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Algorithms for the global design of WDM networks including the traffic grooming

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Abstract

In this paper, we propose a model and algorithms for the global design problem of wavelength division multiplexing (WDM) networks including the traffic grooming. This problem consists in finding the number of fibres between each pair of nodes (i.e. the physical topology), finding the number of transponders at each node, choosing the set of lightpaths (i.e. the virtual topology), routing these lightpaths over the physical topology and, finally, grooming and routing the traffic over the lightpaths. Since this problem is NP-hard, we propose two heuristic algorithms and a tabu search metaheuristic algorithm to find solutions for real-size instances within a reasonable amount of computational time.

Keywords: Wavelength division multiplexing (WDM) networks; Network design; Traffic grooming; Virtual and physical topologies; Mathematical model; Tabu search

1. Introduction

The wavelength division multiplexing technology is widely deployed over different optical networks such as synchronous optical network (SONET) to fully exploit the important optical fibre bandwidth by using multiple wavelengths over a single fibre. This leads to a huge fibre bandwidth, over a terabit per second, available in each fibre. Since, nowadays, a connection between two nodes has rarely more than a gigabit per second, this bandwidth is outsized compared to the requests. This under-exploitation may lead to a premature saturation of the network.

Traffic grooming can be an appropriate solution to bandwidth management. It is a well-known traffic engineering technique which packs low-speed traffic streams into high-capacity optical channels. The traffic streams are composed of connections and each connection is a traffic flow of a certain rate (e.g. OC-3 and OC-12) between two nodes. Thus, the traffic grooming permits grouping of several connections, possibly of different origin-destination node pairs into the same lightpath. This aggregation is allowed by space, frequency and time-division multiplexing. An aggregation involving these levels or granularities is called full grooming. A node performing switching within only its optical devices is a transparent node and a node performing switching only at the electronic level is called opaque. A hybrid node is a translucent node, it is a able to perform switching at the optical

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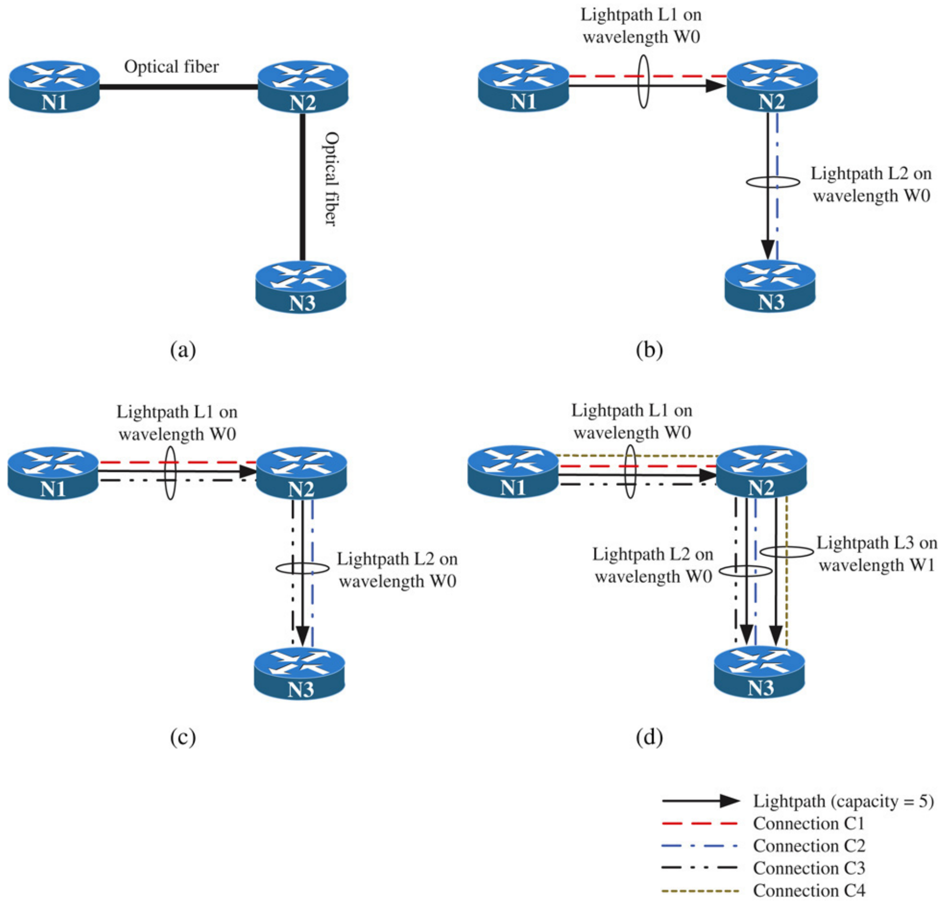

Fig. 1. Illustration of the traffic grooming mechanism.

Table 1
Number of reception and transmission transponders for each node

Node	Number of reception transponders	Number of transmission transponders
N1	1	1
N2	3	3
N3	2	2

or electronic levels. For more details concerning the optical networks, see Goralski [4] and Laude [8].

In order to illustrate the traffic grooming mechanism, an example is provided in Fig. 1. Tables 1 and 2 present respectively the number of transponders for each node and the characteristics of the connections for the example. Fig. 1(a) illustrates the physical topology and Fig. 1(b) illustrates the starting virtual topology where the connection C1 uses the lightpath L1 on the wavelength W0 between the nodes N1 and N2 and the connection C2 uses the lightpath L1 on the wavelength W0 between the nodes N2 and N3. In the case of a network with no grooming facilities, if a connection between the nodes N1 and N3 is requested, this connection is rejected because the lightpath L1 is already in use and all transmission transponders are used at node N1 (see Table 1). This is not the case if the network performs traffic grooming as illustrated in Fig. 1(c) where the connection C3 is groomed with the connection C1 on the lightpath L1 and with the connection C2 on the lightpath L2. Moreover, consider that an additional connection of two capacity units is requested between the nodes N1 and N3. If the node N2 does not perform traffic grooming at the wavelength level, the connection C4 cannot be routed because lightpath L2 does not have enough spare capacity and the wavelength W0 is already used. Otherwise, if the node N2 has full grooming facilities, an additional lightpath L3 may be established between the nodes N2 and N3 on the wavelength W1. Thereby, the connection C4 can be groomed with connections C1, C2 and C3 on the lightpath L1, then

Table 2
Characteristics of the connections

Connection	Origin node	Destination node	Demand
C1	N1	N2	1
C2	N2	N3	2
C3	N1	N3	2
C4	N1	N3	2

it arrives to node N3 on the lightpath L3, as illustrated in Fig. 1 (d).

Several articles are focused on the traffic grooming problem. For a review on traffic grooming related papers, see the paper by Zhu and Mukherjee [12]. Moreover, Zhu and Mukherjee [13] propose a mathematical model for the traffic grooming problem with the objective of maximizing the network throughput. They propose heuristics based on the following problem decomposition: finding the virtual topology, routing the lightpaths, assigning the wavelengths to the lightpaths and, finally, routing the connections. Zhu and Mukherjee [14] propose to model each node using four layers: the access, the lightpath, the wavelength and grooming facilities layers. They suggest an algorithm that builds an auxiliary graph describing the set of nodes and functionalities. A similar approach is explored by Xiang et al. [11] to maximize the single hop traffic grooming. Bouabdallah et al. [1] propose a new concept called distributed aggregation. It consists in grouping into a single lightpath connections from different sources but with the same destination. Another problem decomposition is proposed by Hu and Leida [6]. They solve the grooming and routing subproblems exactly and use a heuristic for the wavelength assignment subproblem. Prathombutr et al. [10] propose an evolutionary algorithm for the traffic grooming problem with multiple objectives. It finds a pareto optimal set by performing population initialization, evolution, selection, crossing and mutation operations.

In this paper, we address the global problem of designing WDM networks that includes the traffic grooming. It consists in finding the number of fibres between each pair of nodes (i.e. the physical topology), finding the number of transponders to install at each node, choosing the set of lightpaths (i.e. the virtual topology), routing these lightpaths over the physical topology and, finally, grooming and routing the traffic over the lightpaths. The literature contains many articles relating to traffic grooming but this global problem has not been considered before. Moreover, the focus of this paper is to provide a general framework by considering the use of many fibres per link, many wavelengths per fibre and many grooming granularities.

This paper is organized as follows. Section 2 presents the mathematical model for the global problem of designing WDM network including the traffic grooming. In Section 3, we propose two heuristic algorithms and a tabu search metaheuristic algorithm. Numerical results are presented and discussed in Section 4 and finally, conclusions follow in Section 5.

2. The global WDM network design problem

2.1. Problem formulation

For the global WDM network design problem, the following information is considered known: (I1) the location of the optical nodes; (I2) the origin-destination connection demand (i.e. the number of OC-3, OC-12, etc.) between each pair of nodes; (I3) the maximum number of optical fibres that can be installed between each pair of nodes; (I4) the maximum number of wavelengths that can be used into a fiber; (I5) the cost of the links between each pair of nodes as a function of the number of fibres including the installation cost (for the patch panels, the patch cords, the labour, etc.); (I6) the cost of the transponders including the installation cost.

We also make the following assumptions concerning the organization of the network: (A1) a link can be installed between each pair of nodes; (A2) a link is composed of one or more fibres; (A3) the number of fibers installed between two nodes cannot exceed the maximum allowed; (A4) an optical fibre can contain multiple wavelengths; (A5) the number of wavelengths used in a fibre cannot exceed the maximum allowed; (A6) each node have full grooming facilities; (A7) an optical cross-connect is installed in each node; (A8) there is no wavelength conversion in the network; (A9) a connection can use more than one lightpath (multi-hop routing); (A10) a connection stays packed during its transport.

The global WDM network design problem consists in finding:

- the number of optical fibres between each pair of nodes (i.e. the physical topology);
- the number of transponders to install at each node;
- the set of lightpaths (i.e. the virtual topology);

as well as:

- routing the lightpaths over the physical topology;
- grooming the traffic and finally;
- routing the traffic over the virtual topology.

The objective is to minimize the cost of the network.

There is a strong interaction between these subproblems and several of them are NP-hard [3] (e.g. the traffic grooming [9] and the wavelength assignment subproblems [2]).

2.2. Notation

The following notation is used throughout this paper. The notation is composed of sets, decision variables, constants and cost parameters.

Sets:

- N, the set of optical nodes;
- T, the set of connection types (e.g. OC-3 and OC-12);
- Ω, the set of wavelengths that can be used on each optical fibre.

Decision variables:

- w_{mn}, the number of lightpaths between nodes m and n;
- x_{ij}, a 0–1 variable such that $x_{ij} = 1$ if and only if node i is connected to node j with a link composed of one or more fibres;
- x_{ij}^{f}, a 0–1 variable such that $x_{ij}^{f} = 1$ if and only if the fibre f is used from node i to node j;
- $x_{ij}^{f\omega}$, a 0–1 variable such that $x_{ij}^{f\omega} = 1$ if and only if wavelength ω is used in fibre f from node i to node j;
- $x_{ijmn}^{f\omega}$, a 0–1 variable such that $x_{ijmn}^{f\omega} = 1$ if and only if wavelength ω is used on fibre f from node i to node j for a lightpath from node m to n;
- y_{mn}^{odkt}, a 0–1 variable such that $y_{mn}^{odkt} = 1$ if and only if the connection k of type t between the origin o and the destination d uses a lightpath between the nodes m and n;
- z^{odkt}, a 0–1 variable such that $z^{odkt} = 1$ if and only if the connection k of type t, between the origin o and the destination d is established.

Constants:

- α^{odt}, the number of connections of type t requested between the nodes o and d;
- β_{ij}, the maximum number of optical fibres that can be installed between nodes i and j;
- δ^{t}, the capacity needed for a connection of type t (i.e. the number of OC-1 for a connection of type t);
- ϵ, the capacity of a wavelength (i.e. the number of OC-1 that a wavelength can carry).

Cost parameters:

- $a_{ij}(n)$, the link cost for n optical fibres between nodes i and j including the installation cost;
- b_i, the transponder cost including the installation cost at node i.

2.3. The mathematical model

The model for the global WDM network design problem, denoted GWNDP, is presented in Appendix.

3. Solution algorithms

In this section, we propose approximate solution algorithms: two heuristic algorithms and a tabu search metaheuristic algorithm for GWNDP.

3.1. Heuristic algorithms

The main steps of the heuristic algorithms (denoted H1 and H2) are: (1) construct a physical topology; (2) construct a virtual topology; (3) route the requests in single hop over the virtual topology and (4) route the remaining requests in multi-hop.

The heuristic H1, partially inspired by [13], takes into account the traffic between the pair of nodes to build the physical and the logical topologies. The heuristic H2 builds a lightpath on each wavelength into each fibre.

Let *floor* be the minimum value of a traffic to be exchanged to set up a fibre between a pair of nodes, volume(k), the volume of the requests of the node pair k and *LP_list*, the list of the lightpaths containing the remaining capacity.

In the heuristics H1 and H2, we also use the function sort() used for sorting the node pairs k in the decreasing order of the volume of remaining requests.

Heuristic H1

Step 1: (Physical topology construction)
  For all node pairs k from i_k to j_k do
  1.1 For $n := \beta_{i_k j_k}$ to 1 do
    If volume(k) $\geq n \times$ floor, build n fibres between the nodes i_k and j_k and go to 2.1.

Step 2: (Virtual topology construction)
  2.1 Set *LP_list* $:= \emptyset$, call the function sort() and set $k := 0$.
  2.2 While volume(k) > 0 and $k < |N|(|N|-1)$ do
    2.2.1 If the links on shortest path found using Dijkstra (at the physical level) from i_k and j_k have enough resources to create a lightpath do
      2.2.1.1 Set up a lightpath from i_k to j_k, update volume(k) $:=$ (volume(k) $- \epsilon)^{+}$, set $k :=$ 0, call the function sort() and update the LP_list.
    Otherwise set $k := k + 1$.

Step 3: (Traffic routing over the virtual topology — single hop)

3.1 Call the function sort().

3.2 For all the lightpaths

3.2.1 For $t := |T|$ to 1 do

If a request of type t exists between the source and the destination of the lightpath and its remaining capacity is sufficient, route this request over the lightpath, update *LP_list*, call the function sort() and set $t := |T|$.

Step 4: (Traffic routing over the virtual topology — multi-hop)

4.1 Set $k := 0$.

4.2 While $k < |N|(|N| - 1)$ do

4.2.1 If volume(k) > 0 and no shortest path is found using Dijkstra (at the lightpath level), set $k := k + 1$ and go to 4.2.

4.2.2 If volume(k) > 0 and a shortest path is found with Dijkstra (at the lightpath level) do

4.2.2.1 For $t := |T|$ to 1 do

If a request of type t exists between this node pair and the remaining capacity is sufficient, route this request over the virtual path, update *LP_list* and set $k := 0$ and $t := 0$.

4.2.2.2 If no request is routed between this node pair, set $k := k + 1$ and go to 4.2.

Note that a shortest path found in Step 4 is defined as a set of lightpaths between the source and the destination. If more than one lightpath exits between a source and a destination, only the lightpath containing the largest remaining capacity is used in the adjacency table of Dijkstra.

For the heuristic H2, we replace the Step 2 of H1 by the Step 2′ presented below.

Heuristic H2

Step 2′: (Virtual topology construction)

2.1 For all the fibres (i, j) and wavelengths on this fibre, create one lightpath from i to j or from j to i.

3.2. Tabu search algorithm

In this section, we propose a tabu search (TS) metaheuristic algorithm for GWNDP, denoted TS. The basic principle of the tabu search is to define a set of possible solutions, and starting from the current solution, to find a better one in its neighbourhood. A neighbourhood is a set of solutions that are found by applying an appropriate transformation of the current solution. In order for the algorithm to move away from a local minimum, the search allows moves resulting in a degradation of the objective function value, thus avoiding the trap of local optimality. To prevent the search from cycling, solutions obtained recently and moves that reverse the effect of recent moves are considered tabu. For more details about tabu search, see Glover and Laguna [5].

The advantage of TS is that several local minima are explored during the search process which is not the case with, for instance, greedy heuristics.

3.3. Neighbourhood, tabu moves and aspiration criterion

Let s be a solution. The neighbourhood of s consists of the solutions obtained by performing all possible moves. Each move consists of adding or removing a single fibre. As a result, the number moves to explore all the neighbourhood is in $O(|N|(|N| - 1))$.

Once a solution is chosen in the neighbourhood to be the next current solution, the affected link is declared tabu for a number of iterations randomly determined according to a uniform discrete distribution on the interval $[L, U]$.

The aspiration criterion can be described as follows. If the use of a tabu link allows us to find a solution of cost lower than any other solution found so far, we remove the tabu on this link.

3.4. Cost function

The cost of a solution is evaluated using the Eq. (A.1) plus a penalty factor for the blocked requests. Thus, the cost function is

$$\sum_{i\in N}\sum_{\substack{j\in N\\ i\neq j}} a_{ij}\left(\sum_{f=1}^{\beta_{ij}} x_{ij}^{f}\right) + \sum_{m\in N}\sum_{\substack{n\in N\\ m\neq n}} (b_m + b_n)\, w_{mn} + \gamma \sum_{o\in N}\sum_{\substack{d\in N\\ o\neq d}} \frac{a_{od}(1)\theta^{odt}\delta^{t}}{\epsilon|F_{od}|} \tag{1}$$

where θ^{odt} is the number of blocked requests of type t from node o to d and γ a penalty factor.

3.5. The tabu algorithm

We now proceed to the detailed description of algorithm TS.

Algorithm TS

Step 1: (Find an initial solution)
Find an initial topology (using for instance the heuristic algorithm H1 or H2 or by considering a fully-meshed topology).

Repeat Steps 2 *to* 3 *for Max_Iter iterations*

Step 2: (Exploring the neighborhood)

2.1 Determine the best move, according to the cost of the resulting solution, while taking into account the tabus and the aspiration criterion. For each move (i.e. the modification of the number of fibres on a link), we find the solution for a fixed topology using heuristic H1 or H2 (i.e. without the physical topology construction step, i.e. the Step 1). The cost of the solution is given by (1).

2.2 Evaluate the tabu for the new transformation, i.e. determine the number of iterations (according to a uniform discrete distribution in the interval $[L, U]$) for which the link used to perform the best move (determined in Step 2.1) is tabu.

Step 3: (Update the best solution)
Retain the current solution as the best solution if its cost is lower than the best solution obtained so far.

In the algorithm TS, the neighbourhood can be fully explored or randomly explored. In the latter case, the percentage of the neighbourhood explored should be specified in the algorithm. Other parameters should be fixed, e.g. the number of iterations (i.e. the Max_Iter parameter) and the interval $[L, U]$.

4. Numerical results

All algorithms are programmed in the C language on a Sun Java workstation under Linux with a AMD Opteron 150 CPU and 2 GB of RAM.

Table 3 presents the cost of the optical network components. Note that for each lightpath, two transponders are needed. Table 4 presents the (x, y) coordinates of the nodes and Table 5 presents the technological parameters. Full grooming is considered and the capacity of each wavelength was set to OC-48.

In this paper, the notation 4/2/0.1 is related to the traffic profile and it means that between two nodes there is a connection demand of up to four OC-3, up to two OC-12 and a probability of 0.1 to have a demand for an OC-48. For this last demand, for each directed pair of nodes, we generate randomly a value between 0 and 1. If this value is less than 0.1, a demand of a whole lightpath (one OC-48) is set between the source and the destination of the current pair of node. Four traffic profile scenarios are considered and presented in Table 6.

For the heuristics H1 and H2, the parameter γ was set to 100 and *floor* to zero. It means that the tabu search begins with a fully-meshed physical topology. For the tabu search algorithm, the best move at each iteration of the search is considered tabu for a number of iterations found randomly in the interval [1,5]. The number of iterations of the search was set to 100.

First, the performance of heuristics H1 and H2 is tested with a fixed physical topology. Fig. 2 presents the network tests N1 and N2. For each link, two fibres are

Table 3
Cost of the network components

Component	Cost
One-fibre link	3750\$/km
Two-fibre link	6250\$/km
Multirate transponder	20 000\$

Table 4
Coordinates of the optical nodes

Node	x (km)	y (km)
1	0	600
2	740	840
3	2080	620
4	3500	0
5	3800	180
6	3960	200

Table 5
Technological parameters

Parameter	Value
$\vert F_{ij}\vert$	2
$\vert \Omega\vert$	2

Table 6
Demand scenarios

Scenario	Traffic profile
1	2/2/0.1
2	4/2/0.1
3	4/4/0.1
4	4/4/0.2

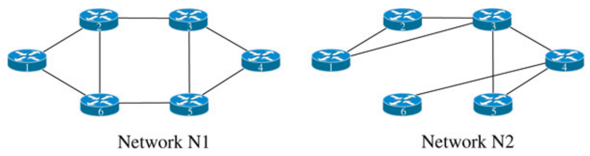

Fig. 2. Networks of the tests.

Table 7
Performance of the heuristics

Network	Scenario	Blocked requests with H1	Blocked requests with H2
N1 (8 links)	1 (138 OC-3)	0.0% (18 LPs)	11.6% (32 LPs)
	2 (189 OC-3)	9.0% (20 LPs)	11.6% (32 LPs)
	3 (231 OC-3)	10.8% (20 LPs)	14.7% (32 LPs)
	4 (321 OC-3)	25.8% (19 LPs)	21.2% (32 LPs)
N2 (7 links)	1 (138 OC-3)	4.3% (16 LPs)	14.5% (28 LPs)
	2 (189 OC-3)	9.0% (17 LPs)	11.6% (28 LPs)
	3 (231 OC-3)	14.3% (18 LPs)	13.4% (28 LPs)
	4 (321 OC-3)	35.5% (15 LPs)	27.4% (28 LPs)

Table 8
Average results from the tabu search combined with H1 and H2

Scenario	TS–H1 cost (k$)	TS–H2 cost (k$)	TS–H1 GAP (%)	TS–H1 GAP (%)
1	40 416	35 774	33.7	18.3
2	40 930	38 415	20.0	12.7
3	52 775	54 410	4.2	7.4
4	65 898	60 602	13.8	4.7

installed. The results of the heuristics are presented in Table 7.

It can be observed that heuristic H2 provides better results when the volume of traffic is important. This can be explained by the fact that when the volume of traffic is larger, while the physical topology remains fixed, the number of the requests that have to be routed in multi-hop, increases. However, because H2 builds more lightpaths, it offers more routing possibilities and better solutions are obtained.

To observe and measure the results of the tabu search algorithm, a five-node network is considered. Five instances are randomly generated and tested for each demand scenario. The results are presented in Table 8. The results are compared to the optimal solutions presented in Table 9 found using the CPLEX Mixed Integer Optimizer 9.0 [7] to solve the mathematical model (see Appendix). For a general resolution of the model involving the selection of the physical topology, we were not able to find optimal solutions for a network sizes larger than five nodes in a reasonable time. Note that the resolution time with CPLEX is large (the resolution time is 178 901 s for the last scenario as illustrated in Fig. 3). That is why we considered only a five-node test network with CPLEX.

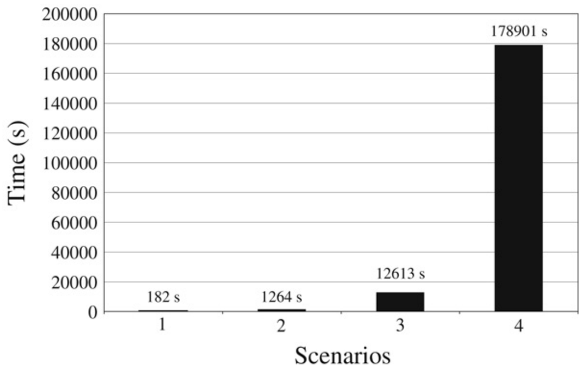

Fig. 3. CPLEX resolution time.

Table 9
Average results from CPLEX

Scenario	Cost (k$)
1	30 228
2	34 089
3	50 643
4	57 886

As can be gathered from the tables, in general, the tabu search combined with H2 performs better than when it is combined with H1. Also, a tendency can be observed concerning the relation between the GAP and the traffic growth for TS–H1 and TS–H2. In fact, with CPLEX, the average proportion of multi-hop routed requests is 49.5%, 48.4%, 29.2% and 23.0% for respectively scenario 1, 2, 3 and 4. H1 and H2 perform better in the two latest scenarios since both algorithms take advantage of single-hop routed requests. However, in average, low gap solutions can be found with the proposed approach.

In addition to the five-node network, networks of 10, 15 and 20 nodes are tested with the four scenarios. The tabu search algorithm is compared to a local search (LS). Both algorithms begin with a fully-meshed physical topology and have the same neighbourhood structure and cost function. For the TS algorithm, the number of iterations was set to 1000.

The results are presented in Table 10. In the provided solutions, all the requests are routed. Once again, the use of H2 provides in general better results than H1. Also, the GAP between the local and the tabu search algorithms can exceed 16% since the TS is "more sophisticated" than the LS. This GAP is more significant with H1.

Table 10
Results from LS and TS algorithms combined with H1 and H2 for different network sizes

Nework size	Scenario	Cost (k$) LS–H1	LS–H2	TS–H1	TS–H2
5	1	45 953	35 774	40 416	35 774
	2	41 872	39 308	40 930	38 415
	3	58 364	56 366	52 775	54 410
	4	67 111	60 602	65 898	60 602
10	1	128 552	103 826	113 283	100 192
	2	145 078	116 797	131 203	110 711
	3	180 156	158 748	173 388	155 197
	4	264 173	220 372	220 080	216 071
15	1	255 467	219 947	246 939	219 947
	2	264 173	257 486	259 021	253 574
	3	475 842	482 684	473 346	473 873
	4	526 706	484 897	501 778	480 892
20	1	560 113	427 942	545 403	427 942
	2	612 810	480 892	605 380	469 844
	3	898 598	816 491	819 730	816 346
	4	1 086 986	940 200	1 015 652	924 501

5. Conclusions

In this paper, we have studied the global problem of designing WDM networks including the traffic grooming. It consists in finding the number of fibres between each pair of nodes (i.e. the physical topology), finding the number of transponders to instal at each node, choosing the set of lightpaths (i.e. the virtual topology), routing these lightpaths over the physical topology and finally, grooming and routing the traffic over the lightpaths. The objective is to find the minimum cost WDM network.

We have presented a mathematical model, two heuristic algorithms and tabu search metaheuristic algorithm for this problem. The results have shown the importance of the physical topology selection and that the tabu-based algorithm can find solutions for real-size instances of the problem.

While H1 is more adapted because it takes into account the traffic to be exchange, H2 provided in general better results even it instals a larger number of lightpaths in the network. This is due to our cost function structure and the important distances between the nodes in which the optical cost was the dominant part of the network cost. We also work on new heuristics in which single hop routing has still priority but with reservation of an amount of the spare capacity in the lightpaths for the high speed connexion demands that have to be routed in multi-hop.

Another research avenue is to explore multicriteria algorithms to take into account several objectives as the blocking in addition to the cost of the network by testing several sets of weightings, to select the most convenient network topologies. Several research avenues are also open at this point. First, we currently explore the use of alternate paths to improve the quality of the heuristics.

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Appendix. Model formulation

The model for the global WDM network design problem is given below.
GWNDP:

$$\min \sum_{i\in N}\sum_{\substack{j\in N\\ i\neq j}} a_{ij}\left(\sum_{f=1}^{\beta_{ij}} x_{ij}^{f}\right) + \sum_{m\in N}\sum_{\substack{n\in N\\ m\neq n}} (b_m+b_n)\,w_{mn} \tag{A.1}$$

subject to
Physical topological constraints

$$\sum_{m\in N}\sum_{\substack{n\in N\\ m\neq n}} x_{ijmn}^{f\omega} \le x_{ij}^{f\omega} \quad \forall_{i,j\in N(i\neq j), f\in F_{ij}, \omega\in\Omega} \tag{A.2}$$

where $F_{ij} = \{1,\ldots,\beta_{ij}\}$.

$$x_{ij}^{f\omega} \le x_{ij}^{f} \quad \forall_{i,j\in N(i\neq j), f\in F_{ij}, \omega\in\Omega} \tag{A.3}$$

$$x_{ij}^{f} \leq x_{ij} \quad \forall_{i,j \in N (i \neq j), f \in F_{ij}} \tag{A.4}$$

$$\sum_{m \in N} \sum_{\substack{n \in N \\ m \neq n}} \sum_{\omega \in \Omega} x_{ijmn}^{f\omega} \leq |\Omega| \quad \forall_{i,j \in N (i \neq j), f \in F_{ij}} \tag{A.5}$$

Additional topological constraints. (A.6)

Virtual topology over physical topology constraints

$$\sum_{\ell \in N \setminus \{m\}} \sum_{f \in F_{m\ell}} \sum_{\omega \in \Omega} x_{m\ell mn}^{f\omega} = w_{mn} \quad \forall_{m,n \in N (m \neq n)} \tag{A.7}$$

$$\sum_{\ell \in N \setminus \{n\}} \sum_{f \in F_{\ell n}} \sum_{\omega \in \Omega} x_{\ell nmn}^{f\omega} = w_{mn} \quad \forall_{m,n \in N (m \neq n)} \tag{A.8}$$

$$\sum_{\substack{i \in N \\ i \neq j}} \sum_{f \in F_{ij}} x_{ijmn}^{f\omega} = \sum_{\substack{\ell \in N \\ j \neq \ell}} \sum_{f \in F_{j\ell}} x_{j\ell mn}^{f\omega} \quad \forall_{j \in N, m,n \in N \setminus \{j\} (m \neq n), \omega \in \Omega} \tag{A.9}$$

$$\sum_{\ell \in N \setminus \{m\}} \sum_{f \in F_{\ell m}} \sum_{\omega \in \Omega} x_{\ell mmn}^{f\omega} = 0 \quad \forall_{m,n \in N (m \neq n)} \tag{A.10}$$

$$\sum_{\ell \in N \setminus \{n\}} \sum_{f \in F_{n\ell}} \sum_{\omega \in \Omega} x_{n\ell mn}^{f\omega} = 0 \quad \forall_{m,n \in N (m \neq n)}. \tag{A.11}$$

Traffic constraints

$$\sum_{m \in N \setminus \{d\}} y_{md}^{odkt} = z^{odkt} \quad \forall_{o,d \in N (o \neq d), t \in T, k \in K^{odt}} \tag{A.12}$$

where $k \in K^{odt} = \{1, \ldots, \alpha^{odt}\}$.

$$\sum_{n \in N \setminus \{o\}} y_{on}^{odkt} = z^{odkt} \quad \forall_{o,d \in N (o \neq d), t \in T, k \in K^{odt}} \tag{A.13}$$

$$\sum_{m \in N \setminus \{\ell\}} y_{m\ell}^{odkt} = \sum_{n \in N \setminus \{\ell\}} y_{\ell n}^{odkt} \quad \forall_{\ell \in N, o,d \in N \setminus \{\ell\} (o \neq d), t \in T, k \in K^{odt}} \tag{A.14}$$

$$\sum_{m \in N} y_{mo}^{odkt} = 0 \quad \forall_{o,d \in N (o \neq d), t \in T, k \in K^{odt}} \tag{A.15}$$

$$\sum_{n \in N} y_{dn}^{odkt} = 0 \quad \forall_{o,d \in N (o \neq d), t \in T, k \in K^{odt}} \tag{A.16}$$

$$\sum_{k=1}^{\alpha^{odt}} z^{odkt} = \alpha^{odt} \quad \forall_{o,d \in N (o \neq d), t \in T} \tag{A.17}$$

$$\sum_{o \in N} \sum_{\substack{d \in N \\ o \neq d}} \sum_{k=1}^{\alpha^{odt}} \sum_{t \in T} \delta^{t} y_{mn}^{odkt} \leq \epsilon w_{mn} \quad \forall_{m,n \in N (m \neq n)} \tag{A.18}$$

$$w_{mn} \in \mathsf{N}, \quad x_{ij} \in \mathsf{B}, \quad x_{ij}^{f} \in \mathsf{B}, \quad x_{ij}^{f\omega} \in \mathsf{B}, \quad x_{ijmn}^{f\omega} \in \mathsf{B}, \quad y_{mn}^{odkt} \in \mathsf{B}, \quad z^{odkt} \in \mathsf{B}. \tag{A.19}$$

The objective function (A.1) represents the cost of the links and of the transponders installed in the network. Constraints (A.2) require that a wavelength is used only if a lightpath uses this wavelength and constraints (A.3) impose to use the fibre f from the node i to node j only if a wavelength is used in this fibre. Constraints (A.4) impose to install a link between nodes i and j only if a fibre is used between these nodes and constraints (A.5) ensure the number of wavelengths used in a fibre be less or equal to the maximum number of wavelengths that can be used in a fibre. Constraints (A.6) include additional topological constraints defined by the network planner (for instance, to increase the network reliability).

Constraints (A.7) necessitate that the number of lightpaths starting from each node is equal to the number of lightpaths having this node as a source and constraints (A.8) require the number of lightpaths terminating at each node is equal to the number of lightpaths having this node as a destination. Constraints (A.9) dictate that a lightpath passing through an intermediate node uses only one wavelength, constraints (A.10) impose that a lightpath cannot enter its source node and constraints (A.11) require that a lightpath cannot leave its destination node.

Constraints (A.12)–(A.18) are traffic constraints. Constraints (A.12) and (A.13) impose that for each connection between each source and destination pair of nodes, a request handled by a lightpath originating from (ending at) the source (the destination) node is set up. Constraints (A.14) state that a request entering to an intermediate node has to leave it. Constraints (A.15) and (A.16) impose that no request entering to (leaving) a node if this node is its source (destination). Constraints (A.17) require that all the requests have to be routed and constraints (A.18) preserve the capacity of the lightpaths. Finally, constraints (A.19) are integrality constraints where $\mathsf{B} = \{0, 1\}$.

Note that GWNDP is NP-hard [3] (e.g. transformation from the traffic grooming [9] or from the wavelength assignment subproblems [2]).

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