Binary trees

Basics of Programming 1

Department of Networked Systems and Services
G. Horváth, A.B. Nagy, Z. Zsóka, P. Fiala, A. Vitéz

27 November, 2024

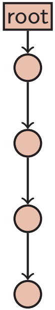
Content

DEPARTMENT OF NETWORKED SYSTEMS AND SERVICES

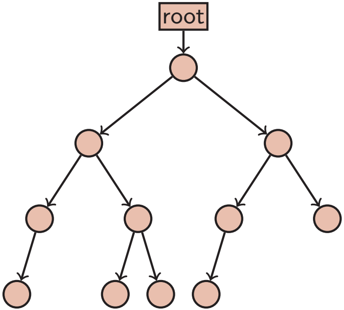
Chapter 1

Binary trees

Trees



$K = 1$ (linked list)



$K = 2$ (binary tree)

- An acyclic graph
- Every node has exactly one incoming edge
- K-ary tree: every node has at most K outgoing edges

Binary trees

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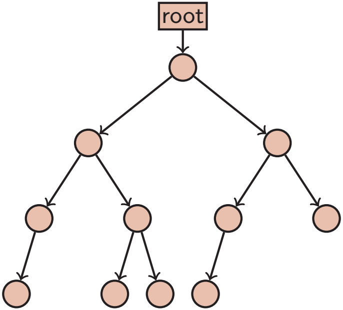


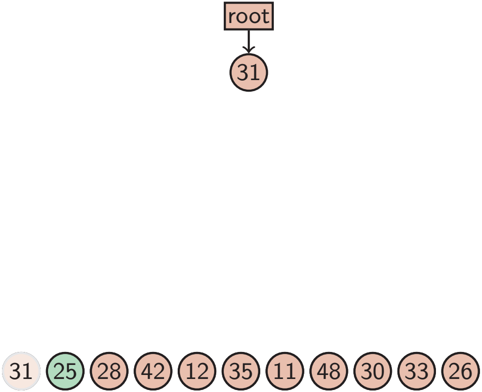
- Declaration of the binary tree data structure

```c
typedef struct tree {
  int data;
  struct tree *left, *right;
} tree_elem, *tree_ptr;
```

link

- Typically we typedef not only the struct, but also the pointer

Binary search trees



- Sub-tree to the left: only elements smaller than the node
- Sub-tree to the right: only elements greater than the node
- The structure of the tree depends on the insertion order of the elements!

Binary search trees

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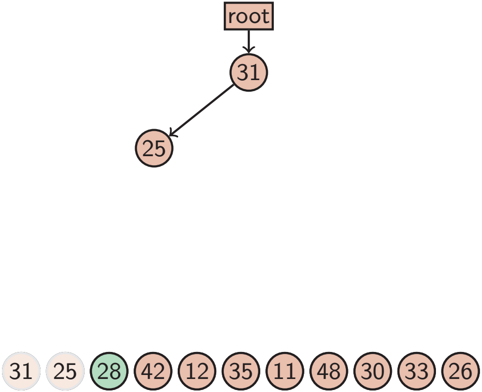

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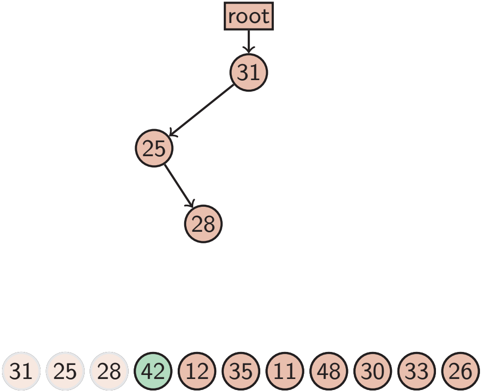

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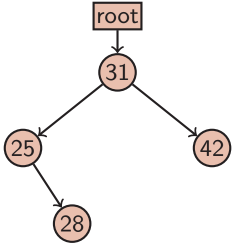

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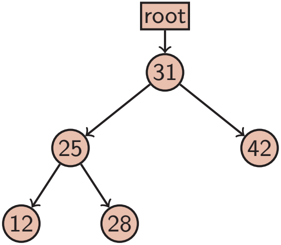

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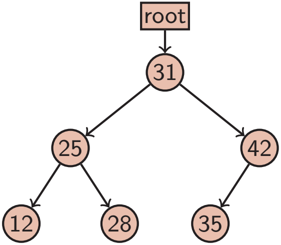

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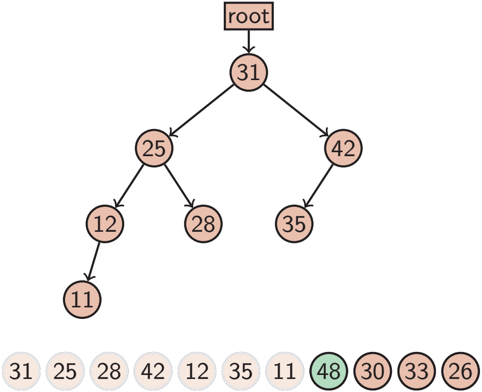

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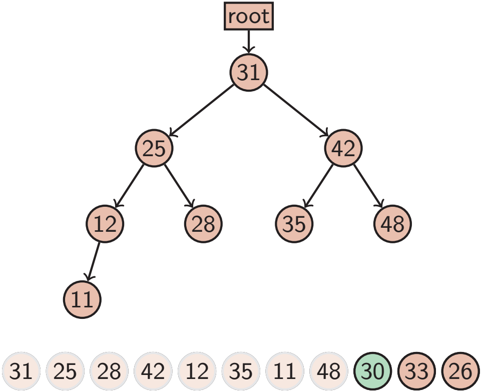

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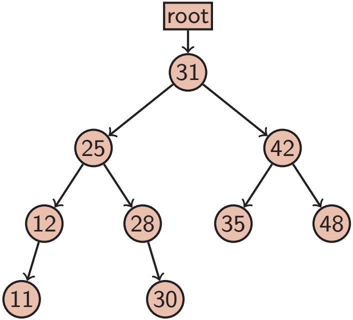

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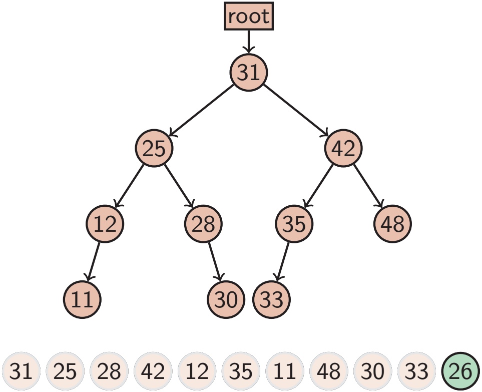

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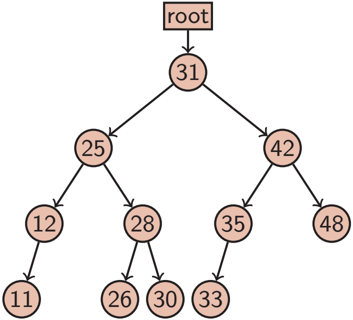

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Searching an element in the tree

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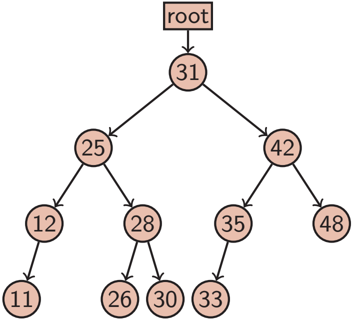


```c
tree_ptr find(tree_ptr root,
              int data)
{
  while (root != NULL &&
    data != root->data)
  {
    if (data < root->data)
      root = root->left;
    else
      root = root->right;
  }
  return root;
}
```

link

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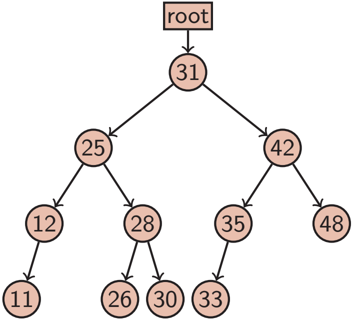


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Searching an element in the tree

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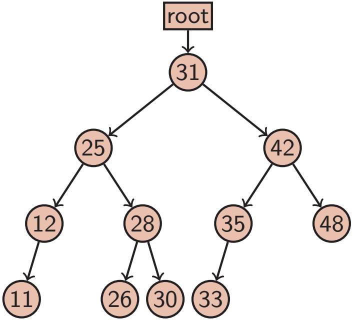


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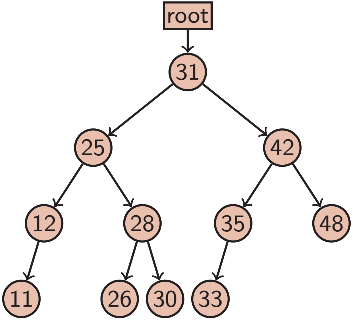
- In a depth-d tree the max. number of steps is d

Searching an element in the tree

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link

- This is not recursive yet
- In a depth-d tree the max. number of steps is d
- If the tree is balanced and has n elements $\Rightarrow\approx \log_2 n$ steps!

In-order traversal

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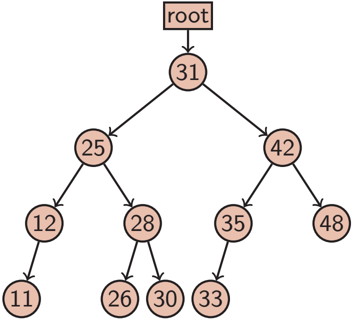


```c
void inorder(tree_ptr root)
{
  if (root == NULL)
    return;
  inorder(root->left);
  printf("%d ", root->data);
  inorder(root->right);
}
```

[redacted]

In-order traversal

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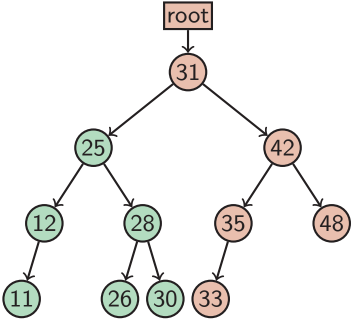


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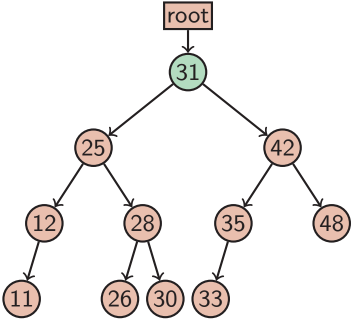
11 12 25 26 28 30

In-order traversal

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11 12 25 26 28 30 31
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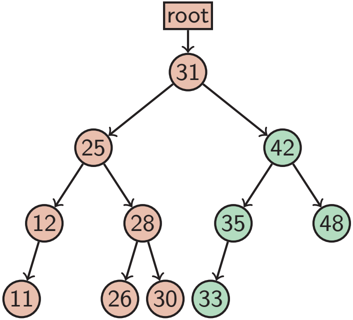
In-order traversal

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11 12 25 26 28 30 31 33 35 42 48

In-order traversal

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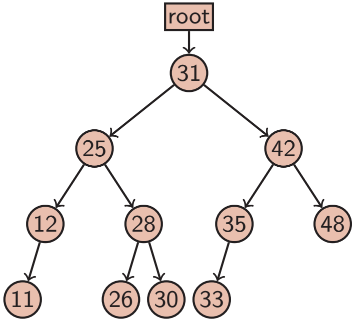


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}
```

11 12 25 26 28 30 31 33 35 42 48

- in-order traversal
  1. left sub-tree
  2. root element
  3. right sub-tree

With this traversal the nodes are visited in increasing order of their values

In-order traversal

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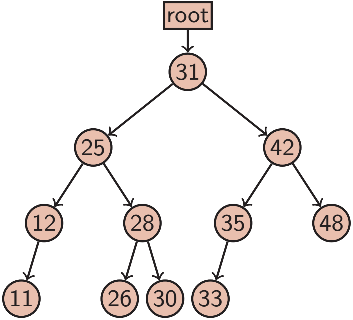
An other implementation of the in-order traversal:

```c
void inorder(tree_ptr root)
{
  if (root->left != NULL)
    inorder(root->left);
  printf("%d ", root->data);
  if (root->right != NULL)
    inorder(root->right);
}
```

But in this case the caller has not make sure that `root != NULL` holds

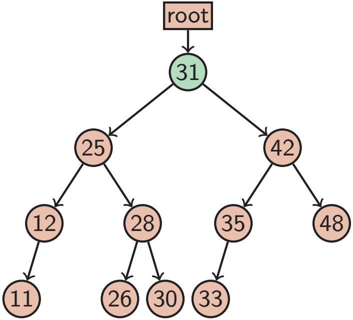
Pre-order traversal




```c
void preorder(tree_ptr root)
{
  if (root == NULL)
    return;
  printf("%d ", root->data);
  preorder(root->left);
  preorder(root->right);
}
```

[redacted]

Pre-order traversal

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  preorder(root->right);
}
```

31

Pre-order traversal

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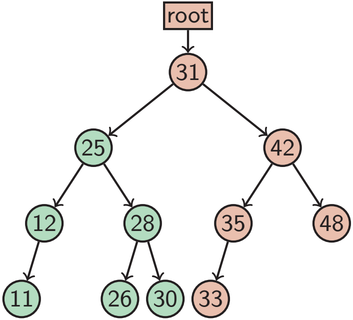


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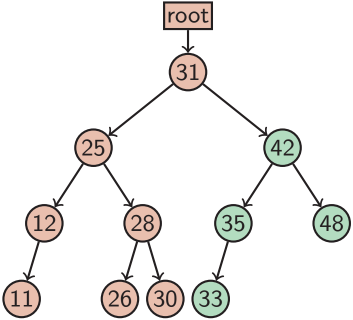
```

```
31 25 12 11 28 26 30
```

Pre-order traversal

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31 25 12 11 28 26 30 42 35 33 48

Pre-order traversal

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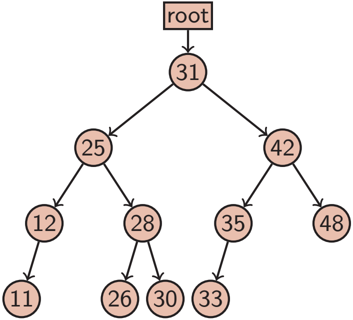


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31 25 12 11 28 26 30 42 35 33 48

- pre-order traversal
  1. root element
  2. left sub-tree
  3. right sub-tree

Saving the elements of the tree in this order, and building it again, the structure of the tree can be fully reconstructed.

Building a tree

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Inserting a new node to the tree

```
tree_ptr insert(tree_ptr root, int data)
{
  if (root == NULL) {
    root = (tree_ptr)malloc(sizeof(tree_elem));
    root->data = data;
  }
  else if (data < root->data)
    root->left = insert(root->left, data);
  else
    root->right = insert(root->right, data);
  return root;
}
```

link

Building a tree

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  }
  else if (data < root->data)
    root->left = insert(root->left, data);
  else
    root->right = insert(root->right, data);
  return root;
}
```

link

Usage of this function:

```
tree_ptr root = NULL;
root = insert(root, 2);
root = insert(root, 8);
...
```

Post-order traversal

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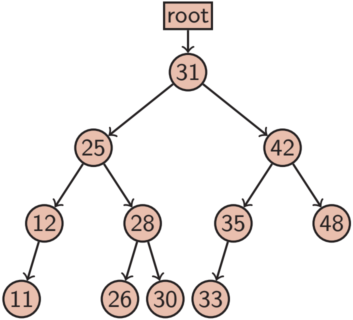


```c
void postorder(tree_ptr root)
{
  if (root == NULL)
    return;
  postorder(root->left);
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  printf("%d ", root->data);
}
```
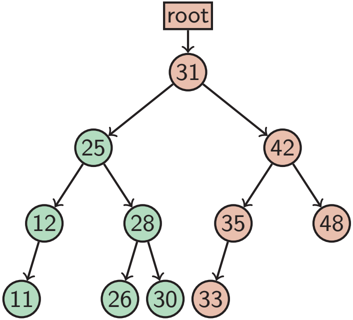

[redacted]

Post-order traversal

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Post-order traversal

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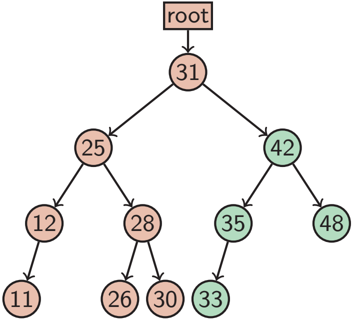


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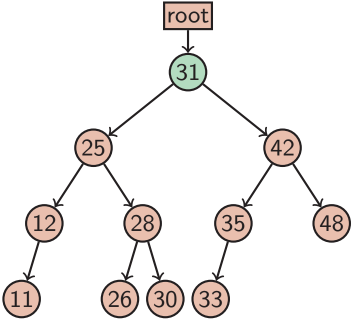
Post-order traversal

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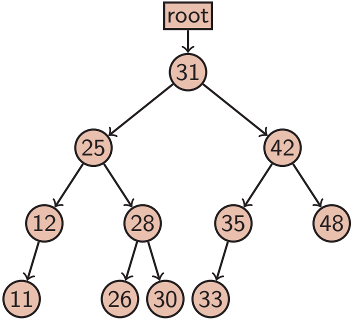


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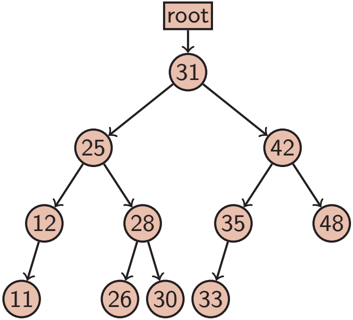
- post-order traversal
  1. left sub-tree
  2. right sub-tree
  3. root element

Post-order traversal

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- post-order traversal
  1. left sub-tree
  2. right sub-tree
  3. root element

In this order the leaves of the tree are visited first → application: releasing/deleting a tree

Deleting a tree by post-order traversal

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```c
void delete(tree_ptr root)
{
  if (root == NULL) /* empty tree: nothing to delete */
    return;
  delete(root->left);     /* post-order traversal */
  delete(root->right);
  free(root);
}
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link

Deleting a tree by post-order traversal

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A program segment (without memory leaks):

```c
tree_ptr root = NULL;
root = insert(root, 2);
root = insert(root, 8);
...
delete(root);
root = NULL;
```

Simple algorithms on binary trees

- Write a recursive function (max. 10 lines), that

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Simple algorithms on binary trees

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Simple algorithms on binary trees

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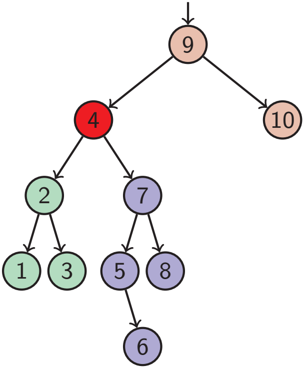
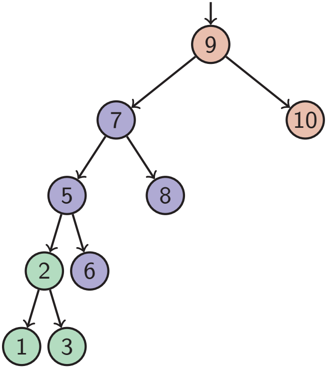
- Write a iterative function (max. 10 lines), that
  - computes the minimum and the maximum of the values stored in the nodes
  - returns the pointer to the node storing the maximal / minimal value of the tree

Deleting an element from a search tree – naively





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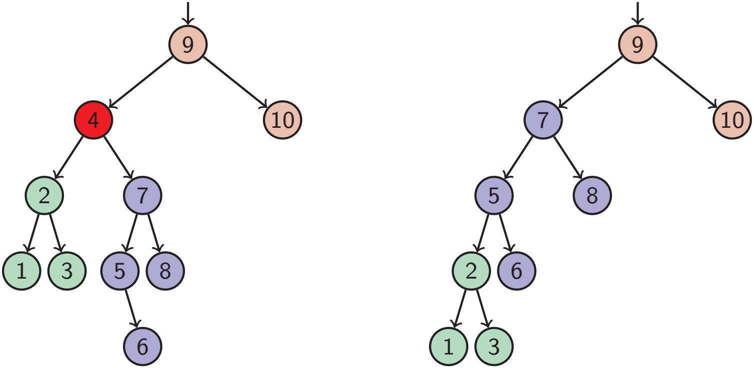
- The left sub-tree is inserted to below the minimal element of the right sub-tree

Deleting an element from a search tree – naively



- The right sub-tree is moved to the place of the deleted node
- The left sub-tree is inserted to below the minimal element of the right sub-tree
- The tree is getting imbalanced!

Deleting an element from a search tree – clever

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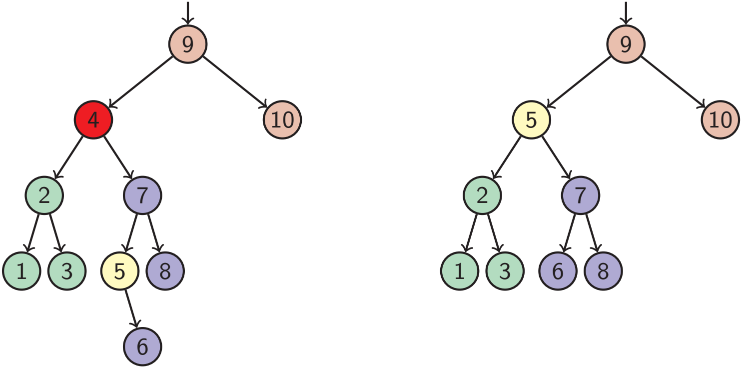

- The minimal element of the right sub-tree is moved to the place of the deleted node
- This element could have only a right sub-tree, it is moved one level up, to its old place

Morse decoding tree

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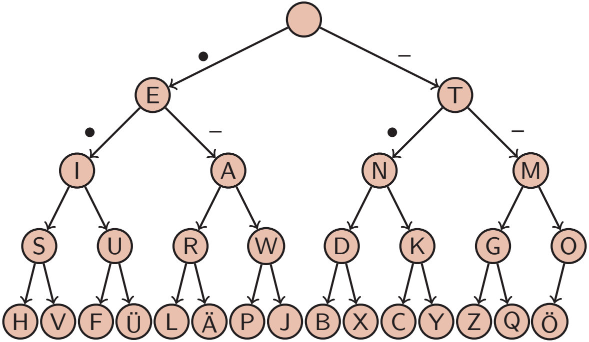
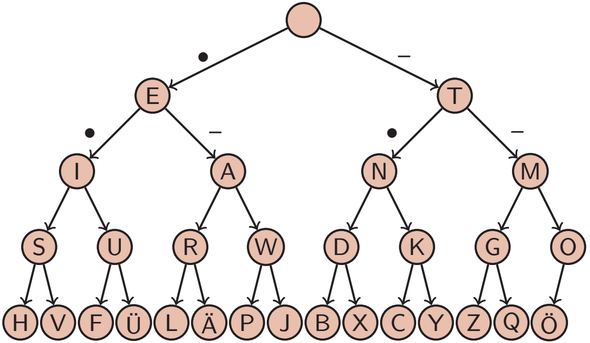


SOSOS: ●●● – – – ●●● – – – ●●●

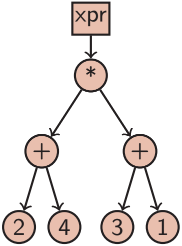
Morse decoding tree



SOSOS: ••• – – – ••• – – – •••

The response: •••• •– •••• •– •••• •–

Evaluating mathematical expressions



- Storing math expressions in a tree
- Leaves $\rightarrow$ numeric constants
- Branches $\rightarrow$ two-operand operators
- In the example: $(2+4)*(3+1)$

Evaluating mathematical expressions

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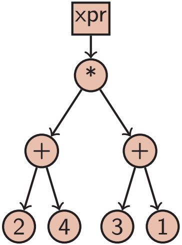

- Storing math expressions in a tree
- Leaves $\rightarrow$ numeric constants
- Branches $\rightarrow$ two-operand operators
- In the example: $(2+4)*(3+1)$

```
int eval(tree_ptr xpr)
{
  char c = xpr->data;
  if (isdigit(c))       /* stopping condition */
    return c - '0';
  if (c == '+')
    return eval(xpr->left) + eval(xpr->right);
  if (c == '*')
    return eval(xpr->left) * eval(xpr->right);
}
```

link

Evaluating functions

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Let us introduce variable x as a leaf node as well:

```c
double feval(tree_ptr xpr, double x)
{
  char c = xpr->data;
  if (isdigit(c))
    return c - '0';
  if (c == 'x')
    return x;
  if (c == '+')
    return feval(xpr->left, x) + feval(xpr->right, x);
  if (c == '*')
    return feval(xpr->left, x) * feval(xpr->right, x);
}
```

link

Evaluating the derivative of a function

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Let us take the derivative of the function! The rules are:

- $c' = 0$
- $x' = 1$
- $(f + g)' = f' + g'$
- $(f \cdot g)' = f' \cdot g + f \cdot g'$

```
double deval(tree_ptr xpr, double x)
{
  char c = xpr->data;
  if (isdigit(c))     /* stopping condition */
    return 0.0;
  if (c == 'x')      /* stopping condition */
    return 1.0;
  if (c == '+')
    return deval(xpr->left, x) + deval(xpr->right, x);
  if (c == '*')
    return deval(xpr->left, x) * feval(xpr->right, x) +
      feval(xpr->left, x) * deval(xpr->right, x);
}
```

link

Thank you for your attention.