Q(x)]$ according to row $i$. The number of zero-eigenvalues of $Q(x)$ is equal to the number of rows which are divideable by $x$ (the number of $\beta_i$'s whose value is 1). □

It is worth emphasizing that for $\beta_i = 0$, the eigenvector corresponding to a zero-eigenvalue is a vector with all elements equal to zero except the $i^{th}$ element which is 1.

### *3.2 Limited Fractional Guard Channel Policy*

The FGC is called the Limited Fractional Guard Channel Policy (LFGC) if the following parameters are applied: $\beta_i = 1$ $(0 \leq i \leq T)$, $\beta_{T+1} = \beta$ and $\beta_i = 0$ $(T+1 < i < c)$. As consequence, the number of zero-eigenvalues is $T+1$. Without the loss of generality, these eigenvalues are $x_0,\ x_1,\ \ldots, x_T$. Since the eigenvector corresponding to $x_i = 0$ is a vector with all elements equal to zero except the $i^{th}$ element which is 1.

$$\mathbf{v}_0 = \mathbf{b}^{(*)} + \sum_{k=T+1}^{c} b_k \boldsymbol{\psi}_k \tag{10}$$

$$\mathbf{v}_j = \sum_{k=T+1}^{c} b_k x_k^j \boldsymbol{\psi}_k \ (j \geq 1), \tag{11}$$

where $\mathbf{b}^{(*)} = \{b_1, \ldots, b_T, 0, \ldots, 0\}$.

## 4 Numerical Results

We have built a simulation for the original and analytically intractable system described in Section 2, where the retrial rate depends [1] on the number of waiting calls in the orbit. In Table 1 we present some illustrative results [2] to show the accuracy of the new method concerning to the blocking probabilities of calls and the average number of occupied channels when the LFGC is applied with $T = c - 1$, $\beta = 0$, $c = 15$, $1/\mu = 120$ s, $\alpha/\mu = 20$, $\lambda_F/\lambda_H = 24$. The offered load is defined by $\rho = \lambda/(c\mu)$. All the computations are performed with machine precision (approximately 2.22045x$10^{-16}$). It is worth emphasizing that the similar observation is obtained with other parameter values as well.

In Figures 2, 3 and 4, we show illustrative results concerning the average number of calls in the orbit, the probabilities that fresh calls and handover calls are forced to the orbit when LFGC policy is applied. It is observed that paramter $\beta$ has a strong impact on handover calls, therefore it can be used for fine-tuning the system.

[1] It is the only difference between the simulation and the proposed approach is the retrial rate of calls waiting in the orbit.

[2] Note that the simulation results are generated with the confidential level of 99%, and the ratio of the half-width of the confidence interval and the mean of collected observations is

Table 1
Comparison with simulation

| $\rho$ | $\theta = 0.1$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Simulation | | | | Analytical | | | |
| | $E_s(I)$ | $E_s(J)$ | $\mathrm{Pr}_s(I = c-1)$ | $\mathrm{Pr}_s(I = c)$ | $E_n(I)$ | $E_n(J)$ | $\mathrm{Pr}_n(I = c-1)$ | $\mathrm{Pr}_n(I = c)$ |
| 0.1 | 1.500250 | 0.000000 | 0.000000 | 0.000000 | 1.5 | $5.6\ 10^{-13}$ | $7.5\ 10^{-10}$ | $6.5\ 10^{-12}$ |
| 0.2 | 3.000170 | 0.000000 | 0.000002 | 0.000000 | 2.99999 | $4.2\ 10^{-9}$ | 0.000002 | $4.8\ 10^{-8}$ |
| 0.3 | 4.500090 | 0.000000 | 0.000161 | 0.000005 | 4.49925 | $4.5\ 10^{-7}$ | 0.000178 | 0.000005 |
| 0.4 | 5.992940 | 0.000000 | 0.002161 | 0.000091 | 5.98737 | 0.00000 | 0.002236 | 0.000078 |
| 0.5 | 7.418670 | 0.000485 | 0.011387 | 0.000611 | 7.41839 | 0.000046 | 0.011449 | 0.000502 |
| 0.6 | 8.712900 | 0.001767 | 0.034116 | 0.002205 | 8.70786 | 0.000165 | 0.033841 | 0.001781 |
| 0.7 | 9.796540 | 0.004544 | 0.071726 | 0.005546 | 9.78445 | 0.000408 | 0.070401 | 0.004325 |
| 0.8 | 10.649000 | 0.008906 | 0.118981 | 0.010615 | 10.6317 | 0.000784 | 0.116733 | 0.008202 |
| 0.9 | 11.302100 | 0.014681 | 0.170403 | 0.017407 | 11.2777 | 0.001282 | 0.167009 | 0.013212 |
| | $\theta = 0.3$ | | | | | | | |
| 0.1 | 1.500250 | 0.000000 | 0.000000 | 0.000000 | 1.5 | $1.9\ 10^{-12}$ | $7.5\ 10^{-10}$ | $1.5\ 10^{-11}$ |
| 0.2 | 3.000600 | 0.000000 | 0.000002 | 0.000000 | 2.99999 | $1.5\ 10^{-8}$ | 0.0000027 | $1.1\ 10^{-7}$ |
| 0.3 | 4.500360 | 0.000000 | 0.000164 | 0.000011 | 4.49932 | 0.0000015 | 0.000178 | 0.000011 |
| 0.4 | 5.994750 | 0.000000 | 0.002234 | 0.000224 | 5.99969 | 0.001129 | 0.002387 | 0.0012942 |
| 0.5 | 7.431050 | 0.001831 | 0.011728 | 0.001494 | 7.4239 | 0.000174 | 0.011521 | 0.0011505 |
| 0.6 | 8.750470 | 0.007213 | 0.035735 | 0.005646 | 8.72407 | 0.000643 | 0.034040 | 0.0040884 |
| 0.7 | 9.862670 | 0.019015 | 0.074871 | 0.014397 | 9.78445 | 0.000408 | 0.070401 | 0.0043254 |
| 0.8 | 10.760900 | 0.038974 | 0.123618 | 0.028256 | 10.6317 | 0.000785 | 0.116733 | 0.0082025 |
| 0.9 | 11.455700 | 0.067699 | 0.175191 | 0.046835 | 11.2777 | 0.001282 | 0.167009 | 0.0132115 |
| | $\theta = 1.0$ | | | | | | | |
| 0.1 | 1.500250 | 0.00 | 0.00 | 0.000 | 1.5 | $1.99\ 10^{-11}$ | $7.5\ 10^{-10}$ | $7.4\ 10^{-11}$ |
| 0.2 | 3.000920 | 0.000000 | 0.000003 | 0.000000 | 3.0 | $2.5\ 10^{-7}$ | 0.000003 | $6.0\ 10^{-7}$ |
| 0.3 | 4.500000 | 0.000073 | 0.000181 | 0.000043 | 4.49999 | 0.000041 | 0.000186 | 0.0000669 |
| 0.4 | 5.999880 | 0.001795 | 0.002508 | 0.000912 | 5.99969 | 0.001129 | 0.002387 | 0.0012942 |
| 0.5 | 7.497120 | 0.017323 | 0.014018 | 0.007419 | 7.49959 | 0.011952 | 0.012508 | 0.0097432 |
| 0.6 | 8.989000 | 0.095756 | 0.043888 | 0.033434 | 8.99538 | 0.071775 | 0.041080 | 0.0373981 |
| 0.7 | 10.453600 | 0.379311 | 0.091433 | 0.102135 | 10.4503 | 0.305958 | 0.0761227 | 0.118219 |
| 0.8 | 11.884100 | 1.244660 | 0.140123 | 0.239130 | 11.8743 | 1.09335 | 0.167975 | 0.20876 |
| 0.9 | 13.250500 | 4.081900 | 0.155641 | 0.468458 | 13.2458 | 4.22319 | 0.153844 | 0.470658 |

For $\rho = 0.9$, $\theta = 0.6$, $1/\mu = 120$ s, $\alpha/\mu = 20$, $\lambda_F/\lambda_H = 24$, $\beta = 0.9$, we plot the computational time versus $c$ in Figure 5. The proposed method is compared against the method proposed by Naoumov et al. [11] (note that the method proposed by Naoumov et al. is considered as one of the fastest approaches to compute the stationary probabilities of QBD processes. Figure 5 shows that our method is much faster than the existing method.

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0.099%.

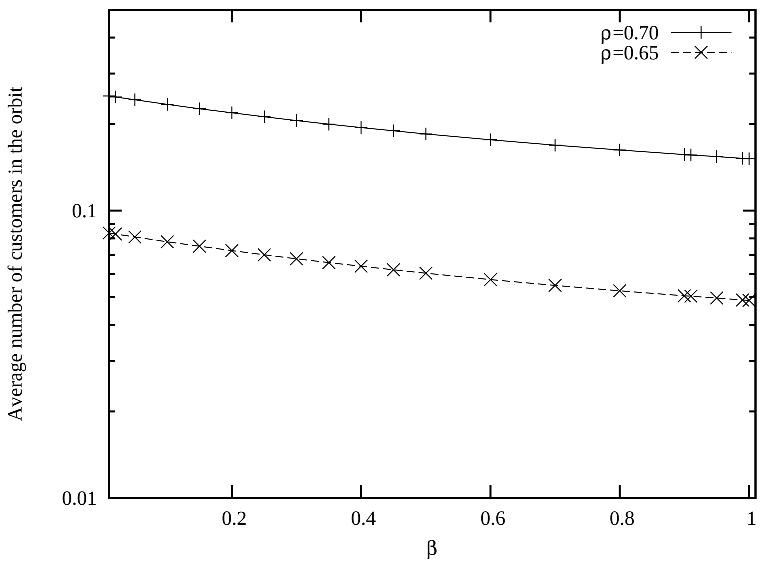


Fig. 2. The average number of calls in the orbit

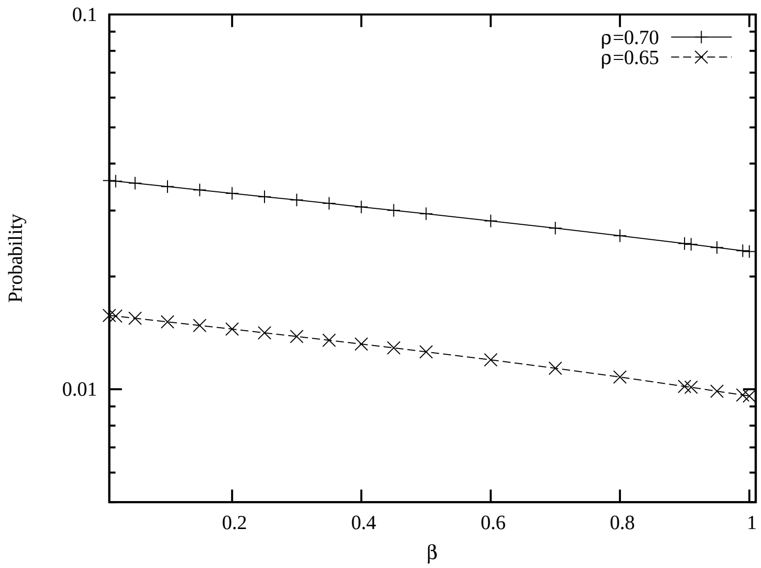


Fig. 3. The probability that fresh calls are forced to the orbit

## 5 Conclusions

We have presented a new retrial queueing problem for wireless cellular networks applying the Fractional Guard Channel (FGC) admission control policy and provided a new algorithm to approximate the retrial queue problem involving the FGC policy. The comparison between analytical and simulation results confirms that our algorithm is accurate and fast to evaluate the performance of the system.

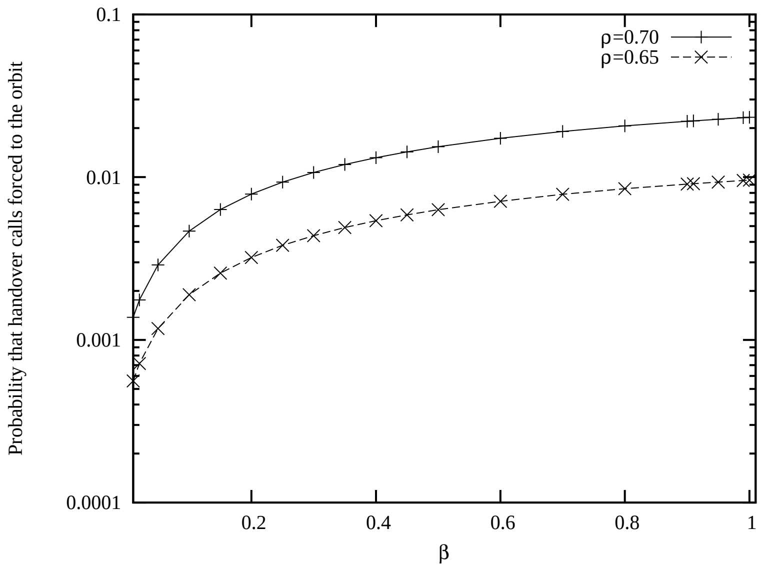


Fig. 4. The probability that handover calls are forced to the orbit

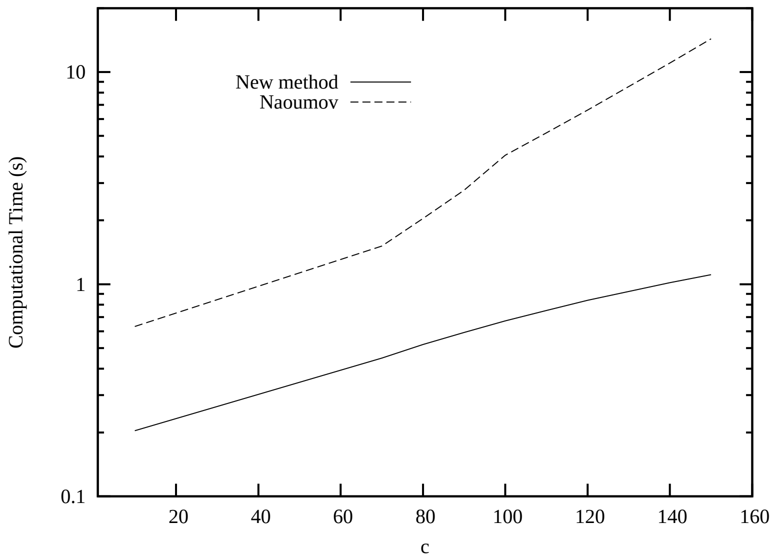


Fig. 5. Computational time

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