Eliminating Rank Reversal Phenomenon in GRA–based Network Selection Method

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Abstract—In order to fulfill the users’ requirements, today’s mobile devices are equipped with multiple interfaces to make the connection possible to different type of networks. To select dynamically the best interface according to different network attributes, such as delay, bandwidth, cost, etc. must be taken into account. Grey Relational Analysis (GRA) is a promising algorithmic approach that can realize dynamic interface selection with multiple alternatives (interfaces) and attributes; however, similarly to some other decision methods, GRA also suffers from rank reversal phenomenon. In this paper alternative solutions are introduced to reduce and eliminate the probability of rank inconsistency, caused by the addition or deletion of an interface. The proposed methods are analyzed analytically and by simulations. Our results confirm that the proposed normalization methods significantly reduce, while one of it eliminates the rank reversal phenomenon.

Index Terms—Network selection, Multiple Attribute Decision-Making, Grey Relational Analysis, Rank reversal

I. INTRODUCTION

The next–generation communication systems can be considered as a composite communication model, where various access systems such as cellular (UMTS, WiMAX, etc.), satellite, Wireless Local Area Network (WLAN) and even wired networks is combined to provide services at any time from any where. In order to utilize the advantages of the heterogeneous network model, mobile devices are now being built as multithomed multi–functioning wireless terminals. In the competitive marketplace of Internet Service Providers (ISP) different network technologies with varying characteristics are available.

In order to keep mobile users always best connected (ABC) [1] numerous Multi Attribute Decision Making (MADM) methods have been utilized for network selection purposes. MADM algorithms are used for determining the ranking of alternatives in terms of their desirability with respect to multiple criteria that can influence the decision. The main goals of these solutions are to avoid frequent handover processes and offer acceptable QoS characteristics for different type of applications. The network selection algorithm must depend on the requested services and the throughput, delay, jitter, cost, signal strength etc. parameters of the reachable networks and be able to determine automatically the best interface that fulfills the user’s requirements.

Numerous MADM approaches, such as the simple additive weighting (SAW) method [2][3], the technique for order preference by similarity to ideal solution (TOPSIS) method [4], has been criticized for its possible rank reversal phenomenon, which means that the relative rankings of two decision alternatives could be reversed when a decision alternative is added or deleted. Rank reversal is a disadvantageous behavior of a network selection method that automatically switches to the best network, because rank inconsistency can cause frequent handovers. Each appearance of a new network or a link disappearance may trigger a handover process increasing delay, causing gap in the transmission or even disconnecting the terminals.

In the recent years a new network selection method has been studied based on Grey System Theory [5]. The theory has been proven to be useful for dealing with poor, incomplete, and uncertain information. Grey Relational Analysis (GRA) is part of Grey System Theory, which is suitable for decision making with complicated interrelationships between multiple factors and variables. GRA has been successfully applied in network selection process as well [6]–[9], however, it also suffers from the rank reversal phenomenon.

In this paper we propose a solution to eliminate the ordering inconsistency of GRA–based network selection algorithm. In order to avoid the effects of the rank reversal, we have analyzed the GRA–based network selection algorithm to find the reason of the phenomenon. We have found that modifying the normalization method, the probability of rank reversal can be significantly decreased or even eliminated. In order to justify our theoretical assumptions, we have examined the efficiency of the proposed solutions.

The rest of this paper is organized as follows. Review of related works in network selection decision methods and the background of GRA–based decision algorithm are presented in Section II. In Section III we introduce our solutions for reducing and eliminating the rank reversal phenomenon for GRA–based network selection. The obtained performance results are presented in Section IV. Finally, we summarize our paper and make the conclusions in the last section.

II. BACKGROUND AND RELATED WORKS

Numerous network selection methods were studied in the past decade. All these decision algorithms are used for determining the ranking of competitive network access possibilities and making the best choice for the user.

In the traditional methods such as [10]–[13], only the radio signal strength (RSS) threshold and hysteresis values are
considered. The simple RSS-based decision is not sufficient because it does not take into account the provided services and features of different access technologies.

Making the decision based on one parameter is usually inadequate, therefore Multi Attribute Decision Making (MADM) algorithms such as SAW, TOPSIS, AHP (Analytic Hierarchy Process), ELECTRE, etc. have been developed and described in [2]–[4], [14]–[18]. These solutions determine the preferred network using technology specific features such as bandwidth, delay, packet loss probability, cost of network usage, etc. Roveri et al. [19] used weight factors to reflect the features of different access technologies.

A robust MADM algorithm must ensure that the best alternative does not change when an alternative, which is not the best, is removed or replaced by another alternative. Therefore, if an algorithm suffers from the ranking abnormality problem, the ranking order is not stable.

A. Grey Relational Analysis (GRA)

Grey System Theory was introduced in [5] to analyze the relational grade for several discrete sequences and select the best sequence. One of the sequences is defined as reference sequence presenting the idea situation. The grey relationship between the reference sequence and the other sequences can be determined by calculating the Grey Relational Coefficient (GRC) according to the level of similarity and variability. The technique is appropriate and effective for network selection purposes as well. The GRA-based network selection method can be implemented following the steps below:

1. Classifying the network parameters (lower-the-better, higher-the-better)
2. Defining the upper and lower bounds of the parameters
3. Normalizing the parameters
4. Calculating the Grey Relational Coefficient (GRC)
5. Ranking the networks according to the GRC values

To calculate the GRC the network parameters must be categorized first. For delay, cost, etc. parameters the smaller-the-better, class is used, while other ones like throughput and signal strength parameters belong to the larger-the-better category. Before calculating the GRC for each parameter of the network, the data need to be normalized to eliminate dimensional units. Assuming that possible networks (\(S_j\), \(S_2\), ..., \(S_N\)) are compared, and each network has \(k\) parameters, the upper bound \((u_i)\) is defined as max\(\{s_1(j), s_2(j), ..., s_k(j)\}\), and the lower bound \((l_i)\) as min\(\{s_1(j), s_2(j), ..., s_k(j)\}\), where \(j = 1, 2, ..., k\). In case of smaller-the-better attribute, the normalized value of \(s(j)\) parameter can be calculated as follows:

\[
\hat{s}^*_j = \frac{u_j - s(j)}{u_j - l_j}
\]  

(1)

Similarly, the normalized value of a larger-the-better parameter:

\[
\hat{s}^*_j = \frac{s(j) - l_j}{u_j - l_j}
\]  

(2)

The attributes of a network can be represented as a row matrix, where the elements of the matrix are the normalized values of \(k\) different network attributes.

\[
S = \begin{bmatrix} s^*_1 & s^*_2 & s^*_3 & ... & s^*_k \end{bmatrix}
\]  

(3)

While \(s^*_j\) parameters are maximized in 1, the most preferable network can be always described as \(s^*_j = 1\), where \(j = 1, 2, ..., k\) and \(k\) is the number of network parameters used for the decision. Utilizing this behavior of the normalizing algorithm, the ideal network can be determined as \(S = [1 1 ... 1]\).

If there are \(N\) competing networks to choose from, the previous row matrix (3) can be extended to an \(N \times k\) matrix, which contains all the parameters that play role in the network selection procedure. The matrix can be determined as follows:

\[
S_g = \begin{bmatrix} s^*_1(1) & s^*_1(2) & s^*_1(3) & ... & s^*_1(k) \\
 & s^*_2(1) & s^*_2(2) & s^*_2(3) & ... & s^*_2(k) \\
 & & s^*_3(1) & s^*_3(2) & s^*_3(3) & ... & s^*_3(k) \\
& & & s^*_n(1) & s^*_n(2) & s^*_n(3) & ... & s^*_n(k) 
\end{bmatrix}
\]  

(4)

Each of the attribute in the matrix is calculated similarly to equation given in (1), if smaller-the-better network attribute is analyzed. If larger-the-better parameter is examined, equation defined in (2) is used to determine the \(N \times k\) matrix elements.

The final step of the GRA-based network selection algorithm is to calculate the Grey Relational Coefficient (GRC). The value of the GRC parameter is calculated by the following equation, where \(w_j\) is the weight of each parameter and \(i (1 \leq i \leq N)\) is the network index:

\[
GRC_i = \frac{1}{\sum_{j=1}^{k} w_j |s^*_j(i) - 1| + 1}
\]  

(5)

The network with the largest GRC is the most desirable one.

B. Rank Reversal Phenomenon in MADM algorithms

Most of the MADM-based ranking algorithms use normalization and upper/lower bounds determinations of the network parameters. Rank reversals in the SAW, TOPSIS, AHP, GRA methods are caused by the changes of normalized attribute values. Some proposals [20],[21] were presented to avoid rank reversal in AHP and TOPSIS, while Barzilai and Golany [22] have proved that no normalization can prevent rank reversal. However, normalization is often necessary for most of the MADM approaches so that different dimensional units can be eliminated. GRA also suffers from the rank reversal as the following example shows. The attributes and its
According to the normalization equations (1) and (2) of the GRA–based decision making, we can find 0 and 1 normalized values for each network attribute. Normalized value 1 means that the network offers the best performance from the given parameter point of view, while 0 means the opposite. By calculating the GRC (5) the ranking order can be determined. In the example presented in Table I, network #3 is the best choice and #2 is the worst one, if the weights of the attributes are considered equal.

Supposing that the worst network (#2) is not reachable anymore, the normalized values and calculated GRCs will be changed as shown in Table II.

### TABLE II

<table>
<thead>
<tr>
<th>Network</th>
<th>Delay (Ms)</th>
<th>Jitter (ms)</th>
<th>Loss rate (%)</th>
<th>Throughput (Mbps)</th>
<th>Cost ($/MB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.25</td>
<td>0.15</td>
<td>0.18</td>
<td>1.0</td>
<td>0.25</td>
</tr>
<tr>
<td>#2</td>
<td>0.23</td>
<td>0.14</td>
<td>0.17</td>
<td>1.0</td>
<td>0.24</td>
</tr>
<tr>
<td>#3</td>
<td>0.24</td>
<td>0.16</td>
<td>0.16</td>
<td>1.0</td>
<td>0.25</td>
</tr>
</tbody>
</table>

After the removal of network #2 the best network was changed to network #1; however, the network parameters were not changed.

### III. ALTERNATIVE NORMALIZATION METHODS FOR GRA–BASED NETWORK SELECTION

Using Grey Relational Analysis (GRA) for network selection purposes has been presented and studied in the recent years. Similarly to other solutions GRA also suffers from rank reversal phenomenon. It can be easily proved that the reason of this observable fact is the normalization method. The result of the Grey Relational Coefficient (GRC) equation (5) will change only if the normalized network attributes also changes, however, we assume that the network parameters are constant.

Although the network parameters, such as bandwidth, delay, loss rate, etc. are considered constant, the normalized value of these attributes can be changed if one of the network is not reachable anymore. This can happen if one of the disappearing networks attributes has the highest or lowest value. In this case the attribute order will not change of course, but the distance between the normalized values may vary significantly as the example bellow shows.

As presented in Fig.1, the normalized values calculated with equation (1) and (2) significantly differ, when the highest element was removed. While the difference between the two normalized values in the middle is only 0.18-0.15=0.03, after the removal of the highest attribute, the difference becomes 0.72-0.6=0.12. Although, the real data was not changed, the significant variance of the normalized values can modify the rank order of the networks.

Our main goal was to reduce the effect of the normalization on the rank order. We propose three different methods to replace the actual normalization procedure described by equations (1) and (2).

Our first normalization solution keeps the normalized values unchanged by determine absolute min-max values of an attribute. By keeping the normalized values unchanged the calculated GRC will be the same even if a network is not reachable anymore, therefore the rank reversal phenomenon is eliminated. The modified normalized value of a smaller-the-better attribute can be calculated as follows:

\[ s'_i(j) = \frac{E_{\text{max}_j} - s_i(j)}{E_{\text{max}_j} - E_{\text{min}_j}} \]  \tag{6}

Similarly, the normalized value of a larger-the-better parameter:

\[ s'_i(j) = \frac{s_i(j) - E_{\text{min}_j}}{E_{\text{max}_j} - E_{\text{min}_j}} \]  \tag{7}

The variable \( E_{\text{min}_j} \) and \( E_{\text{max}_j} \) stand for absolute minimum and maximum of the \( j^{th} \) network parameter, respectively. The \( j^{th} \) parameter value therefore must be always between \( E_{\text{min}_j} \) and \( E_{\text{max}_j} \). For absolute minimum, \( E_{\text{min}_j} = 0 \) can be an adequate option, while for absolute maximum (\( E_{\text{max}_j} \)), the given network attribute behavior must be analyzed.

The second modified normalization method is similar to the previous one, but it uses absolute bound only in the unwanted direction of an attribute. In case of smaller-the-better attribute an absolute maximum is determined, while for larger-the-better parameter an absolute minimum is used. The reason of this solution is that usually those networks are becoming

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**Table I:**

<table>
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<tr>
<th>Network</th>
<th>Delay (Ms)</th>
<th>Jitter (ms)</th>
<th>Loss rate (%)</th>
<th>Throughput (Mbps)</th>
<th>Cost ($/MB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.33</td>
<td>0.1</td>
<td>0.11</td>
<td>0.79</td>
<td>0.978</td>
</tr>
<tr>
<td>#2</td>
<td>0.34</td>
<td>0.14</td>
<td>0.16</td>
<td>0.85</td>
<td>0.1</td>
</tr>
<tr>
<td>#3</td>
<td>0.34</td>
<td>0.15</td>
<td>0.17</td>
<td>0.85</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Table II:**

<table>
<thead>
<tr>
<th>Network</th>
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<td>0.16</td>
<td>0.16</td>
<td>1.0</td>
<td>0.25</td>
</tr>
</tbody>
</table>
unreachable whose attributes are bad. For example, in case of larger-the-better attribute (e.g. signal strength, throughput) that network will disappear whose signal strength and/or throughput become very low. A network with high signal strength and throughput probably will not become unreachable, therefore need to use only absolute minimum needs to be used in these cases in order to reduce the rank reversal probability.

According to the previous assumption, the modified normalized value of a smaller-the-better attribute can be calculated as follows:

$$s'_j(j) = \frac{E_{\text{max}j} - s_j(j)}{E_{\text{max}j} - l_j} \quad (8)$$

Similarly, the normalized value of a larger-the-better parameter is given by:

$$s'_j(j) = \frac{s_j(j) - E_{\text{min}j}}{u_j - E_{\text{min}j}} \quad (9)$$

In the above equations $E_{\text{min}j}$ and $E_{\text{max}j}$ are the absolute minimum and maximum of the $j^{th}$ network parameter that must be determined before the network selection procedure starts. Variable $u_j$ is the maximal value of the $j^{th}$ parameter, and it is defined as $\max\{s_1(j), s_2(j), ..., s_n(j)\}$. The lowest value ($l_j$) is defined as $\min\{s_1(j), s_2(j), ..., s_n(j)\}$.

The secondly introduced normalization technique will not eliminate rank reversal, because the normalized value of a parameter can still change if the best network becomes unreachable. We can assume that in most of the cases the best network will not disappear unexpectedly, therefore the normalized values and the calculated GRC probably will not be changed.

The disadvantage of absolute min-max values is that they must be defined before the network selection starts. If the absolute maximum is significantly higher than the maximal value of the $j^{th}$ parameter ($u_j$) and the absolute minimum is significantly lower than the minimal value of the $j^{th}$ parameter ($l_j$) the distance of calculated normalized values can be very small. If the normalized values are very close, the calculated GRCs, used for the ranking, will be also close to each other.

Using this way of normalization, neither the normalized value of the best attribute will be equal to 1, nor will the normalized value of the worst attribute be equal to 0.

In our third normalization solution our aim was to avoid the usage of absolute min-max values. Similarly to the previous solution we proposed that those networks will become unreachable whose parameters are worse. We have constructed the normalization function in such a way that the normalized value of the best parameter be equal to 1.

The proposed normalization function of a smaller-the-better attribute is as follows:

In order to analyze the efficiency of different normalization methods from the rank reversal phenomenon point of view, we have implemented a simulator tool. Rank reversal depends on the network parameters, disappearing network and normalization method. To compare the different solutions, we have analyzed the frequency of rank order changes caused by network removals. The highest probability of rank reversal can be reached if the worst network is removed, since the disappearance of the worst network is the most realistic.

In our simulator tool we used random variables as network parameters

<table>
<thead>
<tr>
<th>TABLE III</th>
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<tbody>
<tr>
<td>NORMALIZATION TECHNIQUES FOR GRA–BASED NETWORK SELECTION</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technique</th>
<th>Original</th>
<th>Norm 1</th>
<th>Norm 2</th>
<th>Norm 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>smaller-the-better</td>
<td>$u_j - s_j(j)$</td>
<td>$E_{\text{max}j} - s_j(j)$</td>
<td>$E_{\text{max}j} - u_j$</td>
<td>$l_j$</td>
</tr>
<tr>
<td>larger-the-better</td>
<td>$s_j(j) - l_j$</td>
<td>$E_{\text{max}j} - E_{\text{min}j}$</td>
<td>$s_j(j) - E_{\text{min}j}$</td>
<td>$s_j(j)$</td>
</tr>
</tbody>
</table>

In the previous section three different normalization techniques for GRA–based network selection decision were presented. Table III summarizes the original and the proposed solutions.
parameters without weights. According to the defined steps of the GRA method, we have normalized the network parameters (original, norm_1, norm_2, norm_3) and calculated the GRC (Grey Relational Coefficient). As next step we removed the worst network (network with the lowest GRC) and started the GRA process from the beginning. As last step we checked whether the selected network before and after the removal was the same.

We have iterated the described simulation method $10^6$ times in order to have statistically correct results. The ratio of total number of iterations and the number of different decisions was calculated, which can be considered as the probability of rank reversal. Fig.4 shows the obtained results.

![Fig. 4. Measured rank reversal probability in case of 5 network attributes](image)

The number of networks in the simulation was 3 to 10, while the number of network parameters was 5. As the results show, the original GRA performance was very bad. In the case when 5 or less networks were reachable and the worst network was removed, the probability of rank reversal was about 25%. Compared to the original GRA method, our techniques (norm_2, norm_3) reduced the rank abnormality by 65% to 99.9% as shown in Fig.4. Using normalization method with absolute min-max values (norm_1), the rank reversal phenomenon was eliminated. With norm_2 and norm_3, the probability that the best network will change after the worst network removal is significantly lower. The performance of norm_2 is very similar to norm_3 technique.

As it can be observed, the number of accessible networks has impact on the rank reversal probability. By increasing the number of available networks the analyzed ratio decreases, because the probability that the maximal ($u_i$) and minimal ($l_j$) values will change is lower when the worst network is removed.

We have analyzed the relation between the number of parameters and the rank reversal probability. In the simulations we changed the number of attributes from 2 to 10. The results of our simulations are presented in Fig.5 and Fig.6. In Fig.5, the efficiency of the original GRA method is presented. As can be seen, in the worst cases the rank reversal probability can reach even 40%. Using norm_2 and norm_3 technique the rank reversal ratio is decreased. Fig. 6 depicts the performance of norm_2. The performance of norm_3 technique is almost the same.

![Fig. 5. Rank reversal probability of original GRA–based network selection decision](image)

The impact of the number of network attributes is also observable. By increasing the number of network attributes used for decision making, the rank reversal probability increases. When there are more parameters that must be taken into account, the possibility that the removed network has highest or lowest attributes, will be higher.

![Fig. 6. Rank reversal probability of GRA–based network selection decision with norm_2 normalization technique](image)

The results confirm that by using absolute minimal and maximal values defined before for network selection, we can significantly decrease or even eliminate the possibility of the rank reversal phenomenon.

V. CONCLUSIONS

The selection of an optimal network for service delivery became an important issue. MADM algorithms are popular decision making tools, however, rank reversal phenomenon occurs in most of these algorithms. Ranking abnormality can potentially decrease the quality of the results by causing handovers whenever the list of the available networks
changes. In this paper we focused on GRA–based decision making algorithm and showed that using adequate normalization techniques will significantly reduce or even eliminate the rank reversal probability. We have proposed three different normalization solutions to minimize the probability of normalized value changes, when a network becomes inaccessible. To do this, we used absolute min-max values in two of our proposed normalization scheme. The efficiency of the presented normalization methods for GRA–based networks selection decision has been analyzed by simulations. The obtained results show that in case of norm_2 and norm_3 technique we could reduce the rank reversal frequency with 65% to 99.9% when 5 parameters was used for the decision, however, the ratio is similar when the number of network attributes differs from 5. Using norm_1 with absolute min-max values, the unwanted phenomenon was eliminated. As a future work, we would like to analyze the introduced normalization techniques in other MADM algorithms, such as TOPSIS or SAW.

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