Blocking Performance Analysis of Fixed-Paths Least-Congestion Routing in Multifiber WDM Networks

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ABSTRACT

Wavelength-routed all-optical networks have been receiving significant attention for high-capacity transport applications. A good routing and wavelength assignment (RWA) algorithm is critically important to improve the performance of wavelength-routed WDM networks. We study the blocking performance of fixed-paths least-congestion (FPLC) routing in multifiber WDM networks in this paper. A new analytical model based on the link-load correlation is developed to evaluate the blocking performance of the FPLC routing. The analytical model is a generalized model that can be used in both regular (e.g. mesh-torus) and irregular (e.g. NSFnet) networks. It is shown that the analytical results closely match the simulation results, which indicates that the model is adequate in analytically predicting the performance of the FPLC routing in different networks.

Multifiber WDM networks offer the advantage of reducing the effect of the wavelength continuity constraint without using wavelength converters. A wavelength that cannot continue on the next hop on the same fiber can be switched to another fiber using an optical cross-connect (OXC) if the same wavelength is free on one of the other fibers. However, the cost of a multifiber network is likely to be higher than a single-fiber network with the same capacity, because more amplifiers and multiplexer/demultiplexer may be required. The design goal of a multifiber network is to achieve high network performance with the minimum number of fibers.

Two FPLC routing algorithms, wavelength trunk (WT)-based FPLC and lightpath (LP)-based FPLC, are proposed and studied. Our analytical and simulation results show that the LP-based FPLC routing algorithm can use multiple fibers more efficiently than the WT-based FPLC and the alternate path routing. In both the mesh-torus and NSFnet networks, limited number of fibers is sufficient to guarantee high network performance.

Keywords: Wavelength Division Multiplexing, Optical Networks, Least Congestion Routing, and Multifiber Networks

1. INTRODUCTION

With the development of the Internet and World Wide Web, the network bandwidth requirements have increased dramatically in recent years. The research, development, and deployment of wavelength-division multiplexing (WDM) technology are now evolving at a rapid pace[1] to fulfil the increasing bandwidth requirement and deploy new network services. All-optical networks employing wavelength-division multiplexing and wavelength routing are a viable solution for future wide-area networks (WANs) and metropolitan-area networks (MANs). These wavelength-routed WDM networks offer the advantages of protocol transparency and simplified management and processing compared to routing in systems using digital cross-connects.[2]

In wavelength-routed all-optical WDM networks, a *lightpath* is an 'optical communication path' between two nodes, established by allocating the same wavelength throughout the route of the transmitted data.[4] The requirement that the same wavelength must be used on all the links along the selected path is known as the wavelength continuity constraint. Two lightpaths can use the same fiber link, only if they use different wavelengths. A connection request encounters high performance degradation because of the wavelength continuity constraint. Wavelength converters have been proposed to overcome the wavelength continuity constraint. However, the technology of all-optical wavelength conversion is not mature yet. The cost of wavelength converters is likely to remain high in the near future. Using multiple fibers on each link in WDM networks is an alternate solution to conquer the wavelength

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continuity constraint. In multifiber WDM networks, each link consists of multiple fibers, and each fiber carries information on multiple wavelengths. A wavelength that cannot continue on the next hop on the same fiber can be switched to another fiber using an optical cross-connect (OXC) if the same wavelength is free on one of the other fibers. Thus, multiple fibers in WDM networks have the same effect as the limited wavelength conversion.[5] Let F be the number of fibers per link and W be the number of wavelengths per fiber. A network with F fibers per link and W wavelengths per fiber is functionally equivalent to an FW-wavelength network with a partial wavelength conversion of degree F.

Routing and wavelength assignment (RWA) algorithms play a key role in WDM networks. Many researchers have proposed the use of the shortest-path (SP) routing and alternate shortest-path (ASP) routing. In[8,10,11,15] the performance of the SP and ASP routing methods are investigated through approximate analysis and simulation. Since the shortest paths are statically computed and an attempt is made to set up a connection request on fixed paths without acquiring the information of current network status, it is not possible to further improve the network performance in terms of blocking probability by using these routing approaches. Dynamic routing approaches are more efficient than static routing methods. Analytical and simulation results[15–17] show that the dynamic routing method can significantly improve the network performance compared to the SP and ASP. However, most of the research has focused on single-fiber WDM networks. We study the performance of multiple fibers in WDM networks with the fixed-paths least-congestion (FPLC) routing in this paper. In the FPLC, we first statically compute a set of routes to be used for each source-destination pair in a network and store the route information at each source node. Upon arrival of a connection setup request, the least congested route is selected to set up the request. The request is blocked if no channel is free on any route.

Much research has been done in obtaining the call blocking performance of single-fiber WDM networks.[6–8,11,21] A Markov chain (MC) model with the consideration of link-load correlation in[11] is accurate, and has a moderate complexity. As pointed out in,[21] the MC model is an approximate model, because the arrival rates vary with the state of the Markov chain. However, the approximation does not affect the accuracy significantly. There have also been considerable interests to analyze the blocking performance of multifiber WDM networks. The independent wavelength load model[7] is extended to multifiber networks in.[9] The results of this model are not numerically accurate for Poisson traffic because of the assumption that the load on one wavelength is independent of those on the other wavelengths on a link. The link load independence model proposed in[6] is extended to multifiber networks in.[14] However, this independent model is not accurate. It overestimates the blocking performance for $F = 1$ and underestimates it for $F > 1$ in a mesh-torus network.[14] The blocking performance models for first-fit wavelength assignment in[16,17] are also proposed to be applicable in multifiber networks. However, both of these models assume that link loads are independent, which may not be valid for sparse network topologies. Wavelength assignment algorithms for multifiber WDM networks have also been studied in.[18] We use and extend the link-load correlation (LLC) model in[11] to analyze the performance of multifiber WDM networks with the FPLC routing in this paper. To our knowledge, this is the first analytical model that can be used to predict the performance of multifiber networks with the least congestion routing. Our model is a generalized model that can be used in both regular and irregular networks. Multiple fibers per link have the same effect as the limited wavelength conversion. Therefore our analytical model is also applicable in the networks with limited wavelength conversion.

Since the cost of a multifiber network is likely to be higher than a single-fiber network (more amplifiers and multiplexer/demultiplexer), the design goal of a multifiber network is to achieve high network performance with the minimum number of fibers. An important problem in multifiber networks is to decide how many fibers per link are required to guarantee high network performance that is similar to a network with full-range wavelength converters at every node. This fiber requirement may depend on many factors, e.g., the network topology, traffic patterns, the number of wavelengths per fiber, and the routing algorithm employed in the network. A similar problem has been studied in[22] under static traffic. We study the fiber requirement under dynamic traffic in different topologies with different routing algorithms in this paper. Let the fiber-wavelength-ratio (FWR) be the ratio of the number of fibers per link over the number of wavelengths per fiber. We observed that multifiber WDM networks with FPLC routing have similar blocking performance to full-wavelength-convertible networks, as long as the fiber-wavelength-ratio (F/W) is around 25% for the mesh-torus network and 20% for the NSFnet.

This paper is organized as follows. The fundamental ideas of the link-load correlation model are introduced in Section 1.1. The link-load correlation model is extended to analyze the performance of the FPLC routing in multifiber WDM networks in Section 2. An iterative approach is proposed to solve the Erlang-map equation introduced by the FPLC in multifiber networks. The accuracy of the analytical model is assessed in Section 3 by comparing the

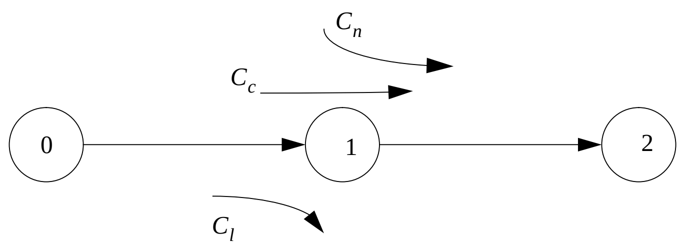


Figure 1. Calls arriving and leaving on a two-hop path.

analytical results to the simulation results. The numerical results show that a small number of fibers per link are sufficient to guarantee high network performance in both the regular mesh-torus networks and the irregular NSFnet. We make our concluding remarks in Section 4.

1.1. Review of the Markov chain (MC) Model for Single-Fiber Networks

In this section, we review the fundamental ideas of the link-load correlation model. For lack of space, we omit explaining of the details of the model and ask the reader to refer to[11] when necessary. The model starts by considering a two-hop path as shown in figure 1. We use hop and link interchangeably throughout this paper. The states of a two-hop path can be modeled using a three-dimensional Markov chain as follows. Let C_l be the number of calls that enter the path at node 0 and leave at node 1. Let C_c be the number of calls that enter the path at node 0 and continue on to the second link. And let C_e be the number of calls that enter the path at node 1. Therefore, the number of calls that use the first link is $C_l + C_c$ and the number of calls that use the second link is $C_c + C_e$. Since the number of calls on a link cannot exceed the total number of available wavelengths, W, we have $C_l + C_c \leq W$, and $C_c + C_e \leq W$. The following conditional probabilities can be derived for the three-dimensional MC on a two-hop path:

$$\begin{aligned} R(n_{f_2}|x_{f_1}, z_{c_2}, y_{f_2}) \;=\; & \Pr\{n_{f_2} \text{ wavelengths are free on a two-hop path} \mid x_{f_1} \text{ wavelengths are free} \\ & \text{on the first hop of the path, } z_{c_2} \text{ wavelengths are busy on both of the link} \\ & \text{carrying calls that continue from the first to the second hop,} \\ & \text{and } y_{f_2} \text{ wavelengths are free on the second hop}\}\ , \end{aligned} \tag{1}$$

$$\begin{aligned} U(z_{c_2}|x_{f_1}, y_{f_2}) \;=\; & \Pr\{z_{c_2} \text{ wavelengths are occupied by continuing calls from the first link to the} \\ & \text{second link} \mid x_{f_1} \text{ wavelengths are free on the first link, and } y_{f_2} \text{ wavelengths} \\ & \text{are free on the second link}\} \\ =\; & P(C_c = z_{c_2} \mid C_l + C_c = W - x_{f_1}, C_e + C_c = W - y_{f_2})\,, \end{aligned} \tag{2}$$

and

$$\begin{aligned} S(y_{f_2}|x_{f_1}) \;=\; & \Pr\{y_{f_2} \text{ wavelengths are free on the second link of a path} \mid x_{f_1} \text{ wavelengths are free} \\ & \text{on the first link of the path}\} \\ =\; & P(C_e + C_c = W - y_{f_2} \mid C_l + C_c = W - x_{f_1})\,. \end{aligned} \tag{3}$$

Here $U(z_{c_2}|x_{f_1}, y_{f_2})$ and $S(y_{f_2}|x_{f_1})$ are functions of the steady-state probability of state (c_l, c_c, c_e) that is given by

$$\pi(c_l, c_c, c_e) = \frac{\frac{(\frac{\lambda_l}{\mu})^{c_l}}{c_l!}\frac{(\frac{\lambda_c}{\mu})^{c_c}}{c_c!}\frac{(\frac{\lambda_e}{\mu})^{c_e}}{c_e!}}{\sum_{j=0}^{F}\sum_{i=0}^{F-j}\sum_{k=0}^{F-j}\frac{(\frac{\lambda_l}{\mu})^{i}}{i!}\frac{(\frac{\lambda_c}{\mu})^{j}}{j!}\frac{(\frac{\lambda_e}{\mu})^{k}}{k!}}, \quad 0 \leq c_l + c_c \leq F, \quad 0 \leq c_c + c_e \leq F, \tag{4}$$

where $\lambda_l, \lambda_c, \lambda_e$ are the arrival rates of calls that leave the first link, continue from the first link to the second link, and enter at the second link, respectively. $1/\mu$ is the expected value of the exponentially distributed call holding time.

Because of the assumption that the load on the lth hop is dependent only on the load on the $(l-1)$th hop, the probability of blocking on the l-hop path can be computed using the results for a two-hop path. This is done by viewing the first $(l-1)$ hops as the first hop and the lth hop as the second hop of a two-hop path.

$$\begin{aligned} T^{(l)}(n_{f_l}, y_{f_l}) &= \Pr\{n_{f_l} \text{ wavelengths are free on an } l\text{-hop path and } y_{f_l} \text{ wavelengths are free on hop } l\} \\ &= \sum_{x_{f_{(l-1)}}=0}^{W} \; \sum_{z_{c_l}=0}^{\min(W-x_{f_{(l-1)}}, W-y_{f_l})} \; \sum_{n_{f_{(l-1)}}=0}^{x_{f_{(l-1)}}} \\ &\quad R(n_{f_l}|n_{f_{(l-1)}}, z_{c_l}, y_{f_l}) U(z_{c_l}|x_{f_{(l-1)}}, y_{f_l}) S(y_{f_l}|x_{f_{(l-1)}}) T^{(l-1)}(n_{f_{(l-1)}}, x_{f_{(l-1)}}) \,. \end{aligned} \tag{5}$$

The probability of blocking on an l-hop path is given by $\sum_{y_{f_l}=0}^{W} T^{(l)}(0, y_{f_l})$. The network-wide blocking probability, P_B is then computed as

$$P_B = \sum_{l=1}^{N-1} \sum_{y_{f_l}=0}^{W} T^{(l)}(0, y_{f_l}) p_l \tag{6}$$

for a network of N nodes, where p_l is the probability that an l-hop path is chosen for routing.

2. ANALYTICAL MODEL FOR THE FPLC ROUTING IN MULTIFIBER WDM NETWORKS

We propose an analytical model to compute the blocking performance of the FPLC in this section. In the FPLC routing, a set of paths is predetermined for each s-d pair*. Upon an arrival of a connection request, the least congested path, i.e., the path that has the maximum number of free wavelength trunks (defined in the next section), is selected to use. If there is a tie, the first shortest path with free wavelength is selected. A wavelength is randomly selected among the free wavelengths to set up the request. The call is blocked if no free wavelength is found on all of the paths. Note that the least congested path may be computed by a central controller in a network as proposed in,[17] or using a distributed algorithm.[12,13] We focus on the performance analysis of the FPLC routing in this paper.

Assumptions and definitions In the analytical model, we assume a Poisson input traffic with an arrival rate λ at each node and an exponentially distributed call holding time with mean $1/\mu$. Two paths are preselected for each source-destination (s-d) pair, and wavelengths are assigned to a connection randomly from the set of free wavelengths on that path. The load on link i of a path given the loads on link $1, 2, \ldots, i-1$, depends only on the load on link $i-1$. Let F be the number of fibers per link and W be the number of wavelengths on each fiber. We assume that F and W are the same on all links and fibers, respectively. We also assume that an incoming request can be switched to any output port using OXC as long as the output port has the same wavelength free regardless of which fiber it is on. If the wavelength is not free on all of the F fibers, the request is blocked on this wavelength. The blocked calls will never return to the network. No wavelength converter is available at any node.

We define a Light Channel (LC) as a wavelength on a fiber on a link. A lightpath (LP), defined in previous section, is a connection between a s-d pair using the same wavelength on all the links of a path. Note that a lightpath consists of several LCs on successive links. The LCs on a path may or may not be on the same fiber. Let a wavelength trunk (WT) λ_i be a collection of the LCs/LPs using wavelength λ_i on all the fibers. We define a WT "free" on a link if the wavelength is free on at least one of the fibers on the link. A WT is "busy" on the link otherwise. A WT is "free" on a path if that WT is free on all of the links constituting the path. A WT is "busy" on the path otherwise.

In the analytical model, we start by analyzing a two-hop path with F fibers on each link and W wavelengths on each fiber. Then the blocking probability on a l-hop path can be computed recursively by viewing the first $l-1$ hops as the first hop and the lth hop as the second hop of a two-hop path. On the two-hop path as shown in Figure 2, we are interested in computing $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$, which is defined as

*We restrict the number of paths for each s-d pair to two for easy discussion in this paper. The analytical model can be easily extended to consider more than two paths. However, more than two paths do not improve significantly the blocking performance as observed in our experiments and also shown in.[17]

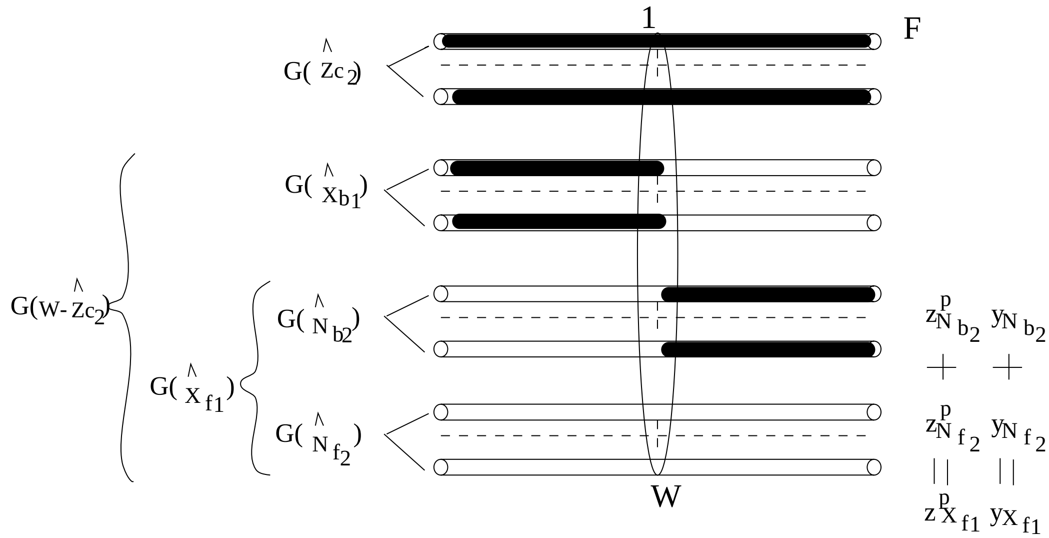


Figure 2. Different wavelength trunks on a two-hop path.

- $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2}) = \Pr\{$the probability that $\widehat{N}_{f_2}$ WTs are free on a two-hop path | $\widehat{X}_{f_1}$ WTs are free on the first hop of the path, y_{f_2} LCs are free on the second hop, and z_{c_2} LCs are busy on both of the links occupied by continuing calls from the first to the second hop[†]$\}$.

The difficulty in computing $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$ results from the continuing calls from the first hop to the second hop. To simplify the computation, we divide the W WTs on the two-hop path into different groups as shown in Figure 2. Each wavelength trunk consists of F fibers. A filled slot in the figure indicates that that wavelength trunk is busy, that is, it is fully occupied on the link. An unfilled slot indicates that the wavelength trunk is free, that is, the wavelength trunk may be partially occupied or free on every fiber. The conditional distribution of continuing calls is computed in each group.

Notations We define the following steady-state probabilities that are used in obtaining $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$.

- $P_1(\widehat{Z}_{c_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2}) = \Pr\{$ $\widehat{Z}_{c_2}$ busy WTs are occupied completely by continuing calls | $\widehat{X}_{f_1}$ WTs are free on the first hop, y_{f_2} LCs are free on the second hop, and z_{c_2} busy LCs continue from the first to the second hop$\}$.

- $P_2(z^p_{X_{b_1}}|z^p_{c_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1}) = \Pr\{z^p_{X_{b_1}}$ continuing calls are in the subgroup $G(\widehat{X}_{b_1})$ | $z^p_{c_2}$ calls are randomly distributed in groups $G(\widehat{X}_{b_1})$ and $G(\widehat{X}_{f_1})$, and no busy WT is occupied completely by $z^p_{c_2}$ continuing calls[‡] $\}$.

- $P_3(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1}) = \Pr\{y_{X_{b_1}}$ LCs are free on the second hop in group $G(\widehat{X}_{b_1})$ | $z^p_{X_{b_1}}$ LCs are in group $G(\widehat{X}_{b_1})$, $z^p_{X_{f_1}}$ LCs are in group $G(\widehat{X}_{f_1})$, and y_{b_2} calls are randomly distributed in groups $G(\widehat{X}_{b_1})$ and $G(\widehat{X}_{f_1})$ $\}$.

- $P_4(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}}) = \Pr\{\widehat{N}_{f_2}$ WTs are free on the two-hop path | all of the WTs are free on the first hop, $y_{X_{f_1}}$ LCs are free on the second hop, and $z^p_{X_{f_1}}$ calls continue from the first hop to the second hop $\}$.

[†]We put a hat on the variables for the number of WTs, to differentiate them from the variables for the number of LCs on a link, throughout this paper.

[‡]Since the busy WTs that are occupied completely by continuing calls are in the $G(\widehat{Z}_{c_2})$ group, the number of continuing calls that only partially occupy a WT in other groups are indicated by z^p.

Computation of $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})$ Let w be the number of considered WTs on a link, $w \leq W$. w is used as a subscript in the expressions of this paper to indicate that the computation of the expressions is on w WTs. The probabilities $P_1(\widehat{Z}_{c_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})_w$, $P_2(z^p_{X_{b_1}}|z^p_{c_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})_w$, $P_3(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})_w$ and $P_4(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}})_w$ can be derived using Figure 2 as follows:

$$P_1(\widehat{Z}_{c_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2})_w = \begin{cases} 0 & \widehat{Z}_{c_2} + \widehat{X}_{f_1} > W \text{ or } y_{f_2} + z_{c_2} > WF\,, \\ \dfrac{\binom{W}{\widehat{Z}_{c_2}} f(z^p_{c_2}, w, F)}{\binom{wF}{z_{c_2}}} & \textit{otherwise}\,, \end{cases} \tag{7}$$

where $z^p_{c_2} = z_{c_2} - \widehat{Z}_{c_2}F$, and $f(z, w, F)$ is the number of ways of distributing z LCs to w WTs such that every WT is free. Note that each WT consists of F fibers. Suppose $j = \lfloor \frac{z}{F} \rfloor$. $f(z, w, F)$ is given by[14]

$$f(z, w, F) = \begin{cases} 0 & z > (w-1)F \\ \binom{wF}{z} & j = 0 \\ \binom{wF}{z} - \sum_{i=1}^{j} \left\{ \binom{w}{i} f(z - iF, w - i, F) \right\} & \text{otherwise.} \end{cases} \tag{8}$$

$$P_2(z^p_{X_{b_1}}|z^p_{c_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})_w = \frac{f(z^p_{X_{b_1}}, \widehat{X}_{b_1}, F) f(z^p_{X_{f_1}}, z^p_{c_2} - \widehat{X}_{b_1}, F)}{f(z^p_{c_2}, w, F)}\,. \tag{9}$$

$$P_3(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1})_w = \frac{\binom{\widehat{X}_{b_1}F - z^p_{X_{b_1}}}{y_{X_{b_1}}} \binom{\widehat{X}_{f_1}F - z^p_{X_{f_1}}}{y_{b_2} - y_{X_{b_1}}}}{\binom{wF - z^p_{X_{b_1}} - z^p_{X_{f_1}}}{y_{b_2}}}\,. \tag{10}$$

$$P_4(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}})_w = \sum_{z^p_{N_{b_2}}=0}^{z^p_{X_{f_1}}} \frac{\binom{\widehat{X}_{f_1}}{\widehat{N}_{f_2}} f(z^p_{N_{b_2}}, \widehat{N}_{b_2}, F) g(\widehat{N}_{f_2}, z^p_{X_{f_1}} - z^p_{N_{b_2}}, y_{X_{f_1}} - y_{N_{b_2}})}{f(z^p_{X_{f_1}}, \widehat{X}_{f_1}, F) \binom{\widehat{X}_{f_1}F - z^p_{X_{f_1}}}{y_{X_{f_1}}}}\,. \tag{11}$$

where $g(\widehat{N}, z, y)$ is the number of ways to distribute z continuing calls and y entering calls to $\widehat{N}$ WTs such that every WT is free. $g(\widehat{N}, z, y)$ is given by

$$\begin{aligned} g(\widehat{N}, z, y) = {} & \binom{\widehat{N}F}{z} \binom{\widehat{N}F - z}{y} - \\ & \sum_{i=1}^{\lfloor \frac{z+y}{F} \rfloor} \sum_{j=0}^{min(iF, z)} \binom{\widehat{N}}{i} \binom{iF}{j} g(\widehat{N} - i, z - j, y - (iF - j))\,. \end{aligned}$$

A closed-form expression of $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ is obtained as

$$\begin{aligned} R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2}) = {} & \sum_{\widehat{Z}_{c_2}=0}^{\lfloor \frac{z_{c_2}}{F} \rfloor} \sum_{z^p_{X_{b_1}}=0}^{z_{c_2} - \widehat{Z}_{c_2}F} \sum_{y_{X_{b_1}}=0}^{(W - \widehat{Z}_{c_2})F - z_{c_2} - y_{f_2}} P_1(\widehat{Z}_{c_2}|\widehat{X}_{f_1}, z_{c_2}, y_{f_2}) \\ & P_2(z^p_{X_{b_1}}|z^p_{c_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1}) P_3(y_{X_{b_1}}|z^p_{X_{b_1}}, z^p_{X_{f_1}}, y_{b_2}, \widehat{X}_{b_1}, \widehat{X}_{f_1}) P_4(\widehat{N}_{f_2}|\widehat{X}_{f_1}, z^p_{X_{f_1}}, y_{X_{f_1}})\,. \end{aligned} \tag{12}$$

Blocking performance of the FPLC routing Given the steady-state distribution $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ of a two-hop path, we can compute the blocking probability on the l-hop path by viewing the first $l-1$ hops as the first hop and the lth hop as the second hop of a two-hop path. Let $P_5^{(l)}(\widehat{N}_{f_l}, y_{f_l})$ be the probability that $\widehat{N}_{f_l}$ wavelengths are free on an l-hop path and y_{f_l} LCs are free on hop l . Following the link-load correlation model reviewed in Section 1.1, $P_5^{(l)}(\widehat{N}_{f_l}, y_{f_l})$ can be derived as

$$P_5^{(l)}(\widehat{N}_{f_l}, y_{f_l}) = \sum_{x_{f_{l-1}}=0}^{FW} \sum_{z_{c_l}=0}^{\min(FW-x_{f_{l-1}}, FW-y_{f_l})} \sum_{\widehat{N}_{f_{l-1}}=0}^{\lfloor \frac{x_{f_{l-1}}}{F} \rfloor} R_{WTLC}(\widehat{N}_{f_l}|\widehat{N}_{f_{l-1}}, z_{c_l}, y_{f_l}) U(z_{c_l}|y_{f_l}, x_{f_{l-1}}) S(y_{f_l}|x_{f_{l-1}}) P_5^{(l-1)}(\widehat{N}_{f_{l-1}}, x_{f_{l-1}}) \ . \quad (13)$$

where $U(z_{c_2}|x_{f_1}, y_{f_2})$ and $S(y_{f_2}|x_{f_1})$ are defined in Eqs. (2) and (3), respectively.

Let $Q_{P_\alpha}(i)$ be the probability that the path P_α connecting s-d pair α has i free wavelength trunks. Let $l(P_\alpha)$ be the length of path P_α. $Q_{P_\alpha}(i)$ is given by

$$Q_{P_\alpha}(i) = \sum_{y_f=0}^{FW} P_5^{l(P_\alpha)}(i, y_f) \ . \quad (14)$$

$Q_{P_\alpha}(i)$ gives the stable-state distribution of the number of free wavelength trunks on a path. Let P_{B_α} be the probability that a connection request of s-d pair α is blocked. In the FPLC routing, a request is blocked if none of the two paths connecting a s-d pair has free wavelength. Thus P_{B_α} is given by

$$P_{B_\alpha} = Q_{P_\alpha^1}(0) Q_{P_\alpha^2}(0) \ . \quad (15)$$

Let $|R|$ be the number of s-d pairs in a network, and P_B be the network-wide average blocking probability. P_B is given by

$$P_B = \sum_\alpha P_{B_\alpha} / |R| \ . \quad (16)$$

Estimation of parameters The analysis in the last two sections can be used to compute the call blocking probability in a multifiber network with the FPLC routing. This analysis assumes that the arrival rates of calls that leave a link, continue from a link on to the next link of a path, and enter at a link, λ_l, λ_c and λ_e respectively, are known. Typically, the traffic in a network is specified in terms of the set of offered loads between s-d pairs. The call arrival rates at links have to be estimated from the arrival rates of calls to node. The complication in estimating the link arrival rates in the FPLC routing is that a path for a request is selected using the current network status. Thus the arrival rate on each link is continuously changing. No steady state is reached in the strict sense when the FPLC is used. We propose to use a technique based on the Erlang Fixed-Point method for Alternate routing[3] to solve this problem. We need the following further notations:

- Let $R_j^{(1)}$ be a set of the first routes that employ link j, and $R_j^{(2)}$ be a set of the second routes that employ link j.
- Let $R_{i,j}^{(1)}$ be a set of the first shortest routes that have a subset of route from link i to j. Let $R_{i,j}^{(2)}$ be a set of the second shortest routes that have a subset of route from link i to j.
- Let $Pr(P_\alpha^1)$ and $Pr(P_\alpha^2)$ be the probabilities that a call for a s-d pair α is set up on the first and second path, P_α^1 and P_α^2, respectively.

In the FPLC, a call request is set up on the first shortest path if the number of free wavelengths on the second shortest path is less than the number of free wavelengths on the first shortest path. Otherwise, it is set up on the second shortest path assuming that the path has at least one free wavelength. Therefore,

$$Pr(P_\alpha^1) = \sum_{i=1}^{F} Q_{P_\alpha^1}(i) (\sum_{n=0}^{i} Q_{P_\alpha^1}(n)), \quad (17)$$

$$Pr(P_\alpha^2) = \sum_{i=1}^{F} Q_{P_\alpha^2}(i)(\sum_{n=0}^{i-1} Q_{P_\alpha^2}(n)). \tag{18}$$

Recall that λ is the call arrival rate at each node. The arrival rate of calls that enter at link i and continue to link j, $\lambda_c(i,j)$, becomes

$$\lambda_c(i,j) = \sum_{P_j \in R_{i,j}^1} \lambda Pr(P_j) + \sum_{P_j \in R_{i,j}^2} \lambda Pr(P_j). \tag{19}$$

The arrival rate of calls that leave from link i, $\lambda_l(i)$, includes calls that use link i as the first or second route, but do not continue to link j,

$$\lambda_l(i) = \sum_{P_i \in R_i^1} \lambda Pr(P_i) + \sum_{P_i \in R_i^2} \lambda Pr(P_i) - \lambda_c(i,j). \tag{20}$$

The arrival rate of calls that enter at link j, $\lambda_e(j)$ includes calls that use link j as the first or second route, but do not include calls that continue from link i to link j,

$$\lambda_e(j) = \sum_{P_j \in R_j^1} \lambda Pr(P_j) + \sum_{P_j \in R_j^2} \lambda Pr(P_j) - \lambda_c(i,j). \tag{21}$$

Given the arrival rates to each link, the conditional probabilities $S(y_f|x_{pf})$ and $U(z_c|y_f, x_{pf})$ can be derived (see[11] for details). The conditional probability, $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ depends only on the number of fibers per link, F, and the number of wavelengths per fiber, W. Let ϵ be a small positive number that is used as convergence criterion. Let J be the number of links in a network. The algorithm given below iteratively computes the approximate average blocking probability.

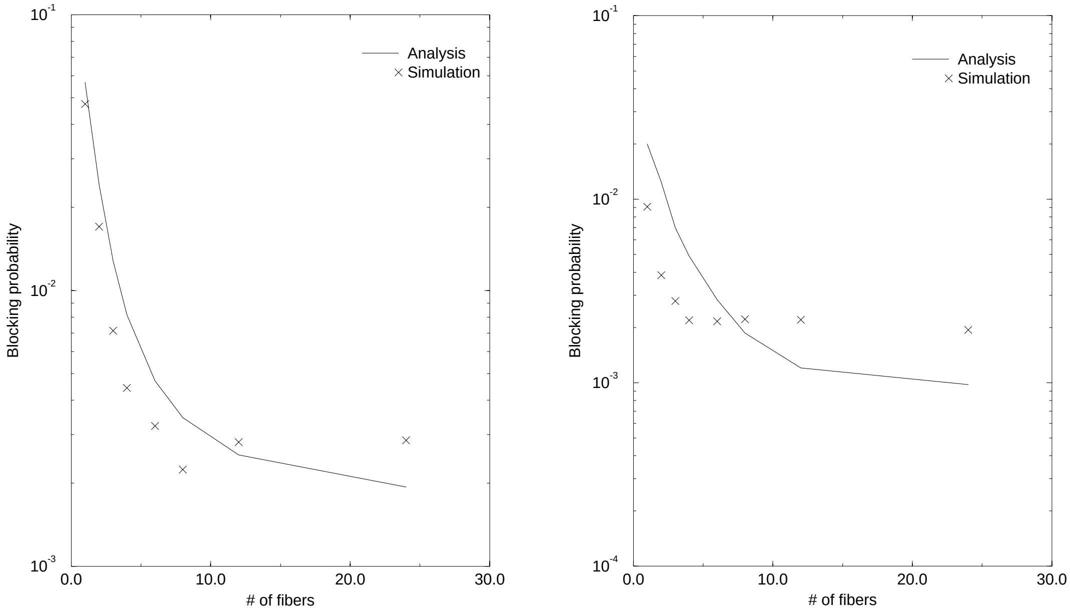
1. Initialization. For each source-destination pair α let $P_{B_\alpha} = 0$. Choose $\lambda_e(i)$, $\lambda_c(i,j)$, and $\lambda_l(i), i, j = 1, \ldots, J$ arbitrarily for all links.
2. Calculate $Q_{P_\alpha}(i)$ for every path of each s-d pair using Eqs. (13) and (14).
3. Calculate the blocking probability $\bar{P}_{B_\alpha}$ for every s-d pair α using Eq. (15). If $\max_\alpha |P_{B_\alpha} - \bar{P}_{B_\alpha}| < \epsilon$ then terminate. Otherwise let $P_{B_\alpha} = \bar{P}_{B_\alpha}$, go to next step .
4. Calculate λ_c, λ_l, and λ_e for each link using Eqs. (19), (20) and (21), then go back to step 2.

Since the arrival rate for each link can be computed individually, this method is suitable for analysis of irregular networks. The method is also applicable to alternate routing approaches with small modifications of Eqs. (17) and (18).

Implementation and complexity analysis Comparing to the link-load correlation model for single fiber networks,[11,20] the analytical model has the same computational complexity except for the computation of the free WT distribution on a two-hop path, $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$. However, $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ does not depend on any network topology and traffic arrival rate. The only parameters needed to compute $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ is the number of fibers per link, F, and the number of wavelengths per fiber, W. Thus $R_{WTLC}(\widehat{N}_{f_2}|\widehat{X}_{f_1}, y_{f_2}, z_{c_2})$ can be computed offline. The results can be used repeatedly in different topologies and traffic patterns, as long as they have the same number of fibers per link and wavelengths per fiber.

3. NUMERICAL RESULTS AND ANALYSIS

In this section, we assess the accuracy of our analysis model by comparing it with the simulation results. The analytical model is applied to a 5×5 mesh-torus network, and an irregular NSF T1 backbone network (NSFnet) with the FPLC routing. We are interested in finding the effect of multifibers on the blocking performance of these networks. The question we attempted to answer is how many fibers are required to yield similar performance as that of a full-wavelength-convertible network.



(a) (b)

Figure 3. Blocking probability versus number of fibers in (a) a 5×5 mesh-torus network and (b) NSFnet using the FPLC routing. Traffic loads are 26 and 17 Erlangs per node for the two networks, respectively. The number of LCs per link is fixed at 24.

In the networks we studied, the link capacity is fixed at 24 light channels, i.e., $FW = 24$ on each link. We vary the number of fibers on each link, F, from $1, 2, 3, 4, 6, 8, 12$ to 24, and the number of wavelengths on each fiber by $W = 24/F$ accordingly. We assume Poisson traffic arrives at each node, and the destination for an arrival request is uniformly distributed among other nodes[§]. Each data point in the simulations was obtained using 10^6 call arrivals. In the approximate analysis of the FPLC routing, multiple iterations are required and the convergence criteria, ϵ, is set to be 10^{-5}. The call blocking probability against the number of fibers per link is plotted in Figure 3.a for a 5×5 mesh-torus network (regular) and in Figure 3.b for the NSFnet (irregular). The traffic loads are 26 and 17 Erlangs per node for the two networks, respectively. From the figure, we observed that the analytical results follow the trend of the simulation results. The analytical results are in good agreement with simulation results for small to moderate number of fibers per link ($F < 8$). The analytical model slightly overestimates the blocking probability when F is large. The analytical results closely match the simulation results, which indicates that the model is adequate in analytically predicting the performance of the FPLC routing in different networks.

We observed from the figures that the network performance of using a full-range wavelength converter (F=24, W=1) at every node is much better than using no wavelength conversion (F=1, W=24) in the mesh-torus network with FPLC routing (more than one order of magnitude). Such performance improvement is not very significant in the irregular NSFnet. However, the network blocking probabilities decrease sharply with the increasing number of fibers per link F in both of the networks, when F is small. The performance improvement becomes less significant after 6 fibers per link are used in the mesh-torus network and 4 fibers per link in the NSFnet. Let the fiber-wavelength-ratio (FWR) be the ratio of the number of fibers per link over the number of wavelengths per fiber. We observed that high network performance using the FPLC routing is guaranteed if we keep the FWR around 25% (6/24) in the mesh-torus network and 18% (4/24) in the NSFnet.

We also noticed from the simulation results in Figure 3 that the best performance is achieved when $F = 8$ in the mesh-torus network and $F = 4$ in the NSFnet. After that, the blocking probability increases slightly with the usage of more fibers. This counter-intuitive observation results from the routing rules. In the FPLC routing, the least congested path is the one that has the maximum number of free wavelength trunks in multifiber networks. Since

[§]The analytical model could also be used for non-uniformly distributed traffic using Eqs. (19), (20) and (21). The uniform distribution assumption is made only for simplicity. Note that link loads in NSFnet are non-uniformly distributed.

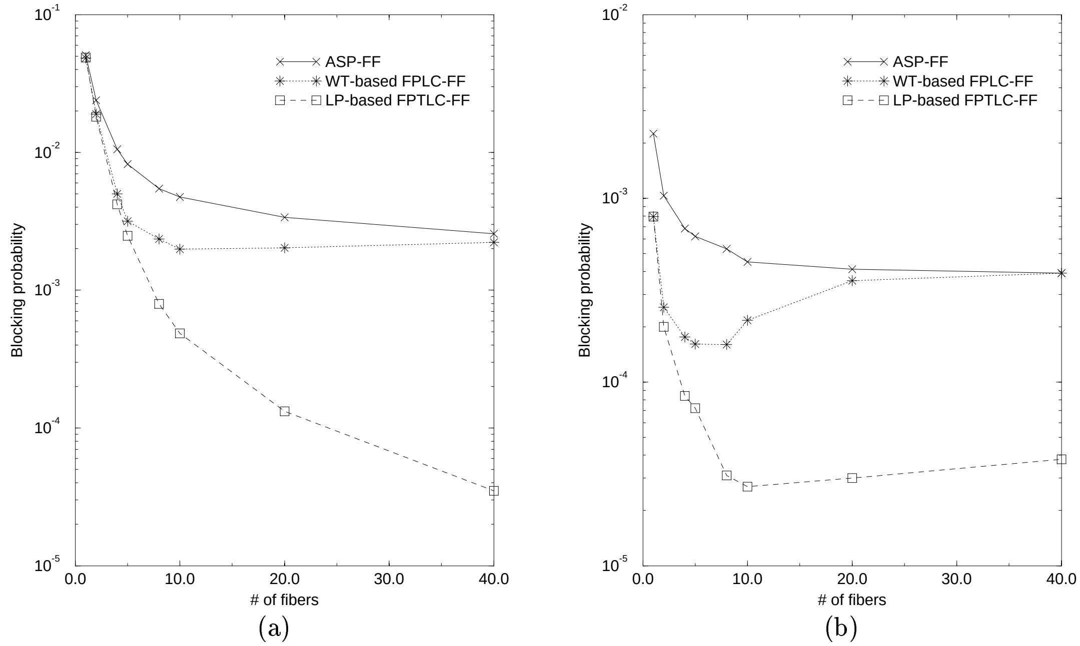


Figure 4. Simulation results of (a) the 5×5 mesh-torus and (b) the NSFnet networks using the FPLC routing. Traffic loads are 48 Erlangs per node for the mesh-torus, and 29 Erlangs per node for the NSFnet. The number of LCs per link is fixed at 40.

a free wavelength trunk may consists of one or more free lightpaths from a source to a destination (the maximum number is F), a path with more free wavelength trunks is not necessary to have more free lightpaths than the other path. Therefore a path with more free resources may not be selected to set up a connection. The routing rule, the first path is selected if the two paths have the same number of free wavelength trunks, leads the FPLC routing to the alternate path routing when $F = 24$, i.e., $W = 1$. It has been shown in[20] that the alternate path routing performs poorly compared to the FPLC routing in the mesh-torus and NSFnet networks. Thus the network performance may be degraded if too many fibers are used in the above FPLC routing. This observation suggests a new FPLC routing algorithm: instead of counting the number of free wavelength trunks on a path, the least congested path should be determined by the number of free lightpaths on a path. We call this new FPLC routing algorithm as an LP-based FPLC, and the previous FPLC algorithm as WT-based FPLC. These two FPLC routing algorithms are compared in Figure 4 using simulation results. The blocking performance of the alternate path routing[20] is also shown in the figure for comparison.

In the simulation, the link capacity is fixed at 40 light channels, i.e., $FW = 40$ on each link. We vary the number of fibers on each link, F, from $1, 2, 4, 5, 8, 10, 20$ to 40, and the number of wavelengths on each fiber by $W = 40/F$ accordingly. We change the wavelength assignment algorithm from random to first-fit,[4] i.e., after a path is determined to use, the first free wavelength with the smallest index is selected to set up a connection. The traffic loads are 48 Erlangs per node for the 5×5 mesh-torus network and 29 Erlangs per node for the NSFnet. We observed from Figure 4 that the LP-based FPLC performs much better than the WT-based FPLC and the alternate path routing in both of the networks when multiple fibers are available. The network blocking probability decreases sharply with the increasing number of fibers per link, F. For the mesh-torus network, the blocking performance is improved continuously with the increasing F, but the rate of the performance improvement decreases. For the NSFnet, the best performance is achieved when $F = 10$. After that the blocking probability increases slightly with the increasing F. This observation suggests that employing more fibers help to improve the network performance in the mesh-torus network with LP-based FPLC. However, in the irregular NSFnet, the benefit of employing multiple fibers is maximized if we keep the FWR around $20\%(8/40)$. From the wavelength-conversion point of view, the results in Figure 4 also suggests that limited wavelength conversion is not only a solution to reduce the cost of wavelength converters, but also a solution to obtain high network performance.

4. CONCLUSIONS

We study the blocking performance of multifiber WDM networks with the FPLC routing. Two FPLC routing algorithms, WT-based FPLC and LP-based FPLC, are proposed and studied. We have proposed a new analytical model based on the link-load correlation to evaluate the blocking performance of such networks. We have shown that the analytical model is accurate for a variety of network topologies by comparing the analytical results with the simulation results. We observed that the LP-based FPLC routing algorithm can use multiple fibers more efficiently than the WT-based FPLC and the alternate path routing algorithms. In both the mesh-torus and NSFnet networks, limited number of fibers is sufficient to guarantee high performance. For the irregular NSFnet, the best performance is achieved when the FWR is around 20%. This observation suggests that determining appropriate number of fibers per link is critically important to achieve high network performance. Since multiple fibers have the same effect as the limited wavelength conversion in WDM networks, our analytical model and conclusions are also applicable in limited-wavelength-convertible networks.

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