Multi-dimensional array – Recursion

Basics of Programming 1

DEPARTMENT OF
NETWORKED SYSTEMS
AND SERVICES

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20 November, 2024

Content

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Chapter 1

Multi-dimensional arrays

Multi-dimensional arrays

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1D array Elements of the same type, stored in the memory beside eachother

Multi-dimensional arrays

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Multi-dimensional arrays

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Multi-dimensional arrays

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3D array 2D arrays of the same size and same type, stored in the memory beside eachother

... ...

Two-dimensional

- Declaration of a 2D array:

```c
char a[3][2]; /* 3row x 2column array of characters */
              /* 3-sized array of 2-sized 1D arrays */
```

a[0][0]	a[0][1]
a[1][0]	a[1][1]
a[2][0]	a[2][1]

Two-dimensional

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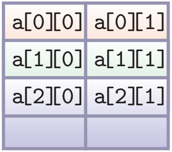
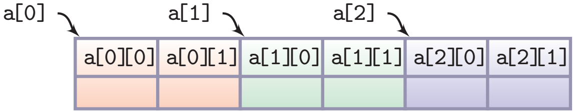


- In C language, storage is done row by row (the second index changes quicker)



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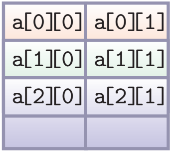

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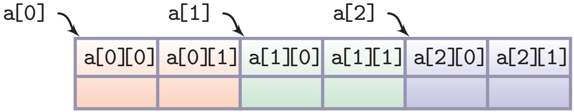

- `a[0]`, `a[1]` and `a[2]` are 2-sized 1D arrays

Taking a 2D array row by row

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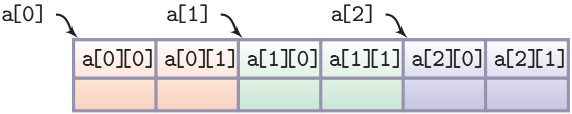
- Filling a 1D array (row) with the given element

```
void fill_row(char row[], size_t size, char c)
{
  size_t i;
  for (i = 0; i < size; ++i)
    row[i] = c;
}
```

- Filling a 2D array row by row

```
char a[3][2];
fill_row(a[0], 2, 'a'); /* row 0 is full of 'a' */
fill_row(a[1], 2, 'b'); /* row 1 is full of  'b' */
fill_row(a[2], 2, 'c'); /* row 2 is full of  'c' */
```



Taking a 2D array as one entity

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- taking as a 2D array – only if number of columns is known

```c
void print_array(char array[][2], size_t nrows)
{
  size_t row, col;
  for (row = 0; row < nrows; ++row)
  {
    for (col = 0; col < 2; ++col)
      printf("%c", array[row][col]);
    printf("\n");
  }
}
```

- Usage of the function

```c
char a[3][2];
...
print_array(a, 3);        /* printing a 3-row array */
```

Taking a 2D array as one entity

- taking 2D array as a pointer

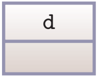
```c
void print_array(char *array, int nrows, int ncols)
{
  int row, col;
  for (row = 0; row < nrows; ++row)
  {
    for (col = 0; col < ncols; ++col)
      printf("%c", array[row*ncols+col]);
    printf("\n");
  }
}
```

- Usage of the function

```c
char a[3][2];
...
print_array((char *)a, 3, 2); /* 3 rows 2 columns */
```

Dynamic 2D array

Let's allocate memory for a 2D array. We would like to use the conventional way of indexing for the array d[i][j]




```
double **d =(double**)malloc(3*sizeof(double*));
d[0] = (double*)malloc(3*4*sizeof(double));
for (i = 1; i < 3; ++i)
  d[i] = d[i-1] + 4;
```

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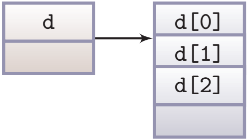


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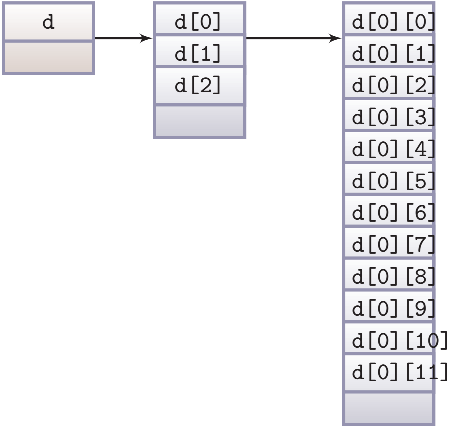


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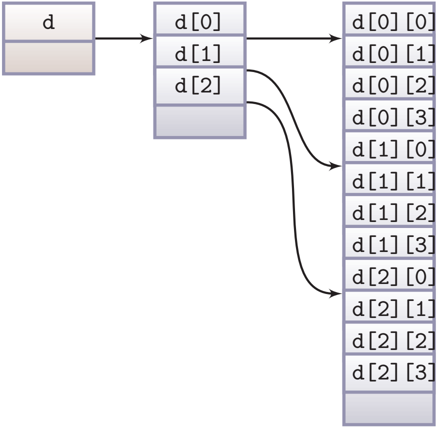


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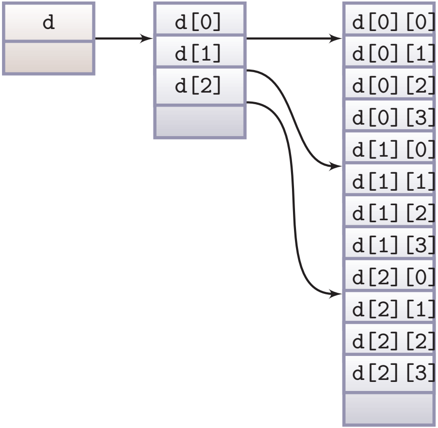

Releasing the array

```
free(d[0]);
free(d);
```

Array of pointers

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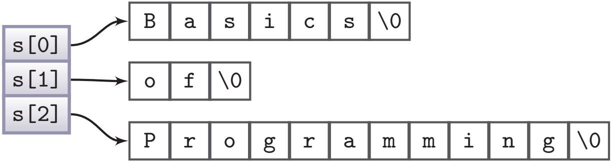
- Defining an array of pointers and passing it to a function

```
char *s[3] = {"Basics", "of", "programming"};
print_strings(s, 3);
```



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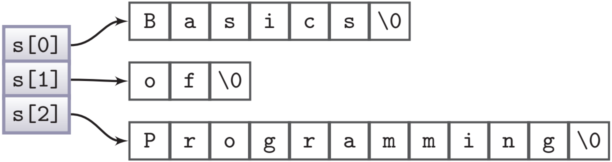

- Taking an array of pointers with a function

```c
void print_strings(char *strings[], size_t size)
/*                 char **strings is also possible  */
{
  size_t i;
  for (i = 0; i < size; ++i)
    printf("%s\n", strings[i]);
}
```

Chapter 2

Recursion

Recursion – definition

Many mathematical problems can be formulated recursively

Recursion – definition

Many mathematical problems can be formulated recursively

- Sum of sequence a_n

$$S_n = \begin{cases} S_{n-1} + a_n & n > 0 \\ a_0 & n = 0 \end{cases}$$

Recursion – definition

Many mathematical problems can be formulated recursively

- Sum of sequence a_n

$$S_n = \begin{cases} S_{n-1} + a_n & n > 0 \\ a_0 & n = 0 \end{cases}$$

- Factorial

$$n! = \begin{cases} (n-1)! \cdot n & n > 0 \\ 1 & n = 0 \end{cases}$$

Recursion – definition

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- Fibonacci numbers

$$F_n = \begin{cases} F_{n-2} + F_{n-1} & n > 1 \\ 1 & n = 1 \\ 0 & n = 0 \end{cases}$$

Recursion – definition

Several everyday problems can be formulated recursively

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$$\text{My ancestor?} = \begin{cases} \text{Ancestor of my father/mother?} \\ \\ \end{cases}$$

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Recursion – definition

DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

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Recursion – definition

Several everyday problems can be formulated recursively

- Is Albert Einstein my ancestor?

$$\text{My ancestor?} = \begin{cases} \text{Ancestor of my father/mother?} \\ \text{Is he my father?} \\ \text{Is she my mother?} \end{cases}$$

- In general

$$\text{Problem} = \begin{cases} \text{Simpler, similar problem(s)} \\ \text{Trivial case(es)} \end{cases}$$

Recursion – outlook

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  - Data structure e.g., linked list, folders of the op. system
  - Geometric constructions e.g., fractals
- We are going to study recursive data structures and recursive algorithms

Recursive algorithms in C

- Factorial

$$n! = \begin{cases} (n-1)! \cdot n & n > 0 \\ 1 & n = 0 \end{cases}$$

$5! = 4! \cdot 5$

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Recursive algorithms in C

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$$n! = \begin{cases} (n-1)! \cdot n & n > 0 \\ 1 & n = 0 \end{cases}$$

$5! = (6 \cdot 4) \cdot 5$

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Recursive algorithms in C

- Factorial

$$n! = \begin{cases} (n-1)! \cdot n & n > 0 \\ 1 & n = 0 \end{cases}$$

$5! = 24 \cdot 5$

Recursive algorithms in C

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- Factorial

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$5! = 24 \cdot 5$

Recursive algorithms in C

- Factorial

$$n! = \begin{cases} (n-1)! \cdot n & n > 0 \\ 1 & n = 0 \end{cases}$$

$5! = 120$

Recursive algorithms in C

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Recursive algorithms in C

DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

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Let us implement it to C!

```
unsigned factorial(unsigned n)
{
  if (n > 0)
    return factorial(n-1) * n;
  else
    return 1;
}
```

Recursive algorithms in C

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- Calling the function

```c
unsigned f = factorial(5); /* it works! */
printf("%u\n", f);
```

Some considerations

- How to imagine recursive functions?

```c
unsigned f0(void) { return 1; }
unsigned f1(void) { return f0() * 1; }
unsigned f2(void) { return f1() * 2; }
unsigned f3(void) { return f2() * 3; }
unsigned f4(void) { return f3() * 4; }
unsigned f5(void) { return f4() * 5; }
...
unsigned f = f5();
```

- Many different instances of the same function coexist simultaneously
- The instances were called with different parameters

Implementing recursion

How can multiple instances of the same function coexist?

Implementing recursion

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How can multiple instances of the same function coexist?

```
/*
  recursive factorial function
 */
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}

int main(void)
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  ...
   factorial(4);
  ...
}
```

register: ??

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0x1FFC: 15
0x2000: 4

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	0x1FFC:	15
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0x1FF8:	3
0x1FFC:	15
n 0x2000:	4
register:	??

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  ...
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}
```

	0x1FF4:	7
n	0x1FF8:	3
	0x1FFC:	15
	0x2000:	4

register: ??

Implementing recursion

DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

How can multiple instances of the same function coexist?

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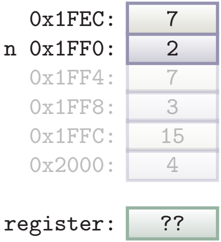

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	Address	Value
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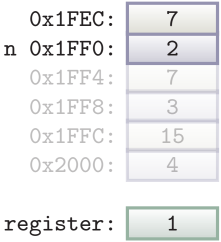

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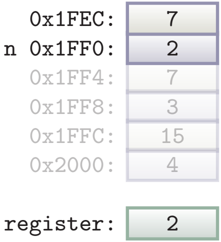

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register: 2

Implementing recursion

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0x1FFC:	15
n 0x2000:	4
register:	24

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Implementing recursion

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- The mechanism of the function calls in C is capable of writing recursive functions

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DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

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- All the data (local variables, return addresses) of the calling functions are stored in the stack
- Whether the function calls itself or an other function makes no difference
- The maximal depth of recursive calls: given by the stack size

Recursion or iteration – factorial

Calculating $n!$ recursively – elegant, but inefficient

```
unsigned fact_rec(unsigned n)
{
  if (n == 0)
    return 1;
  return fact_rec(n-1) * n;
}
```

link

Recursion or iteration – factorial

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Calculating *n*! recursively – elegant, but inefficient

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unsigned fact_rec(unsigned n)
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  return fact_rec(n-1) * n;
}
```

link

and iteratively – boring, but efficient

```
unsigned fact_iter(unsigned n)
{
  unsigned f = 1, i;
  for (i = 2; i <= n; ++i)
    f *= i;
  return f;
}
```

link

Recursion or iteration – Fibonacci

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Calculating F_n recursively – elegant, but way too slow!

```
unsigned fib_rec(unsigned n)
{
  if (n <= 1)
    return n;
  return fib_rec(n-1) + fib_rec(n-2);
}
```

link

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and iteratively – boring, but efficient

```
unsigned fib_iter(unsigned n)
{
  unsigned f1 = 0, f2 = 1, f3, i;
  for (i = 2; i <= n; ++i) {
      f3 = f1 + f2;
      f1 = f2;
      f2 = f3;
  }
  return f2;
}
```

link

Recursion or iteration

1. Every recursive algorithm can be transformed to an iterative one (loops)

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   - There is no general method for this transformation
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   - Easy to do systematically, but usually not efficient

Recursion or iteration

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   - There is no general method for this transformation
2. Every iterative algorithm can be transformed to a recursive one
   - Easy to do systematically, but usually not efficient

There is no universal truth: the choice between recursive and iterative algorithms depends on the problem

Iterative algorithms recursively

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Traversing arrays recursively (without loops)

```
void print_array(int* array, int n)
{
  if (n == 0)
    return;
  printf("%d ", array[0]);
  print_array(array+1, n-1); /* recursive call */
}
```

Iterative algorithms recursively

DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

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```

Traversing strings recursively

```
void print_string(char* str)
{
  if (str[0] == '\0')
    return;
  printf("%c", str[0]);
  print_string(str+1); /* recursive call */
}
```

Iterative algorithms recursively

Traversing arrays recursively (without loops)

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```

Printing number in a given numeral system

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recursively

```
void print_base_rec(unsigned n, unsigned base)
{
  if (n >= base)
    print_base_rec(n/base, base);
  printf("%d", n%base);
}
```

link

Printing number in a given numeral system

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recursively

```c
void print_base_rec(unsigned n, unsigned base)
{
  if (n >= base)
    print_base_rec(n/base, base);
  printf("%d", n%base);
}
```

link

iteratively

```c
void print_base_iter(unsigned n, unsigned base)
{
  unsigned d; /* power of base not greater than n */
  for (d = 1; d*base <= n; d*=base);
  while (d > 0)
  {
    printf("%d", (n/d)%base);
    d /= base;
  }
}
```

link

When the recursive algorithm is definitely better

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The array below stores a labyrinth

```c
char lab[9][9+1] = {
    "+-------+",
    "|       |",
    "+-+ ++ ++",
    "|       |",
    "| + +-+ |",
    "| | |   |",
    "+-+ +-+ |",
    "|     | |",
    "+-----+-+"
};
```

link

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};
```

link

Let us visit the entire labyrinth from start position (x,y)

```c
traverse(lab, 1, 1);
```

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char lab[9][9+1] = {
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```
traverse(lab, 1, 1);
```

We go in every possible direction and visit the yet unvisited parts of the labyrinth

When the recursive algorithm is definitely better

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The simplicity of the recursive solution is striking

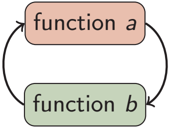
```c
void traverse(char lab[][9+1], int x, int y)
{
  lab[x][y] = '.';          /* mark that we were here */
  if (lab[x-1][y] == ' ') /* go upwards, if needed */
    traverse(lab, x-1, y);
  if (lab[x+1][y] == ' ') /* go downwards, if needed */
    traverse(lab, x+1, y);
  if (lab[x][y-1] == ' ') /* go left, if needed */
    traverse(lab, x, y-1);
  if (lab[x][y+1] == ' ') /* go right, if needed */
    traverse(lab, x, y+1);
}
```

link

It is also possible to do with an iterative algorithm – but it is much more complex

Indirect recursion

Indirect recursion: Functions mutually call each other




```
/* forward declaration */
void b(int); /* name, return type, parameter types */

void a(int n) {
  ...
  b(n); /* b can be called due to the forward decl. */
  ...
}

void b(int n) {
  ...
  a(n);
  ...
}
```

Forward declaration

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Forward declaration will be necessary for recursive data structures

```
/* forward declaration */
struct child_s;

struct mother_s { /* mother type */
  char name[50];
  struct child_s *children[20]; /*pntr. arr. of children*/
};

struct child_s { /* child type */
  char name[50];
  struct mother_s *mother; /*pointer to the mother*/
};
```

Thank you for your attention.