MOBILE COMMUNICATIONS AND QUANTUM TECHNOLOGIES LABORATORY

CHANNEL EQUALIZATION AND MOBILITY MANAGEMENT METHODS IN NEW GENERATION MOBILE COMMUNICATION SYSTEMS

Ph. D. Thesis of Ádám Knapp

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Declaration  I, undersigned Ádám Knapp hereby declare that this Ph.D. dissertation was made by myself, and I only used the sources given at the end. Every part that was quoted word-for-word, or was taken over with the same content, I noted explicitly by giving the reference to the source.

Budapest, Hungary, 26 June 2019

.........................................

Knapp Ádám
Abstract

The radio communication has been significantly evolved in the last decades. The evolution was driven by many forces. On one hand, the development of technology has enabled more and more sophisticated devices and the mass-production of these. On the other hand, the user behaviour and demands have been changed, thus numerous application areas have been emerged as well. Therefore, the wireless and mobile communication networks become real alternatives of the wired information transmission in many cases from technical and from economic aspects as well.

However, many issues originate from the radio communication and the user mobility. Handling of these issues result more efficient operation of such networks. Therefore, this Ph.D. thesis focuses on three related areas.

First, I present a measurement based channel equalization technique. In this case the radio channel’s distortions are equalized at the transmitter side based on the measurements of the receiver side. Analytical expressions for exact bit-error rate are derived that take into account various, widely used fading types. The formulas are validated against simulations. The provided expressions allow to estimate the performance of real wireless systems. The applicability of expressions are detailed in Section 3.3.

Secondly, the investigated problems are related to low-power, wide area networks. In communication systems that use autocorrelation function with one well-determined peak, the correlation peak can be located very precisely. To confirm this, I derived analytical formulas. Using the precise correlation peak detection, I propose a novel, chirp spread spectrum based communication system, where simultaneous data transmission for mobile terminals is provided by pulse position modulation. Finally, I determine the bit-error rate of this system applying the formulas that are presented in Section 2.1.

Thirdly, the mobility management of two-tier LTE/LTE-A network is investigated. During the handover procedure, the system has to specify the new cell, to which the mobile equipment will be assigned. Therefore, the decision mechanism has significant effect on the performance of the handover and, indirectly, on the system as well. I propose a novel han-
dover decision mechanism for two-tier LTE network that takes into consideration different, available parameters of the system to assign the mobile into a new cell. The new cell is chosen to be able to best satisfy the demands of the mobile terminal. The proposed decision mechanism is validated by simulations.
Kivonat

A rádiós kommunikáció jelentős fejlődésen ment át az utóbbi évtizedekben, amit számos tényező hajtott előre. Egyrészt a technológia fejlődése lehetővé tette a kifinomultabb áramkörök létrehozását és nagy mennyiségben való előállítását. Másrészt a felhasználói szokások és velük együtt az igények is megváltoztak, így sok új alkalmazási terület is megnyílt. Ezek mind hozzájárultak ahhoz, hogy a vezetéknélküli és mobil kommunikációs hálózatok mára nagyon sok felhasználási körben valós alternatívát jelentenek a vezetékes információátvitelhez képest mind technikai, mind gazdasági szempontból.

Ugyanakkor számos probléma merül fel a rádiós kommunikációból és a felhasználói mobilitásból fakadóan, melyek kifinomult kezelése lehetővé teszi a hatékonyabb működést. A disszertációmban ezért három területre fókuszálom. Először a mérés alapú csatornakiegyenlítéssel foglalkozom, amikor a vételi oldal mérései alapján az adó oldalon korrekciózzuk a jeleket eliminálándó a rádiós csatorna torzító hatását. Az analitikus vizsgálatok eredményeként a széleskörűen használatos fading modellek figyelembe vételevel bithiba arány összefüggéseket állítok fel, melyeket helyességét és pontosságát szimulációkkal támasztom alá. A képletek segítenek megbecsülni a valós vezetéknélküli rendszerek teljesítőképességét. A képletek felhasználásra konkrét példát is bemutatok a 3.3 fejezetben.

A második problémakör a kis teljesítményű, nagy hatótávolságú rádiós hálózatokhoz kapcsolódik. Az olyan autokorrelációs függvényel rendelkező kommunikációs rendszerekben a korrelációs csúcs nagy pontossággal meghatározható, ahol a függvény egy jól meghatározott csúccsal rendelkezik. Ennek alátámasztására analitikusan levezetett formulákat mutatok be. A korrelációs csúcs detektálás pontosságára építve megalkottam az impulzushelyzet alapú többelhasználós chirp modulációs rendszert, és a 2.1 fejezet képleteit felhasználva meghatározom a rendszer bithiba arányát.

A harmadik problémakör a mobilitásmenedzsmenttel foglalkozik, ami a mobil hálózatok egyik fontos funkciója. A hívásátadás során a rendszernek meg kell határoznia azt az új cellát, ahová a mobil terminált irányítja. Éppen ezért a döntési mechanizmusnak nagy hatása van a hívásátadás és az egész rendszer hatékonyságára. Az általam definiált döntési algoritmus több, a kétrétegű LTE rendszerben elérhető paramétert figyelembe vesz, hogy ki-
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elégítse a felhasználók adatátviteli igényeit. A döntési algoritmus hatékonyságát szimulációs vizsgálatokkal igazoltam.
Köszönetnyilvánítás

Először is szeretnék köszönetet mondani témavezetőmnek Dr. Pap Lászlónak. A folyamatos motiválása mellett ötletei és iránymutatása hatalmas segítséget jelentettek a kutatómunkában és a disszertáció elkészítésében. Külön köszönettel tartozom Dr. Fazekas Péternek, aki a kutatói életpálya felé terelt és akihez bármikor fordulhattam támogatásért és bíztatásért. Köszönöm a kutató- és szerzétársaimnak az eredményes munkát és a közös publikációkat, valamint a Mobil Kommunikáció és Kvantumtechnológiák Laboratórium tagjainak a szakmai légkőrt. Végül, de nem utolsó sorban hálás vagyok a családom és barátaim támogatásáért.
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<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>AFH</td>
<td>Adaptive Frequency-hopping</td>
</tr>
<tr>
<td>AoA</td>
<td>Angel of Arrival</td>
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<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
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<tr>
<td>BFSK</td>
<td>Binary Frequency Shift Keying</td>
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<tr>
<td>BOK</td>
<td>Binary Orthogonal Keying</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<tr>
<td>CDMA</td>
<td>Code-Division Multiple Access</td>
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<tr>
<td>CLPC</td>
<td>Closed-Loop Power Control</td>
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<tr>
<td>CMSS</td>
<td>Chirp Modulation Spread Spectrum</td>
</tr>
<tr>
<td>CSG</td>
<td>Closed Subscriber Group</td>
</tr>
<tr>
<td>CSS</td>
<td>Chirp Spread Spectrum</td>
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<td>DS-CDMA</td>
<td>Direct-Sequence Code-Division Multiple Access</td>
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<td>DSSS</td>
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<td>eNB/eNodeB</td>
<td>Evolved NodeB</td>
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<tr>
<td>EPC</td>
<td>Evolved Packet Core</td>
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<tr>
<td>E-UTRAN</td>
<td>Evolved Universal Terrestrial Radio Access Network</td>
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<tr>
<td>FHSS</td>
<td>Frequency-Hopping Spread Spectrum</td>
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<tr>
<td>FSK</td>
<td>Frequency Shift Keying</td>
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<tr>
<td>GSM</td>
<td>Global System for Mobile Communications</td>
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<tr>
<td>HeNB</td>
<td>Home Evolved NodeB</td>
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<tr>
<td>HF</td>
<td>High Frequency</td>
</tr>
<tr>
<td>HHM</td>
<td>Handover Hysteresis Margin</td>
</tr>
<tr>
<td>HO</td>
<td>Handover / handoff</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
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<tr>
<td>IoT</td>
<td>Internet of Things</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
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<tr>
<td>IPI</td>
<td>Inter-Path Interference</td>
</tr>
<tr>
<td>ISM</td>
<td>Industrial, Scientific and Medical</td>
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<tr>
<td>LDPC</td>
<td>Low-Density Parity-Check</td>
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<tr>
<td>LoRaWAN</td>
<td>Long Range Wide Area Network</td>
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<tr>
<td>LPWAN</td>
<td>Low-Power Wide Area Network</td>
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<tr>
<td>LR-WPAN</td>
<td>Low-Rate Wireless Personal Area Networks</td>
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<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
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<td>LTE-A</td>
<td>Long Term Evolution Advanced</td>
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<td>M2M</td>
<td>Machine-to-Machine</td>
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<td>MAI</td>
<td>Multiple Access Interference</td>
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<tr>
<td>MF</td>
<td>Matched Filter</td>
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<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
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<tr>
<td>MNO</td>
<td>Mobile Network Operator</td>
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<tr>
<td>MRC</td>
<td>Maximal-Ratio Combining</td>
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<tr>
<td>MS</td>
<td>Mobile Station</td>
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<tr>
<td>MTC</td>
<td>Machine Type Communication</td>
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<tr>
<td>NB</td>
<td>NodeB</td>
</tr>
<tr>
<td>NR</td>
<td>New Radio</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal Frequency-Division Multiplexing</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
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<tr>
<td>PAN</td>
<td>Personal Area Networks</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>PPM</td>
<td>Pulse Position Modulation</td>
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<td>PSAM</td>
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<td>PSK</td>
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<td>RSSI</td>
<td>Received Signal Strength Indicator</td>
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<td>RV</td>
<td>Random variable</td>
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<td>SER</td>
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<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
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<td>Spatial Poisson Point Process</td>
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<td>Spread Spectrum</td>
</tr>
<tr>
<td>SSPC</td>
<td>Smart step closed-loop power control</td>
</tr>
<tr>
<td>SUI</td>
<td>Stanford University Interim</td>
</tr>
<tr>
<td>TCPC</td>
<td>Truncated closed-loop power-control</td>
</tr>
<tr>
<td>ToA</td>
<td>Time of Arrival</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>------------------------------------</td>
</tr>
<tr>
<td>TDoA</td>
<td>Time Difference of Arrival</td>
</tr>
<tr>
<td>UE</td>
<td>User Equipment</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
<tr>
<td>UWB</td>
<td>Ultra-wideband</td>
</tr>
<tr>
<td>WAN</td>
<td>Wide Area Network</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
</tr>
</tbody>
</table>
ABBREVIATIONS AND ACRONYMS
### Variables and Symbols

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Amplitude of the elementary chirp signals</td>
</tr>
<tr>
<td>$b$</td>
<td>Binary symbol, $b \in [0, 1]$</td>
</tr>
<tr>
<td>$c$</td>
<td>Threshold value of emitted power</td>
</tr>
<tr>
<td>$d$</td>
<td>Data symbols of BPSK ($d = \pm 1$) and QPSK ($d = \pm 1 \pm j$) modulations</td>
</tr>
<tr>
<td>$E_b$</td>
<td>Bit energy of signal</td>
</tr>
<tr>
<td>$E_s, E_0$</td>
<td>Symbol energy of data carrying and measuring signals</td>
</tr>
<tr>
<td>$f(\cdot)$</td>
<td>Probability density function</td>
</tr>
<tr>
<td>$\text{I}_1(\cdot, \cdot ; \cdot)$</td>
<td>Confluent hypergeometric function</td>
</tr>
<tr>
<td>$g$</td>
<td>Low-pass equivalent complex-valued representation of elementary signals</td>
</tr>
<tr>
<td>$H$</td>
<td>Complex channel coefficient</td>
</tr>
<tr>
<td>$I_0(\cdot)$</td>
<td>Modified Bessel function of the first kind and zeroth order</td>
</tr>
<tr>
<td>$J$</td>
<td>Number of the channel’s output symbols</td>
</tr>
<tr>
<td>$k$</td>
<td>Parameter of Rician fading</td>
</tr>
<tr>
<td>$m$</td>
<td>Parameter of Nakagami fading</td>
</tr>
<tr>
<td>$n, \tilde{n}$</td>
<td>Low-pass equivalent complex-valued representation of AWGN</td>
</tr>
<tr>
<td>$N_0$</td>
<td>Average noise energy</td>
</tr>
<tr>
<td>$\mathbb{E}{\cdot}$</td>
<td>Operator of expected value</td>
</tr>
<tr>
<td>$\text{erf}(\cdot)$</td>
<td>Gauss error function</td>
</tr>
<tr>
<td>$\text{erfc}(\cdot)$</td>
<td>Complementary Gauss error function</td>
</tr>
<tr>
<td>$\exp\cdot$</td>
<td>Exponential function</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability of an event</td>
</tr>
<tr>
<td>$\mathbb{P}{\cdot}$</td>
<td>Operator of probability</td>
</tr>
<tr>
<td>$r, r^*$</td>
<td>Received binary modulated signal with and without channel equalization</td>
</tr>
<tr>
<td>$T, \overline{T}$</td>
<td>Symbol time and symbol measurement time</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Symbol time of chirp modulated signal</td>
</tr>
<tr>
<td>$z, \hat{z}$</td>
<td>Complex fading channel gain and its estimation</td>
</tr>
<tr>
<td>$z_i$</td>
<td>Output signal of $i$-th correlator</td>
</tr>
<tr>
<td>( \delta^2 )</td>
<td>Variance of symmetrical normal distribution</td>
</tr>
<tr>
<td>( \Delta f )</td>
<td>Frequency spreading domain of chirp modulated signal</td>
</tr>
<tr>
<td>( \gamma, \gamma_0, \gamma_s )</td>
<td>SNR of communication channels</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Correlation parameter</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Time delay</td>
</tr>
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</table>
The wireless and mobile communication have been significantly evolved during the last decades. This evolution has been driven by the increasing demands for higher throughput by human users and their changed behaviour. In addition, new group of end users appeared with specific requirements in the last couple of years, namely industry and machines. Internet of Things (IoT), Industrial IoT and Industry 4.0 are such, but similar technologies that focus on these usage domains. The specific requirements of the machines and the industry became determinative in the telecommunication area. Therefore the research and standardization activities take into consideration these requirements, and the next generation networks are planned and prepared to satisfy them more efficiently than nowadays. The requirements of machine-to-machine (M2M) or machine type communication (MTC) in other words include high and constant data throughput, low latency, reliable and robust transmission, high availability, high energy efficiency and low power operations. These are often contradictory from practical aspects, thus specialized communication solutions are developed and started to spread in the world beside the traditional cellular mobile networks.

Low-Power Wide Area Networks (LPWAN) technologies usually handle fixed devices that are often operated from batteries, mobile terminals are not considered. In this thesis I propose a system that is similar to LPWAN, however, it addresses different use cases where terminals are able to move, for example, factory logistic vehicles, even with high speeds, like unmanned aerial vehicles (UAV). Such use cases can be found for fifth generation mobile communication networks (5G) as well. One of the key domains of 5G is M2M communication, however, the complexity, the energy efficiency and the price of receivers makes 5G an uncompetitive technology on the field of LPWAN.

Spread Spectrum (SS) and Closed-Loop Power Control (CLPC) techniques are well-known from decades. SS communication is widely spread, standardized in many ways, and different forms are applied in vast of communication devices. On the one hand, the commu-
communication is based on Direct-Sequence Spread Spectrum (DSSS) technique in IEEE 802.11b standards, as the original Wi-Fi system; in IEEE 802.15.4 standard, which is the physical layer for ZigBee; in Code Division Multiple Access (CDMA) channel access method and in different Global Navigation Satellite Systems (GNSS) like GPS, GLONASS and Galileo. On the other hand, Adaptive Frequency-hopping spread spectrum (AFH) is applied in Bluetooth, which is a variant of Frequency-Hopping Spread Spectrum (FHSS) technique. Finally, Chirp Spread Spectrum (CSS) is important part of IEEE 802.15.4a standard as a supported physical layer in Low-Rate Wireless Personal Area Networks (LR-WPAN) and of LoRaWAN (Long Range Wide Area Network), which is an emerging communications technology for LPWAN as well. The reasons can be found among the advantages of CSS, which include high robustness against channel noise, interference and jamming. Furthermore, the resistance capability against the Doppler effect makes CSS a very good candidate in such mobile communication environment, where support for terminals moving with high speeds and at long ranges is necessary.

The importance of closed-loop power control and channel equalization has been shown by the traditional cellular mobile communication networks. The former handles the so-called near-far problem, which is a quite common issue in mobile networks, while the latter compensates the effect of random fluctuation (fading) appearing in the radio channel. CLPC is part of the operation in 2G GSM as well, however, its significance improved in 3G CDMA-based cellular systems, which utilize CLPC to mitigate in-system interference. The closed-loop power control is also important in fourth generation systems (like 3GPP LTE and its advanced versions) in connection with Multiple Input Multiple Output (MIMO) technologies. It has central role in new directions of mobile communication systems, such as cognitive radio systems. This is motivated by the recent standardization trends towards permitting different heterogeneous communication systems to coexist and share a common wireless channel (e.g. unlicensed spectrum, opportunistic and dynamic spectrum access, spectrum underlay or overlay, etc.). CLPC and channel equalization has been further evolved in the 3GPP New Radio (NR), that is a fifth generation mobile cellular network, by using reference and synchronization signals, i.e. training signals for CLPC and channel equalization, in a more sophisticated way.

Besides handling the prior problems for efficient and reliable communication, the issues related to handovers have to be managed in public cellular networks for higher user experience and for better system performance. One consequence of such networks’ evolution is that the cell sizes are shrinking, while higher user throughput has been achieved. For example the typical cell radius is around several tens of kilometres in 2G GSM, whilst most of 4G LTE cells have ca. 500 m. Furthermore, other deployment scenarios appeared in the recent standards that take into account the coexistence of much smaller cells under the traditional macro cells. In such heterogeneous, two-tier network the other layer beside the macro cells
is collectively called small cells. These can be micro, pico or femto cells depending on their coverage area and feature sets. The latest one is the so-called Home eNodeB in 3GPP terminology that defines a base station deployed by users and not the operator. It is designed to serve a flat or small office and connect to the Mobile Network Operator’s (MNO) network via public Internet connection. Furthermore, outdoor use cases are foreseen as well in that the small cells behave similarly to Wi-Fi access points and provide improved local service. These concepts are further improved in 5G networks. Nevertheless, one of the main challenges is the handover procedure and decision mechanism in this kind of two-tier network.

1.1 Background of the Research

Nowadays, the ICT sector main focus is among others on the 5G and Internet of Things that can be interpreted as network of networks. Besides the well-known public cellular and Wireless Local Area Network (WLAN) technologies, other wireless communication solutions have been emerged that cover specific use case areas, such as LPWAN or IEEE 802.11p for vehicular communication. Therefore, the basics of radio communication are still active research areas, where there are places for novel ideas and approaches.

This section describes the background of fading models and the most important remarks about small cells.

1.1.1 Fading models

Channel equalization is required to cancel or at least mitigate the instantaneous distortions of the radio channel. Traditionally, channel equalization is done at the receiver by using training signals. Usually, synchronization or reference signals are used for this purpose in cellular based systems. By measuring these signals, the receiver can conclude the fluctuation of the channel, and then by using this information, it can restore the originally transmitted data symbols. The random fluctuation of the radio channel may either occur due to multipath propagation, referred to as multipath fading, weather (especially rain), or shadowing from obstacles affecting the radio wave propagation. Fading is often modelled as a random process, which can be categorized into different fading model types. In the present thesis three general and well-known types of fading models are considered:

- Rayleigh fading,
- Rician fading and
- Nakagami fading.
Rayleigh fading is applicable in such cases, where there is no line of sight, i.e. there is no dominant propagation path between the two communicating parties. It is usually a characteristic of dense urban environment. Rayleigh fading models presume that the magnitude of the radio signal fluctuates randomly according to Rayleigh distribution, which is the radial component of the sum of two uncorrelated Gaussian random variables.

Rician (or Ricean or sometimes Rice) fading is also a stochastic model that is applicable for line of sight cases, as well as where there is a strong reflection of signals that is more dominant than the others. In such cases, the amplitude gain is described by Rician distribution that can be interpreted as a generalized Rayleigh distribution. Rician distribution has a $k$ parameter that specifies the ratio between the power of the dominant path and the power of the other, scattered, paths. If $k = 0$, it equals to the Rayleigh distribution.

Nakagami fading is generally very similar to Rician fading model. It is applicable when the received signal is composed from both diffuse and specular scattering, i.e. the electric field is the sum of a strong component (which is not necessarily line of sight) and several contributions with less amplitude. This means that the sum of multiple independent and identically distributed Rayleigh fading signals have a Nakagami distributed signal amplitude. Therefore, it is also a generalization of Rayleigh distribution, which equals to the case when the $m$ parameter of Nakagami distribution is 1. The $m$ parameter relates the amplitudes of strong and weak components. It is relevant to model interference from multiple sources in cellular systems. Note, that Nakagami distribution is quite new (it was first proposed in 1960 [1]). The benefit of using Nakagami model instead of Rician channel is that the Nakagami probability density function has a closed form expression that is simpler to evaluate numerically (it does not contain Bessel functions) and fits better some measurements. However, the specular (strong) component in the Rician channel is deterministic.

Finally, flat or frequency non-selective fading is assumed for analytical investigations meaning that the bandwidth of the signal is smaller than the coherence bandwidth of the radio channel. Thus, all frequency components of the signal have the same magnitude of fading.

### 1.1.2 Small cells

As mentioned, recent trends include the application of small cells. They have the following benefits:

- increasing capacity (locally and system-level);
- improving coverage, particularly for indoor;
- enabling offloading the macro cells;
enhancing user experience, especially increasing the typical available data rates.

Contrarily to Wi-Fi access points that operate in ISM-bands (Industrial, Scientific and Medical), small cells are functioning in the licensed band as macro cells. Besides that, there are already mentioned use cases, where small cells are very similar to Wi-Fi hotspots. The following list summarizes the challenges related to small cells:

- small coverage area due to its limited transmission power;
- consequence of the previous attribute is the frequent, unnecessary handovers;
- dense and uncoordinated deployment, because it is possible to install them by the users;
- uncontrolled interference source due to the unsupervised deployment;
- unreliable backhaul link, because they can be connected to MNO network via public Internet;
- different access control: small cells usually have three main access types:
  - open: everyone is allowed to connect.
  - closed: access granted for equipment that is listed in the closed subscriber group (CSG).
  - hybrid: mixture of previous ones. Non-CSG users get limited bandwidth.

As one can see, it is important to investigate and improve the efficiency of handover procedure and decision mechanism in two-tier heterogeneous cellular networks. There is no strict definition of handover decision mechanism available in the standards. Traditionally, the strongest cell mechanism is applied in practice. During that kind of decision, the signal strength of base stations, i.e. eNodeBs in LTE/LTE-A networks, is taken into account, and the eNodeB with the strongest signal is selected as the target cell for handover. This usually equals to the case when the closest eNodeB is picked due to the attenuation of the used frequency bands. Therefore, the strongest cell decision leads to unnecessary and frequent handovers because of the small coverage area of small cells.

1.2 Motivation and Organization of the Document

The dissertation summarizes all my research results from the recent years. By investigating the channel equalization mechanism, formulas are derived that give good estimation of bit
error and symbol error rates, as well as they are applicable in practice. Similarly, spread spectrum based radio systems are analysed, where results related to correlation peak detection are determined. The outcome of these investigation allowed to construct a novel communication system that combines pulse position modulation with chirp spread spectrum transmission schema. Finally, a practical handover decision algorithm is provided to utilize the small cells even more.

1.2.1 Goal of the Thesis

The goal of the theorems related to measurement based feedback channel equalization is to present a unified analytical method for the accurate error rate analysis of coherent Binary Phase Shift Keying (BPSK) and orthogonal noncoherent Binary Frequency Shift Keying (BFSK) modulations in wireless communication systems with fast closed-loop power control mechanisms subjected to general fading and using a measuring based channel estimation in a separated noisy channel. The main contributions are new exact expressions for error rates of coherent BPSK and orthogonal noncoherent BFSK modulations in different fading environments (including Rayleigh, Nakagami, and Rician fading) and subjected to a non-exact channel transmission parameter measured in a noisy pilot channel. The results of this dissertation can be used to compare the performance of a coherent BPSK or orthogonal noncoherent BFSK wireless network that has a fast closed-loop power control system and uses diversity combining algorithms to parallel solve the near-far problem and channel equalization.

Then, the investigations related to correlation peak detection enables to construct a more precise synchronization system for spread spectrum based communication techniques, as well as to utilize this improved synchronization for network services such as localization. A new correlator is proposed namely the sliding-and-tracking correlator that is based on the cooperation of two correlators. Mathematical model is established to investigate and compare the wrong decision probability of the sliding-and-tracking correlator against the traditional sliding correlator. Furthermore, I also developed a communication system that partially applies these results and partially uses the prior CLPC mechanism. The key feature of the scheme is that it provides simultaneous transmission for multiple mobile terminals using a new communication method. The method combines the pulse position modulation with the chirp spread spectrum technique. This transmission scheme is able to work by itself as LPWAN.

In the last section, I focus on the handover decision algorithms of the two-tier LTE/LTE-A network. Furthermore, I propose a new decision procedure that takes into account several parameters, like the speed of the UEs and the actual load of neighbouring cells. The handover decision algorithm and the related results can support the standardization activities of
future mobile cellular network, particularly the New Radio. It is a candidate fifth generation standard for global usage advocated by many communication companies worldwide under the aegis of the 3rd Generation Partnership Project (3GPP).

1.2. Structure of the Thesis

The rest of the dissertation is organized as follows.

In Chapter 2 channel equalization related researches are presented. Firstly, the relevance of channel equalization is highlighted, then the system model is introduced: in the investigated approach the transmitter performs the equalization based on the feedback measurements of the receiver. Secondly, the numerical calculations are described that led to the theorems. The first thesis presents general bit-error rate formula for the measurement based feedback channel equalization that contains the channel’s fading function as a general parameter. Then, the next theorems provide exact calculations for popular fading model types, namely Rayleigh, Rice and Nakagami fadings. Furthermore, theorems are presented that further improve these results by taking into account the maximum transmission power as an input parameter of the expressions. Thus, the formulas reflect even more practical aspects. In the second section of the chapter, theorems take into consideration the phase shifting characteristic of the channel and give exact BER expressions for BPSK and QPSK modulations. Finally, the investigations led to the optimization problem of necessary pilot signals, and numerical solution is proposed assuming LTE/LTE-A system.

In Chapter 3, after brief introduction of spread spectrum based communication, a novel method is described and analysed for correlation peak detection. Then, exact bit error rate calculations are presented for the detection method. Using these results, new multi-user schemes are defined that are based on chirp spread spectrum. Then, the BER related performance investigations of the proposed system are introduced.

In Chapter 4, the two-tier LTE network is summarized and the challenges regarding to the mobility management are emphasized. Tackling the issue of frequent handovers in the mentioned system, a novel handover decision algorithm is proposed. It uses the velocity of the user equipment, the load and the RSSI of the candidate cells, as well as their access mode. Therefore, it provides load balancing in the network. The algorithm is implemented in a C++ based LTE simulation software that was developed at the department. The performance related results, like overall throughput, average user latency and number of handovers are compared to the legacy handover decision algorithm.

Finally, Chapter 5 concludes the theorems and gives an outlook for further research topics.
This chapter introduces the channel equalization related theorems. First, the measurement based feedback channel equalization is discussed. Bit error rates can be calculated using the proposed formulas, while taking into account the SNR values of the measurement and the normal, data communication channels. Then, symbol error rate expressions are presented that are related to the estimation error of the signal’s phase component. Since the system models are slightly different in the two cases, they are introduced separately.

2.1 Measurement based feedback channel equalization

In the last two decades intensive investigations of wireless systems with closed-loop power control and channel equalization have been carried out. This area in different contexts but always raised in every generation of mobile communication systems. The CLPC solves the so-called near-far problem primarily, however, the channel equalization compensates the effect of random fluctuation (fading) appearing in the radio channel. Traditionally, the prior problem is used to be solved with a high delay closed-loop power control system, while solution of the latter one requires diversity techniques ([2]). The closed-loop power control had an important role in the early mobile technologies (i.e. NMT and GSM), however, its significance increased when the third generation CDMA systems were introduced. These systems require power control for the in-system interference coordination to provide reliable and fair communication. This topic remained in focus since then, numerous papers were published ([3]-[8]) in the recent years. The closed-loop power control is also important in fourth generation systems (Long Term Evolution and its advanced versions) in connection with MIMO technologies ([9]-[13]), as well as in 5G that enables the usage of higher order and more advanced multi-antenna techniques including 128 or even more antenna elements [14]-[16].
2.1.1 Related works

To date, most research have focused on the performance of separated closed-loop power control system and diversity techniques to solve the near-far problem and the channel equalization in presence of white Gaussian noise and some special types of intentional interference. The most relevant to the present dissertation are those that analyse the error rate of the closed-loop power control scheme for the multi-rate services in the third and forth generation wide-band systems. In [17] the authors demonstrate that the long scrambling pseudo-noise code, besides its well-known feature in differentiating users and base stations, can improve power control false command over a frequency-selective fading channel as well. It is shown that the closed-loop power control error is a composite function of the spreading factor, target $E_b/N_0$ and Doppler frequency. Authors of [18] proposed an algorithm that computes the solution to the power control problem with closed-loop effects, and analytical and simulation results show that the algorithm converges under the same conditions as that given in the earlier results. In [19] the authors analyse the system performance of a truncated closed-loop power-control (TCPC) scheme for uplink in direct-sequence/code-division multiple-access cellular systems over frequency-selective fading channels. Closed-form formulas are successfully derived for performance measures, such as system capacity, average system transmission rate, mobile station’s (MS) average transmission rate, MS power consumption, and MS suspension delay. In [20] smart step closed-loop power control (SSPC) algorithm is proposed in a Direct-Sequence Code-Division Multiple Access (DS-CDMA) receiver in the presence of frequency-selective Rayleigh fading. This receiver consists of three stages. In the first stage the desired users’ signal in an arbitrary path is passed and the inter-path interference (IPI) is reduced. Also in this stage, the multiple access interference (MAI) from other users is reduced. Thus, the matched filter (MF) can be used for the MAI and IPI reduction in the second stage. Also in the third stage, the output signals from the matched filters are combined according to the conventional maximal-ratio combining (MRC) principle and then are fed into the decision circuit of the desired user. In [21] the performance of SIR-based CLPC is analytically investigated and an analytical expression of the CLPC under fast fading is also produced. Finally, a quantized-step size power control algorithm, replacing the hard limiter is considered.

2.1.2 System model

Here, a novel channel equalization is presented with a unified analytical method for the accurate error rate analysis of coherent and noncoherent binary modulations. It is assumed that the wireless communication system applies fast closed-loop power control mechanism and uses a measurement based channel estimation in a separated noisy channel. Furthermore, it is affected by general fading.
2.1. MEASUREMENT BASED FEEDBACK CHANNEL EQUALIZATION

The low-pass equivalent complex-valued representation of the binary modulated signal $r^*$ without channel equalization is given at the receiver front end during a signalling interval by

$$r^* = \sqrt{E_s} g^{(b)} z + n, \ t \in (0, T], \quad (2.1)$$

where $E_s$ is the average symbol energy without fading and power control, $T$ is the binary symbol time, $b \in [0, 1]$ is the binary symbol and $\left\{g^{(b)}, \left\|g^{(b)}\right\|^2 = 1\right\}$ are the low-pass equivalent complex-valued representation of the elementary signals (in coherent case $g^{(0)} = -g^{(1)}$, and in noncoherent case $\langle g^{(0)}, g^{(1)} \rangle = 0$ typically), $z$ is the complex fading channel gain and $n$ is the low-pass equivalent complex-valued representation of the additive white Gaussian noise (AWGN) with $E\left[ n \right] = N_0$. $z$ is a random variable that represents the instantaneous amplitude of the received binary signal ($E\left[ |z|^2 \right] = 1$). The distribution of $z$ depends on the fading scenario. In this model, three different types of non-selective slow fading scenarios are considered, namely Rayleigh, Rician and Nakagami fading models. Furthermore, it is supposed that the fading channel gain is estimated in the receiver by measuring an independent pilot channel. The fading channel gain of the communication and measuring (pilot) channel are equivalent.

In the proposed approach the fading channel gain is estimated at the receiver based on the use of an unmodulated pilot signal in an independent pilot (measuring) channel. Thus, the low-pass equivalent complex-valued representation of the measuring signal at the receiver front end during a measuring interval is

$$\tau = \sqrt{E_0 z} + n, \ t \in (0, \bar{T}], \quad (2.2)$$

where $E_0$ is the mean "symbol" energy of the measuring signal, $\bar{T}$ is the measuring time interval and $n$ is the low-pass equivalent complex-valued representation of the additive white Gaussian noise in the measuring channel with $E\left[ n \right] = N_0$. It is assumed that the noises of the communication and the measuring (pilot) channel are independent.

For the fading suppression, i.e. for channel equalization, the transmitted signal is corrected by the estimated fading channel gain in this model. The best estimation of $z$ is given by

$$\hat{z} = \frac{\tau}{\sqrt{E_0}} = z + \frac{n}{\sqrt{E_0}}.$$  

It is easy to show, that $\hat{z}$ is a conditionally complex Gaussian random variable. If $z$ is given, the mean value of $\hat{z}$ is $E\left[ \hat{z} \right] = z$ and the variance of it is given by

$$E\left[ (\hat{z} - E\left[ \hat{z} \right])^2 \right] = \frac{N_0}{E_0} = \frac{1}{\gamma_0}, \quad (2.2)$$
CHAPTER 2. CHANNEL EQUALIZATION

Figure 2.1: The model of the proposed system. The fading channel gain of the measuring (pilot) and the communication channel are the same ($z$), but the noises ($n, \bar{n}$) are independent. The amplitude of the estimated fading channel gain ($|\hat{z}|^2$) is fed back to the transmitter via an error-free digital communication channel. The transmitter equalizes the transmitted signal’s power based on this information.

where $\gamma_0$ is the signal to noise ratio (SNR) of the measuring channel without fading. Note that an unmodulated pilot signal is used during the measurement process, hence the received energy via the pilot channel and therefore the accuracy of the fading channel gain estimation depends on the measuring time interval ($T$) as well. If the measurement process and feedback delay is less than the fluctuation of the fading (that is assumed here), the noise of the pilot channel ($\bar{n}$) can be neglected.

The present model assumes that an error-free digital communication channel is applied to feed back the amplitude of the estimated fading channel gain ($|\hat{z}|^2$) to the transmitter. Then, the transmitter compensates the signal’s power according to this information before radiating it. The closed-loop power control procedure is represented in Figure 2.1.

The low-pass equivalent complex-valued representation of the corrected binary modulated signal is given at the receiver front end during a signalling interval by

$$ r = \sqrt{E_s} g^{(b)}(z) \frac{|z|}{|\hat{z}|^2} + n, \quad t \in (0, T], $$

where $r$ is a conditionally Gaussian RV, if $z, g^{(b)}$ and $\bar{n}$ are given. Using the elementary description of communication systems, the effective conditionally SNR of the communication channel is

$$ \Gamma(\gamma_s|z, \bar{n}) = \frac{E_s}{N_0} \left| \frac{|z|^2}{|z + \frac{\bar{n}}{\sqrt{E_0}}|^2} \right| = \gamma_s \left| \frac{|z|^2}{|z + \frac{\bar{n}}{\sqrt{E_0}}|^2} \right|. $$
where $\gamma_s$ is the SNR of the communication channel without fading. So far, every part of the proposed system is covered and described mathematically. From now on, this system model is called \textit{theoretical} measurement based feedback channel equalization. The novelty of this method is that the equalization is done by the transmitter based on measured and feedback information from the receiver. However, the model has a major issue, namely that in some cases the transmitter has to use infinity power for the equalization. This is due to the fact that the Gaussian random variable $\hat{z}$ can take values close to zero with non-zero probability. To overcome this issue, the system model is slightly modified in the following way to take into account the maximum transmission power of the transmitter. Let us introduce the $h(x)$ function that is a simple threshold function considering an upper limit ($c$) of the power in the transmitter:

$$h(x) = \begin{cases} 
    c & \text{if } |x| \leq c \\
    x & \text{if } |x| > c 
\end{cases} \quad (2.3)$$

So, the previously presented equations will change as follows. First, the low-pass equivalent complex-valued representation of the corrected binary modulated signal is described at the receiver front end during a signalling interval by

$$r = \sqrt{E_s} g^{(b)} \frac{z}{\sqrt{h(|z|^2)}} + n, \ t \in (0, T],$$

where $\sqrt{h(|z|^2)}$ is interpreted as the correction function at the transmitter that depends only on the cardinality of $\hat{z}$. Then, the effective conditional SNR of the communication channel is modified as well:

$$\Gamma(\gamma_s | z, n) = \frac{E_s}{N_0 h \left( |z + \frac{n}{\sqrt{E_0}}|^2 \right)} = \gamma_s \frac{|z|^2}{h \left( |z + \frac{n}{\sqrt{E_0}}|^2 \right)}.$$

This new system model is practically more feasible, therefore it is called \textit{practical} measurement based feedback channel equalization in this dissertation. In the following, theorems will be presented for both system models taking into consideration coherent and noncoherent binary transmissions with the presented fading models. But before that, it is necessary to specify the task in a mathematically accurate manner.

Let us introduce $X$ and $Y$ random variables as

$$X = |z|^2 \quad \text{and} \quad Y = \left| z + \frac{n}{\sqrt{E_0}} \right|^2, \quad (2.4)$$
therefore
\[
\Gamma (\gamma |z, n) = \frac{X}{\gamma} ,
\]
for the theoretical system model and
\[
\Gamma (\gamma |z, n) = \gamma \frac{X}{h(Y)} = \gamma \frac{X}{\gamma'} ,
\]
for the practical system model, where
\[
h (y) = y' = \begin{cases} 
  c & \text{if } y \leq c \\
  y & \text{if } y > c 
\end{cases}
\]
(2.6)

Now, let us determine the \( f_{Y|X} (y|x) \) conditional probability density function of \( Y \), if \( X \) is given. After some simple mathematical manipulations, one can get the following result:
\[
f_{Y|X} (y|x) = \gamma_0 \exp (-\gamma_0 (x + y)) I_0 (2\gamma_0 \sqrt{xy}) ,
\]
(2.7)

where \( I_0 (.) \) is the modified Bessel function of the first kind and zeroth order and \( \gamma_0 \) is the SNR of the measurement channel as mentioned in Eq. 2.2. In addition
\[
f_{XY} (x, y) = \gamma_0 \exp (-\gamma_0 (x + y)) I_0 (2\gamma_0 \sqrt{xy}) f_X (x) .
\]
Therefore the PDF of \( Y \) is given by
\[
f_Y (y) = \int_0^\infty \gamma_0 \exp (-\gamma_0 (x + y)) I_0 (2\gamma_0 \sqrt{xy}) f_X (x) \, dx ,
\]
where \( f_X (x) \) is the PDF of the fading channel gain:
\[
f_X (x) = \exp (-x) \text{ in the case of Rayleigh},
\]
(2.8)
\[
f_X (x) = (1 + k) \exp (-k - (1 + k)x) I_0 \left( 2\sqrt{k (1 + k)x} \right) \text{ in the case of Rician},
\]
(2.9)
\[
f_X (x) = \frac{x^{m-1}}{(m-1)!} m^m \exp (-mx) \text{ in the case of Nakagami fading models}.
\]
(2.10)

Furthermore,
\[
f_{XY} (x, y) = f_{Y|X} (y|x) f_X (x)
\]
(2.11)
for the theoretical system model, and
\[
f_{XY'} (x, y') = \begin{cases} 
  A (x) \delta (y' - c) f_X (x) & \text{if } y' = c \\
  f_{Y|X} (y'|x) f_X (x) & \text{if } y' < c 
\end{cases}
\]
(2.12)
for the practical system model, where
\[
A (x) = \int_0^c f_{Y|X} (y|x) \, dy.
\]
2.1.3 Derivation of the average error rate for coherent binary transmission in fading channels

In this section, a method is presented for efficient computation of the average error probability of the coherent binary channels with the above mentioned feedback channel equalization by reducing the number of improper integrals. The method is capable of decreasing the complexity of the mathematical problem and increasing the accuracy of the calculation. A key to the proposed method is to transform the conditional error probability function of the fading channel into a special form, which enables using closed integral formulas and simplifies the calculation of the average error rate.

**Theorem 2.1.** The following analytical expression determines the average bit error probability in the theoretical measurement based feedback channel equalization model assuming coherent binary transmission and taking into account the general fading model:

\[
P_b(\gamma_s) = \frac{1}{\pi} \int_0^\infty \int_0^\infty \frac{x}{y \cos^2(\theta)} \gamma_0 \exp(-\gamma_0 (x+y)) I_0(2\gamma_0 \sqrt{xy}) f_X(x) \, d\theta \, dx \, dy,
\]

where \(f_X(x)\) is the PDF of the general fading channel gain, \(\gamma_0\) is the SNR of the measurement channel and \(\gamma_s\) is the SNR of the communication channel.

**Proof.** It is well known from the literature that the bit error rate of a binary coherent communication system is given by ([22])

\[
P_b(\gamma_s | z, n) = \frac{1}{2} \text{erfc} \left( \sqrt{\text{SNR}} \right),
\]

where \(\text{erfc} (\cdot)\) is the complementary Gauss error function, \(\text{SNR}\) is given in Eq. 2.5 and based on the well-known lemma ([23]):

\[
\frac{1}{2} \text{erfc} \left( \sqrt{\text{SNR}} \right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp \left( -\frac{\text{SNR}}{\cos^2(\theta)} \right) \, d\theta.
\]

One can get the following general expression of the conditional error probability of the coherent binary system:

\[
P_b(\gamma_s | z, n) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp \left( -\frac{\gamma_s x}{y \cos^2(\theta)} \right) \, d\theta.
\]
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Now, use the $Y$ and $X$ random variables from Eq. 2.4, where $Y$ is a conditionally Rician distributed RV (if $z$ is given) with PDF from Eq. 2.7, as $\bar{n}$ is a complex Gaussian RV with independent uniformly distributed real and imaginary parts. Using the conditional error probability function of the coherent receiver Eq. 2.15, one can obtain the error function of $P_b$ after calculating the average, i.e. the expected value according to the $Y$ and $X$ random variables:

$$P_b(\gamma_s) = \frac{1}{\pi} \int \mathbb{E} \left[ \exp \left( -\frac{\gamma_s x}{y\cos^2(\theta)} \right) \right] d\theta,$$

where

$$\mathbb{E} \left[ \exp \left( -\frac{\gamma_s x}{y\cos^2(\theta)} \right) \right] = \int \int \exp \left( -\frac{\gamma_s x}{y\cos^2(\theta)} \right) f_{XY}(x,y) dx dy,$$

therefore the error probability can be calculated by the following triple integral:

$$P_b(\gamma_s) = \frac{1}{\pi} \int \int \int \exp \left( -\frac{\gamma_s x}{y\cos^2(\theta)} \right) f_{XY}(x,y) d\theta dx dy.$$

Finally, one can get the expression of Theorem 2.1 by substituting Eq. 2.11 in the formula above.

Before going into the details of the different fading models, a similar theorem is presented related to the practical system model.

Theorem 2.2. The following analytical expression determines the average bit error probability in the practical measurement based feedback channel equalization model assuming coherent binary transmission and taking into account the general fading model:

$$P_b(\gamma_s) = \frac{1}{\pi} \int \int \int \exp \left( -\frac{\gamma_s x}{\bar{n}'\cos^2(\theta)} \right) f_{XY'}(x,y') d\theta dx dy'.$$

(2.16)

where $y'$ and $f_{XY'}(x,y')$ are given as before by Eq. 2.6 and Eq. 2.12, i.e.

$$h(y) = y' = \begin{cases} c & \text{if } y \leq c \\ y & \text{if } y > c \end{cases}$$

and

$$f_{XY'}(x,y') = \begin{cases} A(x) \delta(y'-c)f_X(x) & \text{if } y' = c \\ f_{Y'|X}(y'|x)f_X(x) & \text{if } y' > c \end{cases}$$
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where

\[ A(x) = \int_{0}^{c} f_{Y|X}(y|x) \, dy. \]

**Proof.** The expression can be derived by using \( y = y' \) and \( Y = Y' \) in the steps of the previous proof. ■

By these theorems, the average bit error probabilities are available in case of coherent binary transmission considering general fading model. Now, the already presented fading models are investigated and the average bit error probability functions are determined for Rayleigh, Rician and Nakagami fading models.

### 2.1.3.1 Average error rate for coherent binary transmission in Rayleigh fading channel

This subsection presents the average bit error probabilities for the theoretical and practical system models supposing coherent binary transmission and Rayleigh fading as channel model. For the Rayleigh fading model, the PDF of Rayleigh distribution is used as earlier presented in Eq. 2.8, i.e. \( f_X(x) = \exp(-x) \). Basically, the task is to substitute this PDF into Eq. 2.13 and Eq. 2.16, and transform them into a more convenient form.

**Theorem 2.3.** The average bit error probability function is given by the following expression in the **theoretical** measurement based feedback channel equalization model assuming coherent binary transmission and Rayleigh fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0}{\pi} \int_{0}^{\pi} \int_{0}^{\infty} \frac{1}{1 + \gamma_0 + \frac{\gamma}{y \cos^2(\theta)}} \exp \left( -\gamma_0 y \frac{1 + \frac{\gamma}{y \cos^2(\theta)}}{1 + \gamma_0 + \frac{\gamma}{y \cos^2(\theta)}} \right) \, dy \, d\theta. \tag{2.17}
\]

**Proof.** After substituting \( f_X(x) = \exp(-x) \) into Eq. 2.13, it is required to solve the following integral:

\[
\int_{0}^{\infty} \exp \left( -x \left( 1 + \gamma_0 + \frac{\gamma}{y \cos^2(\theta)} \right) \right) I_0(2\gamma_0 \sqrt{xy}) \, dx.
\]

Use the following identity from [24, Eq. (4.86)]

\[
\int_{0}^{\infty} \exp(-at)I_0(2\sqrt{bt}) \, dt = \frac{1}{a} \exp \left( \frac{b}{a} \right)
\]
with the next variables:
\[ t = x, \quad a = 1 + \gamma_0 + \frac{\gamma_s}{y \cos^2(\theta)} \quad \text{and} \quad b = \gamma_0^2 y. \]

Thus, one can get the Eq. 2.17.

**Theorem 2.4.** The average bit error probability function is given by the following expression in the practical measurement based feedback channel equalization model assuming coherent binary transmission and Rayleigh fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \frac{1}{1 + \gamma_0 + \frac{\gamma_s}{h(y) \cos^2(\theta)}} \exp \left( -\gamma_0 y \frac{1 + \frac{\gamma_s}{h(y) \cos^2(\theta)}}{1 + \gamma_0 + \frac{\gamma_s}{h(y) \cos^2(\theta)}} \right) dy \, d\theta, \quad (2.18)
\]

where the integral by \( y \) should be calculated in two intervals: \([0, c]; h(y) = c\) and \((c, \infty); h(y) = y\).

**Proof.** To prove this theorem, one can get the following, modified integral after the substitution that takes into account the maximum transmission power of the transmitter. Then, the same steps has to be performed as during the previous proving:

\[
\int_0^{\infty} \exp \left( -x \left( 1 + \gamma_0 + \frac{\gamma_s}{h(y) \cos^2(\theta)} \right) \right) I_0 \left( 2\gamma_0 \sqrt{xy} \right) dx.
\]

\[ \blacksquare \]

### 2.1.3.2 Average error rate for coherent binary transmission in Rician fading channel

Similarly to the Rayleigh fading model, in this subsection the average bit error probabilities are provided for the theoretical and practical system models supposing coherent binary transmission and Rician fading as channel model. In this case, the PDF of Rician distribution is applied as described in Eq. 2.9.

**Theorem 2.5.** The average bit error probability function is given by the following expression in the theoretical measurement based feedback channel equalization model assuming coherent binary transmission and Rician fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0}{\pi} \sum_{l=0}^{\infty} \frac{k^l (1+k)^l}{l!} \int_0^{\infty} \frac{1}{1 + \gamma_0 + \frac{\gamma_s}{(1+k) + \gamma_0 + \frac{\gamma_s}{y \cos^2(\theta)}}} \exp \left( -\gamma_0 y \frac{1 + \frac{\gamma_s}{(1+k) + \gamma_0 + \frac{\gamma_s}{y \cos^2(\theta)}}}{1 + \gamma_0 + \frac{\gamma_s}{h(y) \cos^2(\theta)}} \right) dy \, d\theta, \quad (2.19)
\]
where $k$ is the parameter of the Rician distribution and $\frac{1}{\Gamma(\cdot, \cdot)}$ is the confluent hypergeometric function.

**Proof.** Again, after substituting Eq. 2.9 into the generalized formula Eq. 2.13, the calculation is simplified to the handling of the following integral:

$$
\int_0^\infty \exp \left( -x \left( (1+k) + \gamma_0 + \frac{\gamma_0^2}{y \cos^2(\theta)} \right) \right) I_0 \left( 2 \sqrt{k(1+k)x} \right) I_0 \left( 2 \gamma_0 \sqrt{xy} \right) dx.
$$

Now, using the serial expression of $I_0(.)$ as

$$
I_0 \left( 2 \sqrt{k(1+k)x} \right) = \sum_{l=0}^{\infty} \frac{k^l (1+k)^l}{(l!)^2} x^l,
$$

then, the above integral modifies as follow:

$$
\sum_{l=0}^{\infty} \frac{k^l (1+k)^l}{(l!)^2} \int_0^\infty x^l I_0 \left( 2 \gamma_0 \sqrt{xy} \right) \exp \left( -x \left( (1+k) + \gamma_0 + \frac{\gamma_0^2}{y \cos^2(\theta)} \right) \right) dx.
$$

Furthermore, applying Eq. 6.643.2 and Eq. 9.220.2 from [23], one can get the Eq. 2.19. ■

**Theorem 2.6.** The average bit error probability function is given by the following expression in the practical measurement based feedback channel equalization model assuming coherent binary transmission and Rician fading model:

$$
P_b(\gamma_s) = \frac{\gamma_0}{\pi} \int_0^{\infty} \frac{\gamma_s^2 y}{(1+k) + \gamma_0 + \frac{\gamma_0^2 y}{h(y) \cos^2(\theta)}} \exp \left( -\gamma_0 y \right) \exp \left( -k \right) \frac{\exp \left( -\frac{\gamma_0 y}{h(y) \cos^2(\theta)} \right)}{\left( (1+k) + \gamma_0 + \frac{\gamma_0^2 y}{h(y) \cos^2(\theta)} \right)^{l+1}} dy d\theta,
$$

(2.20)

where the integral by $y$ should be calculated in two intervals: $\{0, c\}$; $h(y) = c$ and $\{(c, \infty)\}; h(y) = y$.

**Proof.** As before, in case of practical system model the same steps lead to this theorem, just it is necessary to use

$$
\frac{\gamma_s}{h(y) \cos^2(\theta)}
$$

instead of

$$
\frac{\gamma_s}{y \cos^2(\theta)}.
$$

Note that substituting $k = 0$ yields the formulas of Rayleigh fading. This meets the expectations, since the Rician fading model is the generalization of the Rayleigh fading model.
2.1.3.3 Average error rate for coherent binary transmission in Nakagami fading channel

This subsection describes the average bit error probabilities for the theoretical and practical system models assuming coherent binary transmission and Nakagami fading as channel model. For the Nakagami fading model, the PDF of Nakagami distribution is used from Eq. 2.10.

**Theorem 2.7.** The following exact formula determines the average bit error probability in the theoretical measurement based feedback channel equalization model supposing coherent binary transmission and Nakagami fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0 m^m}{\pi} \int_0^\infty \int_0^\infty \exp\left(-\gamma_0 y\right) \left(m + \gamma_0 + \frac{\gamma}{y \cos^2(\theta)}\right)^m {}_1F_1\left(m, 1; \frac{\gamma_0^2 y}{m + \gamma_0 + \frac{\gamma}{y \cos^2(\theta)}}\right) dy d\theta, \tag{2.21}
\]

where \( m \) is the parameter of the Nakagami distribution and \(_1F_1(., . ; .)\) is the confluent hypergeometric function.

**Proof.** Here, the following integral has to be taken into account after substituting Eq. 2.10 in 2.13:

\[
\int_0^\infty x^{m-1} \exp\left(-x \left(m + \gamma_0 + \frac{\gamma}{y \cos^2(\theta)}\right)\right) I_0\left(2\gamma_0 \sqrt{xy}\right) dx.
\]

By using the equations of [23, Eq. 6.643.2 and Eq. 9.220.2], one can get Eq. 2.21 as result. \( \blacksquare \)

**Theorem 2.8.** The following exact formula determines the average bit error probability in the practical measurement based feedback channel equalization model supposing coherent binary transmission and Nakagami fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0 m^m}{\pi} \int_0^\infty \int_0^\infty \exp\left(-\gamma_0 y\right) \left(m + \gamma_0 + \frac{\gamma}{h(y) \cos^2(\theta)}\right)^m {}_1F_1\left(m, 1; \frac{\gamma_0^2 y}{m + \gamma_0 + \frac{\gamma}{h(y) \cos^2(\theta)}}\right) dy d\theta, \tag{2.22}
\]

where the integral by \( y \) should be calculated in two intervals: \([0, c] ; h(y) = c\) and \((c, \infty) ; h(y) = y\).
Proof. Again, for the practical system model one can perform the same steps to get theorem’s equation by using
\[ \frac{\gamma_s}{h(y)\cos^2(\theta)} \] instead of \[ \frac{\gamma_s}{ycos^2(\theta)}. \]

Note that using \( m = 1 \) both presented expressions equal to the Rayleigh fading model related formulas. This is also expected thanks to the characteristic of the Nakagami distribution.

To summarize briefly, Eq. 2.17 and Eq. 2.18, Eq. 2.19 and Eq. 2.20, and Eq. 2.21 and Eq. 2.22 are new exact expressions based on Eq. 2.13 and Eq. 2.16 for the average error probabilities of the coherent binary fading channels with measurement based feedback channel equalization over different fading channels, namely Rayleigh, Rician and Nakagami channels.

2.1.4 Derivation of the average error rate for noncoherent binary transmission in fading channels

This section presents a novel method for efficient computation of the average error probability of the noncoherent binary channels with the earlier described feedback channel equalization similarly to the coherent case. Basically, the same logic is followed here as in coherent cases. First, the general formulas are determined for the average error rate, then these expressions are used to calculate the exact probability functions of the different fading models.

Theorem 2.9. The average bit error probability function is given by the following formula in the theoretical measurement based feedback channel equalization model supposing noncoherent binary transmission and taking into account the general fading model:
\[
P_b(\gamma_s) = \frac{1}{2} \int_0^\infty \int_0^\infty \exp \left( -\frac{\gamma_s}{2y} \right) f_{XY}(x,y) \, dx \, dy.
\] (2.23)

Proof. It is well known from the literature that the bit error rate of a noncoherent binary communication system with orthogonal elementary signals is given by ([22])
\[
P_b(\gamma_s|z,\pi) = \frac{1}{2} \exp \left( -\frac{\text{SNR}}{2} \right),
\]
where SNR is given in Eq. 2.5 and one can calculate the following general expression of the conditional error probability of the noncoherent binary system:
\[
P_b(\gamma_s|z,\pi) = \frac{1}{2} \exp \left( -\frac{\gamma_s}{2y} \right),
\] (2.24)
Now, let us use the $Y$ and $X$ random variables from Eq. 2.4, where $Y$ is a conditionally Rician distributed random variable (if $z$ is given) with PDF Eq. 2.7, as $\overline{\pi}$ is a complex Gaussian RV with independent uniformly distributed real and imaginary parts. By applying the conditional error probability function of the noncoherent receiver Eq. 2.24, one can obtain the following error function of $P_b$ after calculating the average according to the $Y$ and $X$ random variables:

$$P_b(\gamma_s) = \frac{1}{2} \mathbb{E} \left[ \exp \left( -\frac{\gamma_s x}{2y} \right) \right],$$

where

$$\mathbb{E} \left[ \exp \left( -\frac{\gamma_s x}{2y} \right) \right] = \int_0^\infty \int_0^\infty \exp \left( -\frac{\gamma_s x}{2y} \right) f_{XY}(x,y) \, dx \, dy.$$  

Thus, the error probability can be calculated by the presented double integral of the theorem Eq. 2.23.

**Theorem 2.10.** The average bit error probability function is given by the following expression in the practical measurement based feedback channel equalization model supposing noncoherent binary transmission and taking into account the general fading model:

$$P_b(\gamma_s) = \frac{1}{2} \int_0^\infty \int_0^\infty \exp \left( -\frac{\gamma_s x}{2h(y)} \right) f_{XY'}(x,y') \, dx \, dy', \quad (2.25)$$

where $y'$ and $f_{XY'}(x,y')$ are described in Eq. 2.6 and Eq. 2.12.

**Proof.** Here, the same expected value are determined taking into account the maximum transmission power of the transmitter. Therefore,

$$P_b(\gamma_s | z, \overline{\pi}) = \frac{1}{2} \exp \left( -\frac{\gamma_s x}{2h(y)} \right),$$

where

$$\mathbb{E} \left[ \exp \left( -\frac{\gamma_s x}{2h(y)} \right) \right] = \int_0^\infty \int_0^\infty \exp \left( -\frac{\gamma_s x}{2h(y)} \right) f_{XY}(x,y) \, dx \, dy.$$  

This leads to Eq. 2.25
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2.1.4.1 Average error rate for noncoherent binary transmission in Rayleigh fading channel

This subsection presents the average bit error probabilities for the theoretical and practical system models supposing noncoherent binary transmission and Rayleigh fading as channel model. Exact formulas are determined for both the theoretical and the theoretical measurement based feedback channel equalization model.

Theorem 2.11. The average bit error probability function is given by the following expression in the theoretical measurement based feedback channel equalization model assuming noncoherent binary transmission and Rayleigh fading model:

\[ P_b(\gamma_s) = \frac{\gamma_0}{2} \int_0^\infty \frac{1}{1 + \gamma_0 + \frac{\gamma_s}{2y}} \exp \left( -\gamma_0 y - \frac{1 + \frac{\gamma_s}{2y}}{1 + \gamma_0 + \frac{\gamma_s}{2y}} \right) dy. \]  (2.26)

Proof. Here, \( f_X(x) \) is exponentially distributed, i.e. \( f_X(x) = \exp(-x) \). Using this PDF and the identity of [23, Eq. 6.614.1], Eq. 2.26 can be derived.

Theorem 2.12. The average bit error probability function is given by the following expression in the practical measurement based feedback channel equalization model supposing noncoherent binary transmission and Rayleigh fading model:

\[ P_b(\gamma_s) = \frac{\gamma_0}{2} \int_0^\infty \frac{1}{1 + \gamma_0 + \frac{\gamma_s}{2h(y)}} \exp \left( -\gamma_0 y - \frac{1 + \frac{\gamma_s}{2h(y)}}{1 + \gamma_0 + \frac{\gamma_s}{2h(y)}} \right) dy, \]  (2.27)

where the integral by \( y \) should be calculated in two intervals: \( \{0, c\}; h(y) = c \} \) and \( \{(c, \infty); h(y) = y \} \).

Proof. The same method can be used as for the previous case to derive the final formula.

2.1.4.2 Average error rate for noncoherent binary transmission in Rician fading channel

In this subsection the average bit error probabilities are given for the theoretical and practical system models assuming noncoherent binary transmission and Rician fading as channel model.

Theorem 2.13. The following exact formula determines the average bit error probability in the theoretical measurement based feedback channel equalization model supposing coherent
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binary transmission and Rician fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0}{2} \sum_{l=0}^{\infty} \frac{k^l (1+k)^l}{l!} \int_{0}^{\infty} \frac{\exp(-\gamma_0 y) \exp(-k)}{(1+k) + \gamma_0 + \frac{\gamma_s}{2}}^{l+1} \left(1 + \frac{\gamma_0 y}{(1+k) + \gamma_0 + \frac{\gamma_s}{2}}\right) dy.
\]

(2.28)

\textbf{Proof.} In this case \( f_X(x) \) is distributed according to the non-central chi-square distribution, i.e.

\[
f_X(x) = (1+k) e^{-k} e^{-(1+k)x} I_0 \left(2 \sqrt{k(1+k)x}\right).
\]

Substituting the PDF of \( f_X(x) \) in the general bit error rate formula Eq. 2.23 and performing the same steps as in case of the coherent pair of this theorem, i.e. Theorem 2.1, one can get the presented expression Eq. 2.28. \( \blacksquare \)

\textbf{Theorem 2.14.} The following exact formula determines the average bit error probability in the \textit{practical} measurement based feedback channel equalization model assuming noncoherent binary transmission and Rician fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0}{2} \sum_{l=0}^{\infty} \frac{k^l (1+k)^l}{l!} \int_{0}^{\infty} \frac{\exp(-\gamma_0 y) \exp(-k)}{(1+k) + \gamma_0 + \frac{\gamma_s}{2}}^{l+1} \left(1 + \frac{\gamma_0 y}{(1+k) + \gamma_0 + \frac{\gamma_s}{2}}\right) dy,
\]

(2.29)

where the integral by \( y \) should be calculated in two intervals: \{[0, c] ; h(y) = c\} and \{(c, \infty) ; h(y) = y\}.

\textbf{Proof.} Similarly to the proof of Theorem 2.13, after following the same deduction, Eq. 2.29 is received. \( \blacksquare \)

It is noted that applying \( k = 0 \) in the presented equations, they lead to the Rayleigh fading related formula (Eq. 2.11 and Eq. 2.12) in noncoherent case as well.

\textbf{2.1.4.3 Average error rate for noncoherent binary transmission in Nakagami fading channel} 

In this subsection the average bit error probabilities are determined for the theoretical and practical system models supposing noncoherent binary transmission and Nakagami fading model.
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**Theorem 2.15.** The average bit error probability is given by the following expression in the *theoretical* measurement based feedback channel equalization model supposing noncoherent binary transmission and Nakagami fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0 m^m}{2} \int_0^\infty \frac{\exp(-\gamma_0 y)}{(m + \gamma_0 + \frac{\gamma_s^2}{2h(y)})} m F_1 \left( m, 1; \frac{\gamma_0^2 y}{m + \gamma_0 + \frac{\gamma_s^2}{2h(y)}} \right) dy.
\]  
(2.30)

*Proof.* Here, \( f_X(x) \) is a gamma distributed random variable with the following probability density function:

\[
f_X(x) = \frac{x^{m-1}}{(m-1)!} m^m e^{-mx}.
\]

After substituting of this PDF and using Eq. 6.631.1 from [23], Eq. 2.30 is received.

**Theorem 2.16.** The average bit error probability is given by the following expression in the *practical* measurement based feedback channel equalization model supposing noncoherent binary transmission and Nakagami fading model:

\[
P_b(\gamma_s) = \frac{\gamma_0 m^m}{2} \int_0^\infty \frac{\exp(-\gamma_0 y)}{(m + \gamma_0 + \frac{\gamma_s^2}{2h(y)})} m F_1 \left( m, 1; \frac{\gamma_0^2 y}{m + \gamma_0 + \frac{\gamma_s^2}{2h(y)}} \right) dy,
\]

(2.31)

where the integral by \( y \) should be calculated in two intervals: \([0, c] ; h(y) = c\) and \((c, \infty) ; h(y) = y\).

*Proof.* As earlier, the same steps have to be performed as for the theoretical system model to get Eq. 2.31.

Note that using \( m = 1 \), the above expression equal to the Rayleigh fading case (Eq. 2.11 and Eq. 2.12) here too. Furthermore, both the Rician and Nakagami fading related formulas contain the same confluent hypergeometric function with slightly different parameters. This shows the similarities of the two fading models as mentioned in Section 1.1.1. Nevertheless, the computational complexity is almost the same due to eradicating of the Bessel function from the Rician fading model.

To summarize, Eq. 2.23, Eq. 2.25, Eq. 2.26, Eq. 2.27, Eq. 2.28 and Eq. 2.29, as well as Eq. 2.30 and Eq. 2.31 are novel exact formulas for the average error probabilities of the noncoherent binary transmission using measurement based feedback channel equalization over different fading channels.
2.1.5 Derivation of the average SNR when using the power limitation of the practical system model

The closed-loop power control affects the average transmission power. This means that the average SNR decreases depending on the actual statistic of the fading channel gain, the estimation error of the channel parameter and the upper limit of the transmission power \( c \). This subsection describes this effect in case of different fading types.

In the practical measurement based feedback channel equalization model, the instantaneous transmission power is calculated as follows:

\[
E_t = \frac{E_s}{h(y)} = \frac{E_s}{h \left( z + \frac{\bar{n}}{\sqrt{E_0}} \right)^2} = \frac{E_s}{y'},
\]

where \( E_s \) is the average symbol energy without fading and power control, \( y' \) is a realization of the \( Y' \) random variable defined in Eq. 2.6. In this case the average SNR is given by the following expected value:

\[
\mathbb{E}[\text{SNR}] = \mathbb{E} \left[ \frac{E_t}{N_0} \right] = \mathbb{E} \left[ \frac{E_s}{N_0 \cdot h(Y)} \right] = \gamma_s \mathbb{E} \left[ \frac{1}{Y'} \right],
\]

from which the average change of the SNR can be calculated with the following expression:

\[
\mathbb{E} \left[ \frac{\text{SNR}}{\gamma_s} \right] = \mathbb{E} \left[ \frac{1}{Y'} \right] = \int_0^\infty \frac{1}{y'} f_{Y'}(y') \, dy' = \int_0^\infty \frac{1}{y} f_Y(y) \, dy + \int_0^\infty \frac{1}{c} f_Y(y) \, dy = (2.32)
\]

\[
= \int_0^c \int_0^\infty \frac{1}{c} f_{XY}(x,y) \, dy \, dx + \int_c^\infty \int_0^\infty \frac{1}{c} f_{XY}(x,y) \, dy \, dx.
\]

Theorem 2.17. The average SNR change is given by the following formula assuming the practical measurement based feedback channel equalization model and Rayleigh fading as channel model:

\[
\mathbb{E} \left[ \frac{\text{SNR}}{\gamma_s} \right] = \int_0^\infty \frac{1}{h(y)} \frac{\gamma_0}{1 + \gamma_0} \exp \left( -\frac{\gamma_0}{1 + \gamma_0} y \right) \, dy.
\]

Proof. In this case, \( f_X(x) \) is exponentially distributed, i.e. \( f_X(x) = \exp(-x) \). Substituting this PDF in Eq. 2.32 and using [23, Eq. 6.614.1], one can calculate Eq. 2.33. \( \blacksquare \)
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Theorem 2.18. The average SNR change is described by the following expression supposing the practical measurement based feedback channel equalization model and Rician fading as channel model:

\[
\mathbb{E}\left[ \frac{\text{SNR}}{\gamma_0} \right] = \int_0^\infty \sum_{l=0}^\infty \frac{k^l (1+k)^l}{l!} \frac{\gamma_0 \exp(-\gamma_0 y) \exp(-k)}{h(y)} \\
\frac{1}{((1+k)+\gamma_0)^{l+1}} \text{I}_1(l+1,1;\frac{\gamma_0^2 y}{(1+k)+\gamma_0}) \, dy.
\]

(2.34)

Proof. In this case \( f_X(x) \) is a random variable with the non-central chi-square distribution, i.e. \( f_X(x) = (1+k) e^{-k} e^{-((1+k)x)} I_0 \left(2\sqrt{k(1+k)}x\right) \). By applying the identity of Eq. 6.631.1 from [23], the presented formula (Eq. 2.34) is received. ■

Theorem 2.19. The following formula expresses the average SNR change in the practical measurement based feedback channel equalization model assuming Nakagami fading as channel model:

\[
\mathbb{E}\left[ \frac{\text{SNR}}{\gamma_0} \right] = \int_0^\infty \frac{1}{h(y)\gamma_0 (m+\gamma_0)^m} \exp(-\gamma_0 y) \text{I}_1\left(m,1;\frac{\gamma_0^2 y}{m+\gamma_0}\right) \, dy,
\]

(2.35)

Proof. Now, \( f_X(x) \) is a gamma distributed random variable with the following probability distribution function

\[
f_X(x) = \frac{x^{m-1}}{(m-1)!} m^m e^{-mx}.
\]

Substituting this PDF in Eq. 2.32 and then using Eq. 6.631.1 from [23], one is able to get Eq. 2.35. ■

2.1.6 Results for measurement based channel equalization

This subsection introduces numerical and simulation results, as well as it draws the most important conclusions. Figure 2.2—Figure 2.9 shows average Bit Error Rate (BER) probabilities against the average SNR of the communication channel in the interval of \([-20,30]\) dB. It is known from Section 2.1.2 that the average bit error rate depends on the fading channel gain \(z\), the SNR of the measurement (pilot) channel \(\gamma_0\) and the parameter of the transmission power limiting factor \(c\). Therefore the results focus on the effect of these dependencies.

Figure 2.2 and Figure 2.3 show the average BER curves of Rayleigh fading channel against the average SNR in coherent and noncoherent case, respectively. The average BER
curves of Rician fading channel in coherent case are shown in Figure 2.4, when the fading parameter is $k = 10$. In addition, Figure 2.5 represents the average BER curves of Nakagami fading channel in coherent case against the average SNR, when the fading parameter is $m = 4$. In these figures the SNR of the measuring channel is $\gamma_0 = [0, 30]$ dB and the power limiting parameter is $c = 0.001$. A straightforward consequence of the proposed system is that the average BER reduces, when the SNR of the measuring channel increases and therefore the uncertainty of the fading channel gain estimation decreases.

Average probabilities of BER curves of Rayleigh fading channel against the average SNR in case of coherent and noncoherent case are represented in Figure 2.6 and Figure 2.7. Figure 2.8 shows the average BER curves of Rician fading channel in noncoherent case and Figure 2.9 depicts the average BER curves of Nakagami fading channel in coherent case against the average SNR. The fading parameters are the same as before ($k = 10$, $m = 4$). The SNR of the measuring channel is $\gamma_0 = 30$ dB and the power limiting factors are $c = [0.001, 1]$. Both in coherent and noncoherent case, the average BER reduces with the decrease of parameter $c$, which means that by enabling higher transmission power, the probability of bit error also decreases.

Note that the BER curve of the diversity is also represented. Because of using two channel (one for communication and one for the measuring) in the proposed system, it is better to compare the results against the diversity technique. The Figure 2.2—Figure 2.4 and Figure 2.6—Figure 2.8 show that it is possible to achieve better BER results instead of using diversity in some scenarios. That is true both coherent and noncoherent cases. This is expected when the fluctuation of the fading is slower than the overall value of the measuring time interval ($\bar{T}$) and the feedback delay.

One can assume a base station (BS) with given transmission power, which is able to communicate to the cell edge, and a simple path loss model. It is easy to understand that in this scenario one can transmit information with a low transmission limiting factor $c$ to the proximity of the BS. The parameter $c$ is growing proportionately with the distance. However, based on the results it is better to use measurement based feedback channel equalization technique in close range instead of diversity.

The numerical results are validated by simulations. The simulation process is implemented in MATLAB, and $10^8$ transmission was performed in each scenario. It is visible that the calculated values are very close to the simulated ones. The absolute differences between them are less than $10^{-3}$ in the positive SNR domain for each scenario.

As mentioned before, an errorless and fast (faster then the fluctuation of the fading) feedback channel is assumed that is a strict requirement. The effects of the delay and the error in the feedback channel are complicated problems, these need further investigations.
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![Graph 2.2](image)

Figure 2.2: Validation of numerical results in the case of coherent binary transmission. Average BER of Rayleigh fading channel against the average SNR, when the SNRs of the measuring channel are $\gamma_0 = [0, 30]$ dB and the power limiting factor $c = 0.001$.

![Graph 2.3](image)

Figure 2.3: Numerical results in the case of noncoherent binary transmission. Average BER of Rayleigh fading channel against the average SNR, when the SNRs of the measuring channel are $\gamma_0 = [0, 30]$ dB and the power limiting factor is $c = 0.001$. 
Figure 2.4: Validation of the numerical results in the case of coherent binary transmission. Average BER of Rician fading channel against the average SNR, when the SNRs of the measuring channel are $\gamma_0 = [0, 30]$ dB, $k = 10$ and the power limiting factor is $c = 0.001$.

Figure 2.5: Validation of the numerical results in the case of coherent binary transmission. Average BER of Nakagami fading channel against the average SNR, when the SNRs of the measuring channel are $\gamma_0 = [0, 30]$ dB, $m = 4$ and the power limiting factor is $c = 0.001$. 
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![Figure 2.6: Validation of the numerical results in the case of coherent binary transmission. Average BER of Rayleigh fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 30$ dB and the power limiting factors are $c = [0.001, 1]$.](image)

![Figure 2.7: Numerical results in the case of noncoherent binary transmission. Average BER of Rayleigh fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 30$ dB and the power limiting factors are $c = [0.001, 1]$.](image)
Figure 2.8: Numerical results in the case of coherent binary transmission. Average BER of Rician fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 30$ dB, $m = 4$ and the power limiting factors are $c = [0.001,1]$.

Figure 2.9: Numerical results in the case of coherent binary transmission. Average BER of Nakagami fading channel against the average SNR, when the SNR of the measuring channel is $\gamma_0 = 30$ dB, $m = 4$ and the power limiting factors are $c = [0.001,1]$. 
To summarize, the first part of this chapter presented the measurement based feedback channel equalization technique and for that a novel general analytical method. The analysed system estimates the fading channel gain in an independent noisy pilot (measuring) channel and then feedbacks this information to the transmitter that uses it for equalizing the signal. With this method, error rate analysis of coherent and noncoherent binary wireless transmission was performed in the presence of non-selective slow fading and additive white Gaussian noise. Exact expressions was determined assuming Rayleigh, Rician and Nakagami fading models. It is shown that the proposed system is able to achieve BER gain against diversity technique in some scenarios, while the used channel bandwidth and the average transmission power is the same. The numerical results are validated by simulations.

The next subsection arranges the presented theorems into theses.

2.1.7 Theses related to the measurement based feedback channel equalization

**THESIS I.1. [J1]** I presented an analytical calculation method to determine the bit error rate for such transmission scheme that applies measurement based feedback channel equalization. The method takes into consideration the estimation error of the signal’s amplitude that originates from general fading channel.

Lists of theorems: Theorem 2.1, Theorem 2.9

**THESIS I.2. [J1]** I derived exact formulas to calculate the BER of measurement based feedback channel equalization for Rayleigh, Rice and Nakagami fading models, as well as assuming that the transmission power can be infinite.

Lists of theorems: Theorem 2.3, Theorem 2.5, Theorem 2.7, Theorem 2.11, Theorem 2.13, Theorem 2.15

**THESIS I.3. [J1]** I extended the system model and the calculations to take into account the maximum transmission power as a constraint.

Lists of theorems: Theorem 2.2, Theorem 2.10

**THESIS I.4. [J1]** I transformed the exact expressions according to this new system model, and determined the BER of measurement based feedback channel equalization for Rayleigh, Rice and Nakagami fading models.

Lists of theorems: Theorem 2.4, Theorem 2.6, Theorem 2.8, Theorem 2.12, Theorem 2.14, Theorem 2.16, Theorem 2.17, Theorem 2.18, Theorem 2.19
2.2 Phase error correction analysis of channel equalization

In the previous section, the amplitude gain and its fluctuation were investigated in the presence of channel estimation error supposing binary transmission. Nevertheless, this section presents the analysis of the channel estimation error as well, but focusing on the behaviour of the signal’s phase. This effect was investigated assuming two basic modulation techniques, namely the BPSK and QPSK modulations that have exact, closed-form expressions for the channel capacity too.

In practice, reference signals are transmitted for appropriate communication to the receiver periodically, which is able to estimate the channel distortions. Therefore, the accuracy of channel equalization has a major effect on the quality of communication, i.e. how precisely the receiver is capable of recovering the transmitted data in different channel propagation circumstances. Numerous wireless systems apply this kind of channel equalization, which is called Pilot Symbol Assisted Modulation (PSAM). PSAM has been researched in the last two decades [25]-[29]. However, due to new emerging and even more complex technologies, further investigations are necessary to improve their capabilities [30]-[33]. This is particularly true in term of 5G mobile networks that require even more sophisticated solutions.

Here, the statistical behaviour of the impact of channel estimation error on Symbol Error Ratio (SER) is analysed using pilot signals in a flat, non-frequency selective fading channel. As corollaries of theorems, the effective channel capacities for both BPSK and QPSK modulations are determined. The term of effective channel capacity means the ratio of pilot signals against the data signals. While pilot signals are required for proper channel estimation, they do not carry any useful information to the receiver. Therefore, this issue is rephrased as an optimization problem and further investigated in this section. At the end, the results are compared to the state-of-the-art LTE/LTE-A standard that uses QPSK modulated pilot signals for channel equalization.

2.2.1 System model

The previous system model is slightly modified. The received signal over a flat-fading channel can be expressed as:

\[ r^* = Hg + n, \]

where \( g \) is the low-pass equivalent complex-valued representation of the modulated signal, \( H \) is the complex channel coefficient and \( n \) is the low-pass equivalent complex-valued representation of additive white Gaussian noise. After channel estimation, the equalized signal can be written as:

\[ r = HH\hat{g} + n, \]
where \( r \) is the equalized signal and \( \hat{H} \) is the complex conjugate of the channel coefficient, which equals:

\[
\hat{H} = |H| \exp(-j \arg(H) + \phi),
\]

where \( |H| = z \) and \( \arg(H) \) are the amplitude gain and the phase shift of the channel, respectively, and \( \phi \) is the phase shift due to the estimation error. Note that it is known from the literature [25] that the channel estimation error causes additional phase shift, which can be modelled by rotating the decision domains. This effect is shown in Figure 2.10 in case of QPSK modulation. Similar effect is observable when investigating BPSK modulation, but there are only two decision domains instead of four.

The data symbols of BPSK modulated signals are \( d = \pm 1 \), while the data symbols of QPSK modulated signals are \( d = \pm 1 \pm j \). For practical reasons the QPSK symbols’ positions are \( \pi/4 \) rotated in \( H \) resulting in \( \pm \sqrt{2} \) and \( \pm j \sqrt{2} \) points as the circles depict in Figure 2.10. These shifts have to be corrected before use.

From now on, the positive real 1 symbol is supposed as pilot signal in case of BPSK modulation and the \( \sqrt{2} \) signal for QPSK modulation.

In the following subsection the impact of the estimation error is investigated, i.e. how the symbol error probability changes depending on \( \phi \). Then the effective channel capacity based on the phase shift is determined for both BPSK and QPSK modulations.

### 2.2.2 Derivation of symbol error probabilities

Since the noise is responsible for the phase shift of the estimation error, its statistical behaviour is analysed in the followings. The noise is represented by a complex Gaussian ran-
dom variable that is split into real and imaginary components by $X$ and $Y$:

$$X = \sqrt{E_s} + n_{Re},$$
$$Y = n_{Im},$$

where $E_s$ is the transmitted symbol energy and $\mathbb{E}(n_{Re}^2) = \mathbb{E}(n_{Im}^2) = N_0/2$.

From that, the joint probability density function of $X,Y$ is as follows:

$$f_{X,Y}(x,y) = \frac{1}{\pi N_0} \exp \left( -\frac{(x - \sqrt{E_s})^2 + y^2}{N_0} \right).$$

For further calculations, the $X(x)$ and $Y(y)$ RVs are transformed to polar coordinates of $R(r)$ and $\Phi(\phi)$:

$$X = R \cos(\Phi), \quad x = r \cos(\phi),$$
$$Y = R \sin(\Phi), \quad y = r \sin(\phi).$$

After the substitutions, the joint PDF $f_{R,\Phi}(r,\phi)$ is rewritten as:

$$f_{R,\Phi}(r,\phi) = \frac{r}{\pi N_0} \exp \left( -\frac{r^2}{N_0} \right) \exp \left( -\frac{E_s}{N_0} \right) \exp \left( \frac{2r \sqrt{E_s} \cos(\phi)}{N_0} \right).$$

Thus, the PDF of $\Phi$ is calculated as follows:

$$f_\Phi(\phi) = \exp \left( -\frac{E_s}{N_0} \right) \int_0^\infty \frac{r}{\pi N_0} \exp \left( -\frac{r^2}{N_0} \right) \exp \left( \frac{2r \sqrt{E_s} \cos(\phi)}{N_0} \right) dr. \quad (2.36)$$

To simplify this equation, the next theorem describes the same PDF without using integration.

**Lemma 2.1.** The phase rotating behaviour of the channel can be expressed by the following probability density function using polar coordinate system (similar result is also available in [26]):

$$f_\Phi(\phi) = \frac{1}{2\pi} \exp \left( -\gamma_s \cos^2(\phi) \right) \sqrt{\frac{\pi}{2}}$$

$$\left[ \sqrt{\frac{2}{\pi}} \exp \left( -\gamma_s \cos^2(\phi) \right) + \sqrt{2\gamma_s} \cos(\phi) \right] [1 - \text{erf} \left( -\sqrt{\gamma_s} \cos(\phi) \right)] \right]. \quad (2.37)$$

where $\gamma_s = E_s/N_0$ means the signal-to-noise ratio and erf ($\cdot$) is the Gaussian error function.
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Proof. First, Eq. 2.36 is transformed by using Equation 3.462.1 from [23]

$$\int_0^{\infty} x^{\nu-1} \exp\left(-\beta x^2 - \gamma x\right) dx =$$

$$= (2\beta)^{-\nu/2} \Gamma(\nu) \exp\left(\frac{\gamma^2}{8\beta}\right) D_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}}\right)$$

$$\text{Re} \{\beta\} > 0, \text{Re} \{\nu\} > 0$$

with the following substitutions, where $D_{-\nu}(dzs)$ represents a parabolic cylinder function:

$$x = r; \quad \beta = \frac{1}{N_0}; \quad \gamma = -\frac{2\sqrt{E_s} \cos(\phi)}{N_0}; \quad \nu = 2.$$

Then, [23, Eq. 9.254.2] are applied

$$D_{-2}(dzs) = \exp\left(\frac{dzs^2}{4}\right) \sqrt{\pi} \left\{ \sqrt{2} \exp\left(-\frac{dzs^2}{2}\right) - dzs \left[ 1 - \text{erf}\left(\frac{dzs}{\sqrt{2}}\right) \right] \right\},$$

where $dzs$ equals

$$dzs = -\frac{2\sqrt{E_s} \cos(\phi)}{\sqrt{2} N_0 \sqrt{N_0}}.$$

Based on these, the second part of Eq. 2.36 is expressed as follows:

$$\frac{1}{\pi N_0} \int_0^{\infty} r \exp\left(-\frac{r}{N_0} + \frac{2r\sqrt{E_s} \cos(\phi)}{N_0}\right) dr =$$

$$= \frac{1}{\pi N_0} \left[ N_0 \exp\left(\frac{4E_s \cos^2(\phi)}{N_0} \right) D_{-2}\left(-\frac{2\sqrt{E_s} \cos(\phi)}{N_0}\right) \right] =$$

$$= \frac{1}{2\pi} \exp\left(\frac{E_s}{2N_0} \cos^2(\phi)\right) D_{-2}\left(-\sqrt{\frac{2E_s}{N_0}} \cos(\phi)\right) =$$

$$= \frac{1}{2\pi} \exp\left(\frac{E_s}{2N_0} \cos^2(\phi)\right) \sqrt{\pi} \left\{ \sqrt{2} \exp\left(-\frac{E_s}{N_0} \cos^2(\phi)\right) + \frac{2E_s}{N_0} \cos(\phi) \left[ 1 - \text{erf}\left(-\sqrt{\frac{E_s}{N_0}} \cos(\phi)\right) \right] \right\} =$$

$$= \frac{1}{2\pi} \exp\left(\frac{\gamma \cos^2(\phi)}{N_0}\right) \sqrt{\pi} \left\{ \sqrt{2} \exp\left(-\frac{\gamma \cos^2(\phi)}{N_0}\right) + \sqrt{2\gamma} \cos(\phi) \left[ 1 - \text{erf}\left(-\sqrt{\frac{\gamma}{N_0}} \cos(\phi)\right) \right] \right\},$$
Finally, the probability density function of $\Phi$ is determined by Eq. 2.37.

After this, the symbol error rates of BPSK and QPSK modulations are derived using basic trigonometric functions. Based on the presented exact SER formulas, the more accurate upper bound of the channel capacity, i.e. the effective channel capacity is determined as corollaries for both modulation techniques. These upper bounds take into consideration the signal to noise ratio, as well as the number of pilot signals, which is closely related to the transmitted energy that is used for channel estimation. In order to have more precise estimation, more energy, i.e. more pilot signals have to be sent by the transmitter. Contrarily, the number of data signals decreases that results less available, useful channel capacity.

### 2.2.2.1 Symbol error rate for BPSK modulation

This subsection presents the symbol error rate formula for BPSK modulation case. First, the theorem is described, and then a general method is introduced as part of the proof. This method is repeated to get the QPSK modulation related results as well. Therefore, it will not discussed there in details.

One can see that two SNR values are used both in Eq. 2.38 and in Eq. 2.41. The reason of this will be explained later and further discussed in Section 2.2.3. Now, it is enough to keep in mind that the SNR of data signals ($\gamma_s$) and the SNR of reference symbols ($\gamma_0$) are distinguished.

**Theorem 2.20.** The next formula expresses the symbol error rate assuming BPSK modulation in the pilot symbol assisted transmission system, when the presented signal phase equalization is used:

\[
P_{s,BPSK}(\gamma_s, \gamma_0) = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2} + \phi} \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2} + \phi} \exp\left(-\gamma_0 \frac{\sin^2\left(\frac{\pi}{2} + \phi\right)}{\sin^2\left(\frac{\pi}{2} + \phi - \theta\right)}\right) f_\Phi(\phi) d\theta d\phi + \]

\[
+ \left(1 - \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2} + \phi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2} + \phi} \exp\left(-\gamma_0 \frac{\sin^2\left(-\frac{\pi}{2} + \phi\right)}{\sin^2\left(-\frac{\pi}{2} + \phi - \theta\right)}\right) f_\Phi(\phi) d\theta d\phi\right),
\]

(2.38)

where $\gamma_s$ and $\gamma_0$ are the signal to noise ratios of the data and the pilot symbols and $f_\Phi(\phi)$ is the probability density function of the signals’ phase rotation (Eq. 2.37).

**Proof.** Let us start from the definition of symbol error rate. It means the probability of the event, when the receiver makes a wrong decision during the demodulation. In case of BPSK
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demodulation, it is only possible, if the equalized phase rotation of the received symbol is so significant that it moves in the wrong decision domain. To calculate this probability, the size of the wrong decision domain has to be determined taking into account the phase shift $\phi$. This can be performed by launching an auxiliary half-line from the constellation point related to the symbol value 1 as it is depicted in the I-Q diagram of the BPSK modulation in Figure 2.11. This auxiliary half-line is separated by the decision domain into two parts indicated by $r_1$ and $r_2$. Their length can be calculated by using basic geometric relationships as

$$
\frac{\sqrt{E}}{r_1} = \left| \frac{\sin \left( \frac{\pi}{2} + \phi - \theta \right)}{\sin \left( \frac{\pi}{2} + \phi \right)} \right|,
$$

where $\phi \in [-\pi/2, \pi/2]$ and $\theta \in [\pi/2 + \phi, 3\pi/2 + \phi]$. From that, $r_1$ can be easily expressed, while $r_2 = \infty$. Similarly, in the complementary case:

$$
\frac{\sqrt{E}}{r_2} = \left| \frac{\sin \left( -\frac{\pi}{2} + \phi \right)}{\sin \left( -\frac{\pi}{2} + \phi - \theta \right)} \right|,
$$

where $\phi \in [\pi/2, 3\pi/2]$ and $\theta \in [-\pi/2 + \phi, \pi/2 + \phi]$, furthermore $r_1 = \infty$. So, based on these, the size of the wrong decision related domain can be written as the sum of integrals

![Figure 2.11: Illustration of the calculation steps of Theorem 2.20](image-url)
along the auxiliary half-line. The following integrals describe this calculation for both the $[-\pi/2, \pi/2]$ and $[\pi/2, 3\pi/2]$ domains:

$$
\frac{1}{2\pi} \int_{\frac{\pi}{2} + \phi}^{\frac{3\pi}{2} + \phi} \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{\pi}{2} + \phi \right)}{\sin^2 \left( \frac{\pi}{2} + \phi - \theta \right)} \right) d\theta \quad \text{if } \phi \in \left[ -\frac{p_i}{2}, \frac{p_i}{2} \right],
$$

$$
1 - \frac{1}{2\pi} \int_{-\frac{\pi}{2} + \phi}^{\frac{\pi}{2} + \phi} \exp \left( -\gamma_0 \frac{\sin^2 \left( -\frac{\pi}{2} + \phi \right)}{\sin^2 \left( -\frac{\pi}{2} + \phi - \theta \right)} \right) d\theta \quad \text{if } \phi \in \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right].
$$

Thus, the overall probability of the wrong decision is determined as

$$
p_{-1}(\phi) = \frac{1}{2\pi} \int_{\frac{\pi}{2} + \phi}^{\frac{3\pi}{2} + \phi} \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{\pi}{2} + \phi \right)}{\sin^2 \left( \frac{\pi}{2} + \phi - \theta \right)} \right) d\theta +
$$

$$
\left( 1 - \frac{1}{2\pi} \int_{-\frac{\pi}{2} + \phi}^{\frac{\pi}{2} + \phi} \exp \left( -\gamma_0 \frac{\sin^2 \left( -\frac{\pi}{2} + \phi \right)}{\sin^2 \left( -\frac{\pi}{2} + \phi - \theta \right)} \right) d\theta \right).
$$

Finally, the dependence of $\phi$ is eliminated by using Eq. 2.37 resulting the expression of the theorem (Eq. 2.38).

**Corollary 2.20.1.** The theoretical capacity of a BPSK modulated and pilot symbol assisted communication system is expressed as follows, when the presented signal phase equalization is used:

$$
C_{BPSK} = 1 + p_{-1} \log p_{-1} + (1 - p_{-1}) \log (1 - p_{-1}). \quad (2.39)
$$

**Proof.** It is known [22] that the channel capacity of the strong symmetric channel is expressed as

$$
C = \log_2 (J) + \sum_{i=1}^{J} p_i \log_2 (p_i), \quad (2.40)
$$

where $J$ is the number of the channel’s output symbols and $p_1, p_2, ..., p_J$ are the transition probabilities. Based on this, the exact channel capacity of BPSK modulated transmission is Eq. 2.39.
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2.2.2 Symbol error rate for QPSK modulation

The QPSK modulation related symbol error rate is presented in this section.

**Theorem 2.21.** The symbol error rate of QPSK modulation is determined by the following formula assuming pilot symbol assisted communication system, when the presented signal phase equalization is used:

\[
P_{s,QPSK}(\gamma_s, \gamma_0) = \int_{-\pi/2}^{\pi/2} p_s(\phi) f_\Phi(\phi) d\phi = \int_{-\pi/2}^{\pi/2} p_{-\sqrt{2}}(\phi) f_\Phi(\phi) d\phi + \int_{-\pi/2}^{\pi/2} p_{j\sqrt{2}}(\phi) f_\Phi(\phi) d\phi + \int_{-\pi/2}^{\pi/2} p_{-j\sqrt{2}}(\phi) f_\Phi(\phi) d\phi,
\]

(2.41)

where \( \gamma_s \) and \( \gamma_0 \) are the signal to noise ratios of the data and the pilot symbols, \( f_\Phi(\phi) \) means the probability density function of the signals’ phase rotation (Eq. 2.37), and \( p_{-\sqrt{2}}(\phi), p_{j\sqrt{2}}(\phi), p_{-j\sqrt{2}}(\phi) \) are presented in Eq. 2.43.

**Proof.** Here, the same method is applied as in case of BPSK modulation. The main difference is that there are four elementary symbols (\( \pm \sqrt{2} \) and \( \pm j\sqrt{2} \)) in QPSK modulation, from which only one is related to the good decision, while three correspond to the wrong choice. It is assumed that the \( \sqrt{2} \) value is related to the good decision, therefore the overall symbol error ratio \( p_s \) is determined as the sum of \( -\sqrt{2} \) and \( \pm j\sqrt{2} \) symbols’ probabilities depending on \( \phi \). Thus, \( p_s \) is formally equals to

\[
p_s(\phi) = p_{-\sqrt{2}}(\phi) + p_{j\sqrt{2}}(\phi) + p_{-j\sqrt{2}}(\phi),
\]

(2.42)
where $p_{-\sqrt{2}}(\phi), p_{j\sqrt{2}}(\phi), p_{-j\sqrt{2}}(\phi)$ are given as

\[ p_{j\sqrt{2}}(\phi) = \frac{1}{2\pi} \int_{\frac{3\pi}{4} + \phi}^{\frac{5\pi}{4} + \phi} \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{\pi}{4} + \theta \right)}{\sin^2 (\frac{3\pi}{4} + \phi + \theta)} \right) d\theta + \frac{1}{2\pi} \int_{\frac{\pi}{4} + \phi}^{\frac{3\pi}{4} + \phi} \left[ \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{3\pi}{4} + \phi \right)}{\sin^2 (\frac{3\pi}{4} + \phi + \theta)} \right) - \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{3\pi}{4} + \phi \right)}{\sin^2 (\frac{\pi}{4} + \phi + \theta)} \right) \right] d\theta, \]

\[ p_{-\sqrt{2}}(\phi) = \frac{1}{2\pi} \int_{\frac{3\pi}{4} + \phi}^{\frac{7\pi}{4} + \phi} \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{3\pi}{4} - \phi \right)}{\sin^2 (\frac{\pi}{4} - \phi + \theta)} \right) d\theta, \]

\[ p_{-j\sqrt{2}}(\phi) = \frac{1}{2\pi} \int_{\frac{5\pi}{4} + \phi}^{\frac{9\pi}{4} + \phi} \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{\pi}{4} - \phi \right)}{\sin^2 (\frac{-\pi}{4} - \phi + \theta)} \right) d\theta + \frac{1}{2\pi} \int_{\frac{-\pi}{4} + \phi}^{\frac{3\pi}{4} + \phi} \left[ \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{\pi}{4} - \phi \right)}{\sin^2 (\frac{-\pi}{4} - \phi + \theta)} \right) - \exp \left( -\gamma_0 \frac{\sin^2 \left( \frac{3\pi}{4} - \phi \right)}{\sin^2 (\frac{3\pi}{4} - \phi + \theta)} \right) \right] d\theta. \]

Now, by using the PDF of $\phi$ again, one can get Eq. 2.41.

**Corollary 2.21.1.** Based on Eq. 2.40, the exact channel capacity of QPSK modulated transmission is the following:

\[ C_{QPSK} = 2 + \sum_i p_i \log p_i, \]  

(2.44)

where $p_i \in \{ p_{-\sqrt{2}}, p_{j\sqrt{2}}, p_{-j\sqrt{2}}, 1 - p_s \}$.

So far, the phase equalization of the data signals are described assuming pilot signal assisted modulation. During the demodulation of the reference symbols, estimation error appears due to the AWGN noise in the channel. Then, this estimation error distorts the proper equalization of data signals as well. Therefore, two SNR value is considered in the presented formulas. This approach is very similar to the measurement based feedback channel equalization.
2.2.3 Optimization problem of PSAM systems

As one can see from Eq. 2.38 and Eq. 2.41, the definition of SER includes two parameters. As in case of the measurement based feedback channel equalization, the signal to noise ratios are differentiated for the data symbols and for the pilot, i.e. reference signals. However, the traditional PSAM approach applies the reference signals in the same channel beside the data carrying symbols, e.g. as in 4G Long Term Evolution. According to this model, the SNR values are fully correlated, practically they are the same. This observation leads to an optimization problem that is discussed in this subsection.

Let us introduce the following quantity of the radio channel in the pilot symbol assisted communication system:

\[ R \leq \frac{n-m}{n} C(m), \] (2.45)

where \( n \) is the overall number of channel symbols, \( m \) is the number of pilot signals, \( C(m) \) is the capacity of the channel (depending on \( m \)) and \( R \) indicates the effective channel capacity, i.e. the upper bound of available rate. Basically, the effective channel capacity determines, how many information bits per symbol can be transmitted through the channel in average.

Eq. 2.45 describes the relationship between the number of data and reference signals. With other words, if there are given amount of symbols \( n \), they have to be divided into data \( (n-m) \) and pilot \( m \). In the presented model, the accuracy of the channel estimation depends on the energy that is carried by reference signals, i.e. more reference signals result better reception, and thus higher channel throughput. However, if too many (and unnecessary) pilot symbols are used, the throughput decreases. Therefore, the goal is to find the optimal number of reference signals \( m \) that maximizes the effective channel capacity.

**Lemma 2.2.** Assuming QPSK modulation based transmission only, the Long Term Evolution technology is comparable against the current system model. Furthermore, the exact number of necessary pilot signals can be determined taking into account the signal to noise ratio to maximize the throughput.

*Proof.* LTE/LTE-A downlink communication is based on Orthogonal Frequency-Division Multiplexing (OFDM) scheme. The 3GPP specifications determine the number and positions of pilot signals in the so-called Physical Resource Block (PRB) meaning a static pilot pattern. This pattern is shown in Figure 2.12. Depending on the Cyclic Prefix (CP) length, being normal or extended, every slot contains seven or six OFDM symbols, respectively. A group of twelve adjacent subcarriers of a slot makes a PRB. The QPSK modulated reference signals are almost uniformly distributed on the so-called time-frequency or resource grid helping the receiver to estimate channel conditions more accurate.
Therefore, one can assume \( n \) length series of QPSK modulated resource elements, from which \( m \) number of signals is used as pilot. For the sake of simplicity, the positions of the \( m \) reference signal in the series are neglected, just the effect of transmitted symbol energy is investigated. Furthermore, the communication channel is considered as a strong symmetric channel.

One subcarrier in frequency domain and one radio frame with normal cyclic prefix in time domain are assumed as the basis of the comparison, which makes \( n = 140 \). Then, in case of the standard, the number of average pilot symbols has to be calculated on the same, aforementioned basis. That equals to \( m = \frac{1}{21} \times 140 = 6.667 \) (see Figure 2.12), while it depends on the actual SNR of the channel in the investigated case, i.e. the pilots are dynamically allocatable.

Based on these, Eq. 2.45 is applicable, and Eq. 2.44 from Corollary 2.21.1 can be substituted in place of \( C(m) \). This means:

\[
R \leq \frac{n-m}{n} C(m) = \frac{n-m}{n} \left( 2 + \sum_i p_i \log p_i \right).
\]

The connection between the different SNR values can simply be given as \( \gamma_0 = m \cdot \gamma_s \), i.e. the SNR of pilot signals is directly proportional to the SNR of the channel. Based on numerical calculations, the optimal \( m \) can be easily found. The results are presented in the next subsection.
2.2. PHASE ERROR CORRECTION ANALYSIS OF CHANNEL EQUALIZATION

2.2.4 Results for measurement based channel equalization for phase error correction

This section introduces the numerical results of the statistical analysis and the comparison of LTE/LTE-A standard against the proposed solution.

The overall symbol error probability \( p_s \) based on Eq. 2.42 is depicted in Figure 2.13. For practical reasons the rotation of \( \phi \) representing the estimation error is in \([-\pi/4, \pi/4]\) range. It is clearly visible that the SER is symmetric to the phase shift, which is the expected behaviour. Furthermore, if the decision domain of the \( \sqrt{2} \) symbol is fully rotated by the noise into another symbol’s (i.e. into one of \( \pm j\sqrt{2} \) symbols) decision domain, then the symbol error probability equals to 0.5 meaning that the demodulator have to guess the value of the received symbol. However, using additional reference symbols the SNR increases in proportion to the more transmitted symbol energy. Therefore, the effective channel capacity \( (R) \) approximates its theoretical maximum value, but unnecessary pilot signals just occupy valuable resource slots and reduces the average bit per symbol (bps) ratio.

To find the optimal amount of reference symbols that maximizes the effective channel capacity, numerical calculations were carried out. The alteration of the effective channel capacity against the number of pilots is shown in Figure 2.14. Note that simple \( \gamma \) is used in the figure to describe the SNR of the channel, since in LTE/LTE-A the data and the measurement/pilot channels are not separated. The curves illustrate that the effective channel capac-

![Figure 2.13: Symbol error rate when \( \phi = [-\pi/4, \pi/4] \)](image)
ity has a maximum value at the optimal pilot number, where the equalization is the best and just the necessary amount of pilots are used. Before the maximum point, $R$ increases rapidly, while the estimation error decreases. After reaching its maximum, the estimation error stays at its minimum, but the effective channel capacity begins to reduce due to the unnecessary pilots. However, in high SNR domain one reference signal is enough to maximize $R$, thus only the diminishing part is visible.

To confirm the meaning of the optimization, the results are compared to the LTE/LTE-A standard. Figure 2.15 depicts the differences between the standard and the proposed solution. The left axis represents the number of reference symbols, while the right axis illustrates the effective channel capacity in function of SNR. The optimal amount of reference signals are selected, where the effective channel capacity is at its maximum. As the SNR increases and with that the effective channel also rises, the optimal pilot number decreases. At 0 dB, in case of the standard, constant 6.667 pilot symbols are used as average from the available 140, which achieves 0.559 average bit per symbol effective channel capacity. Contrarily, using the optimal amount of reference symbols (which equals to 28) achieves 0.773 bps, which means about 38% gain. At 15 dB, LTE/LTE-A provides 1.905 bps, while in the optimal case only one pilot symbol applied resulting in 1.985 bps effective channel capacity, which is a significant benefit as well. In the previous case, LTE/LTE-A achieves the minimal estimation error with the 6.667 average pilots, but the unnecessary ones occupy resources from data symbols.

Figure 2.14: Available rate in function of number of pilot symbols when $\gamma = [0, 15] \ dB$
To summarize, the impact of the channel estimation error is investigated in this section. The derived formulas can be used to compare LTE/LTE-A system channel equalization to the presented solution. SER formulas related to BPSK and QPSK modulations are introduced. The latter expressions are used for optimizing the amount of pilot signals on subcarrier level to maximize the effective channel capacity. In such situations, where there are too few reference signals, they do not be able to efficiently reduce the estimation error, but in case of too many pilot, they just occupy resource slots instead of using them for data transmission. The results indicate that the effective channel capacity can achieve a maximum value by tuning the number of pilot symbols. Furthermore, the results are compared to the standard, and they show that the optimal amount of reference signals achieves higher average bit per symbol ratio on the radio link up to about 38% in different channel conditions.

Such investigations are important to steer future standardization activities. The calculated results basically determine the minimum amount of reference signals taking into consideration the actual SNR of the channel. The results could be included in LTE/LTE-A standards, since the described static pilot signal pattern is one of the defects of the 4G system. However, due to compatibility reasons, the later versions of LTE did not change this pattern. To
see the full picture, it have to be noted that in 3GPP 5G New Radio (5G NR), the reference signals are more flexible and can be dynamically allocated [34] that reflects the validity of this presented approach.

The next subsection groups the presented theorems and defines the thesis.

2.2.5 Thesis related to the phase error correction analysis of channel equalization

THESIS I.5. [C1] *I described an analytical calculation method to determine the symbol error rate for BPSK and QPSK modulations taking into account the estimation error of the signal’s phase assuming Rayleigh fading channel.*

Lists of lemmas, theorems and corollaries: Lemma 2.1, Theorem 2.20, Corollary 2.20.1, Theorem 2.21, Corollary 2.21.1, Lemma 2.2
3 Analysis of spread spectrum methods

This chapter presents all the investigations and results related to spread spectrum based communication. The chapter is separated into two main sections. First, the correlation peak detection of chirp spread spectrum based communication systems are analysed. By using the proposed sliding-and-tracking correlators in the receiver, faster and more reliable correlation peak detection can be achieved, than with traditional correlators. Then, a novel chirp spread spectrum technique is proposed that utilizes the results of the previous analysis. The new scheme combines the traditional CSS based communication with pulse position (PP) modulation. The system provides simultaneous data transmission to multiple mobile terminals in an efficient and sophisticated way. The next section describes the related works.

3.1 Related works

Chirp signals were first introduced in World War II as in radar technology [35] due to its unique properties of pulse compression and precise ranging. Chirp modulation, or with other words linear frequency modulation was patented by Sidney Darlington in 1954 for digital communication purposes, and with significant later work performed by Winkler in 1962. She proposed using one pair of linear chirps that have opposite sign of chirp rates for binary signalling. This scheme is called later as Binary Orthogonal Keying (BOK) [36]. In 1971, Gott and Newsome [37] described an experimental communication system employing chirp modulation in the High Frequency (HF) band in order to apply chirp signals as wideband sweep signals. They also measured this method’s bit error rate in additive white Gaussian noise and carrier interference. Their work showed the benefits of the chirp based communication scheme over narrowband transmission systems. One of the most important advantages that they highlighted is the protection against signal-fading loss because of multipath propagation in HF bands, which appears most important. In addition, it also has the
benefit of being capable of resisting carrier and impulse interference, as well as Doppler shift. It is also interesting that this work emphasized that the receiver synchronization may be readily achieved so that the system can avoid additional complexity for synchronization with reduced performance.

In 1973, Berni and Gregg [38] also investigated the usage of chirp modulation for digital signalling, and compared the BER performance of chirp BOK, phase-shift keying (PSK), and frequency-shift keying (FSK) modulation schemes in coherent, partially coherent, Rayleigh, and Rician transmission channels. Theoretically, chirp modulation is found to be outstanding in the partially coherent and frequency-selective fading cases over certain ranges of channel conditions. Zaytsev and Zhuravlev [39] analysed a binary noncoherent chirp system applying AWGN channel. The authors highlighted that the system error rate approaches that noncoherent (orthogonal) FSK for large time-bandwidth products.

In 1974, Cook presented the chirp multi-access for the first time [40]. His work examined the utility of pairs of linear chirps that have different chirp rates as spread spectrum signals by assigning them to several users. By this, the proposed scheme allows multiple access within a common frequency band. However, for a given time-bandwidth product, multiple access interference limits the number of users simultaneously accessing the shared communications channel. One of the unique attributes of this system is that the bandwidth of each user is different.

El-Khamy et al. [41] extended Cook’s work in 1996. They proposed a scheme that uses chirp signals with same power and same bandwidth for multiple access communication. Furthermore, the authors derived closed form expressions and approximate analytical expressions for the MAI and BER of this novel chirp modulation spread spectrum (CMSS) technique. The same authors in [42] further developed their work and proposed a new scheme that combines their CMSS technique with frequency hopping. They showed that the performance of the transmission system is higher by this hybrid spread spectrum technique especially in multipath fading dispersive channels. The performance of the presented technique is limited in terms of bit-error rate and bandwidth, when the number of users increases. In addition, the interference between the users increases when there is an increase in the number of users which yields to a degradation in performance. In 2002, Hengstler [43] proposed a new multi access for chirp direct modulation. In this work, each user will be assigned to one chirp signal from a set of chirp signals. Those chirp signals are calculated in a way that the cross-correlation between them is minimized. Based on Hengtsler work, a new method is proposed by Ju and Barkat in 2004 [44] to design a set of orthogonal chirp signal using Fraction Fourier transform. This technique slightly improves the bit-error-rate performance, but it uses much larger bandwidth and requires higher system complexity. Furthermore, authors of [45] and [46] also take into consideration the orthogonality to enhance the performance of their proposed CSS based communication systems.
3.2 Correlation peak detection

In the last decade new wireless technologies have tried to tackle the exponential increasing demand for higher data rates and higher mobility. Some technologies, e.g. Wireless Local Area Network, 3G and 4G have been evolved using different sophisticated techniques e.g. Multiple Input Multiple Output and new coding procedures, like turbo codes, Low-Density Parity-Check (LDPC) codes to boost up existing capacities of wireless links, and some technologies just emerge, like UWB and cognitive radios to share and reuse the radio spectrum, therefore optimize its utilization.

For proper operation, the receiver has to synchronize with the incoming signal. This means timing synchronization, when the receiver needs to determine at which time instants the incoming signal has to be sampled, and carrier synchronization, when the receiver needs to adapt the frequency and phase of its local carrier oscillator with those of the received signal, too. The accuracy of synchronization has a major effect on the performance of the communication independently from the technology. However, if an unfriendly, but realistic wireless environment is assumed with low signal-to-noise ratio, fading, multipath propagation, in-system and out-system interference, then it is easy to understand, that the acquisition of synchronization parameters is difficult. Therefore synchronization should be considered in general as a challenging task.

Here, a simple synchronization scheme is proposed that applies two correlators, and its statistical behaviour related to correlation peak detection is analysed using binary chirp modulation via AWGN channel. The so-called sliding-and-tracking correlators enable to acquire overlapping noise components during the measurements, which means that these components are not statistically independent. The advantage of this property is taken into account during the calculation of correlation in order to reduce the error probability of correlation peak detection. The proposed correlator ensures higher accuracy for synchronization than using the traditional sliding correlator. The findings can be adapted for spread spectrum modulation based communication systems as well, where the autocorrelation function of the spectrum spreading code has only one well-determined peak, e.g. DS and FFH based technologies.

Other possible application areas of the proposed method include the Ultra-wideband (UWB) and WLAN-based localization systems. There are several techniques available for localization [47], e.g. AoA (Angle of Arrival), ToA (Time of Arrival) and TDOA (Time Difference of Arrival), which require precise synchronization. Therefore the accuracy of positioning are closely related to the accuracy of synchronization.

The following subsection describes the system model. After that, formulas are derived for both coherent and noncoherent cases to show the properties and benefits of the proposed sliding-and-tracking correlators. Finally, numerical results are presented.
3.2.1 System model

During the investigation, chirp spread spectrum based communication is assumed, more specifically binary chirp system that uses two elementary signals. The analysis focuses on the reception of the elementary signals. However, it easy to see that the investigations are applicable in any communication scheme for correlation peak detection that has autocorrelation function with one well-determined peak, e.g. direct sequence spread spectrum techniques.

The elementary signals assigned to each bit in the applied binary chirp system are $g_{c1}(t), g_{c2}(t)$ with $[-T_c/2, T_c/2)$ holders. The two low-pass equivalent complex-valued representation of the elementary chirp signals is as follows:

\[
\begin{align*}
\text{for 0: } & \quad g_{c1}(t) = A \exp \left( j2\pi \frac{\Delta f}{2T_c} t^2 \right), \quad t \in \left[ -\frac{T_c}{2}, \frac{T_c}{2} \right] \\
\text{for 1: } & \quad g_{c2}(t) = A \exp \left( -j2\pi \frac{\Delta f}{2T_c} t^2 \right), \quad t \in \left[ -\frac{T_c}{2}, \frac{T_c}{2} \right]
\end{align*}
\]

(3.1)

where $A$ is the amplitude of the elementary signals, $\Delta f$ is the chirp modulated signal’s frequency spreading domain and $T_c$ is the frequency varying sinusoid signal’s symbol time. The instantaneous frequency changing of the signals is presented on Figure 3.1. The units of the time (x-axis) and frequency domain (y-axis) are $10^{-5}$ second and $10^5$ Hz, respectively.

Figure 3.1: Instantaneous frequency of the chirp signal in function of time, when $T_c = 1, \Delta f = 200$
3.2. **CORRELATION PEAK DETECTION**

![Diagram of correlation peak detection](image)

**Figure 3.2:** The integration domains of the different scenarios

Now, the basic idea of the proposed sliding-and-tracking correlators is depicted in Figure 3.2. As one can see, two, not independently operating correlators are applied that have overlapping time windows. So, the accumulated noise will appear in both correlators in the overlapped time. This feature enable to handle the additive white Gaussian noises as dependent random variables. In the next subsections, the statistical investigations are presented considering coherent and noncoherent reception.

### 3.2.2 Statistical analysis assuming noncoherent reception

In this section, the model of the noncoherent receiver, the joint PDF of the correlators’ output signals and the decision error probability for both sliding-and-tracking and sliding correlators are presented.
3.2.2.1 Noncoherent synchronization system

The noncoherent synchronization system is shown in Figure 3.3. The goal of the receiver is the detection of the correlation function maximum at $\tau = 0$ position. This means noncoherent measuring of the correlation function’s values that belong to 0 and $\tau$ delays, and then deciding on the greater of them. It is assumed that the transmitter sends a training sequence, e.g. $g_{c1}(t)$ to the receiver till the synchronization is not finished. Note that the synchronization is possible without training sequence. The statistical properties of the depicted scheme are investigated in two scenarios.

In the first scenario, it is supposed that 0 and $\tau$ delays of $z_{1}(0)$ and $z_{1}(\tau)$ values are generated in overlapping time windows, i.e. two correlators are used, a sliding and a tracking correlator at the same time. This is illustrated by Figure 3.2a. The integration domain of the correlators are normalized to $[-T_c/2, T_c/2]$, but it is assumed that the incoming signal of one correlator is received in appropriate synchronous position and the other correlator gets it with $\tau$ delay. In this system, the two correlation calculations are evaluated in overlapping time windows, where the AWGN noises are not independent from each other, i.e. the results of the correlation are dependent random variables.

In the second scenario, it is supposed that the measurements of 0 and $\tau$ delays are performed in non-overlapping time windows, i.e. sliding correlators are applied for the detecting the maximum position. Figure 3.2b shows this second case. The integration domain of the correlators are also normalized to $[-T_c/2, T_c/2]$. Furthermore, it is assumed that the incoming signal of one correlator is received in appropriate synchronous position, but the other one receives it with $\tau - nT_c$ delay, i.e. the two correlation calculations are evaluated in non-overlapping time windows, thus the results of the correlation are independent RVs.

The synchronous position is determined by choosing the greater signal from the result of the two correlation calculations. The null comparator is used for this purpose, i.e. it compares the correlators’ output signals as shown in Figure 3.3.
3.2. CORRELATION PEAK DETECTION

The operation of the synchronizing scheme can be described next expression:

\[ z_1(\tau) = |v_1(\tau)|^2 = \left| \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} r_{ekv}(t + \tau) g_{c_1}^*(t) \, dt \right|^2 = \]

\[ = \left| \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} r_{ekv}(t + \tau) A \exp \left( -j2\pi \frac{\Delta f}{2T_c} t^2 \right) \, dt \right|^2, \]

where

\[ r_{ekv}(t + \tau) = A g_1(t + \tau) + A g_1(t - T_c + \tau) + n(t + \tau), \]

and \( n(t) = n_1(t) + jn_2(t) \) is an independent complex Gaussian distributed stochastic process with zero expected value and \( N_0 \) power density per complex dimension. Thus, the formula of \( z_1(\tau) \) changes by the following way after the substitutions:

\[ z_1(\tau) = |v_1(\tau)|^2 = \left| \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} r_{ekv}(t) g_{c_1}^*(t) \, dt \right|^2 = \left| A \int_{-\frac{T_c}{2} - \tau}^{\frac{T_c}{2} - \tau} g_{c_1}(t + \tau) g_{c_1}^*(t) \, dt + \right. \]

\[ + \left. A \int_{\frac{T_c}{2} - \tau}^{\frac{T_c}{2}} g_{c_1}(t - T_c + \tau) g_{c_1}^*(t) \, dt + A \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} n(t + \tau) g_{c_1}^*(t) \, dt \right|^2, \; (3.2) \]

where \( v_1(\tau) \) is the complex output signal of the integrator at the end of the integration time.

As a reminder, the task is to determine the probability of \( z_1(0) < z_1(\tau) \) with given signal-to-noise ratio \((\gamma = E_b/N_0)\), i.e. according to the appropriate synchronous position, we choose the \( \tau \) value based on the given measurement. To simplify the calculation, the second part of Eq. 3.2 is neglected due to the properties of the chirp modulation. So, the following expression is received:

\[ z_1(\tau) = |v_1(\tau)|^2 \approx \left| A \int_{-\frac{T_c}{2}}^{\frac{T_c}{2} - \tau} g_{c_1}(t + \tau) g_{c_1}^*(t) \, dt + A \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} n(t + \tau) g_{c_1}^*(t) \, dt \right|^2 = \]

\[ = \left| A \int_{-\frac{T_c}{2} - \tau}^{\frac{T_c}{2} - \tau} \exp \left( j2\pi \frac{\Delta f}{2T_c} t^2 \right) \exp \left( -j2\pi \frac{\Delta f}{2T_c} t^2 \right) \, dt + \right. \]

\[ + \left. A \int_{\frac{T_c}{2} - \tau}^{\frac{T_c}{2}} n(t + \tau) \exp \left( -j2\pi \frac{\Delta f}{2T_c} t^2 \right) \, dt \right|^2. \; (3.3) \]
CHAPTER 3. ANALYSIS OF SPREAD SPECTRUM METHODS

The expected value of noncoherently received complex signal can be derived from the first part of Eq. 3.3, while the second part of Eq. 3.3 gives the stochastic part of the complex Gaussian distributed random variable.

First, calculate the aforementioned expected value as follows:

\[
E \{ v_1(\tau) \} = A^2 \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} \exp \left( j2\pi \frac{\Delta f}{T_c} (t + \tau) \right) \exp \left( -j2\pi \frac{\Delta f}{2T_c} \tau^2 \right) dt =
\]

\[
= A^2 \exp \left( j2\pi \frac{\Delta f}{2T_c} \tau^2 \right) \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} \exp \left( j2\pi \frac{\Delta f}{T_c} \tau t \right) dt =
\]

\[
= A^2 \exp \left( j2\pi \frac{\Delta f}{2T_c} \tau^2 \right) \left[ \exp \left( j2\pi \frac{\Delta f}{T_c} \tau \frac{T_c}{2} \right) \right]_{-\frac{T_c}{2}}^{\frac{T_c}{2}} =
\]

\[
= A^2 \exp \left( j2\pi \frac{\Delta f}{2T_c} \tau^2 \right) \frac{\exp \left( j2\pi \frac{\Delta f}{T_c} \tau \left( \frac{T_c}{2} - \tau \right) \right) - \exp \left( -j2\pi \frac{\Delta f}{T_c} \tau \left( \frac{T_c}{2} - \tau \right) \right)}{j2\pi \frac{\Delta f}{T_c} \tau} =
\]

\[
= A^2 \frac{j\pi \Delta f \tau \left( \frac{T_c}{2} - \tau \right)}{j2\pi \frac{\Delta f}{T_c} \tau} =
\]

\[
= 2E_b \left( 1 - \frac{\tau}{T_c} \right) \frac{\sin \left( \pi \Delta f \tau \left( 1 - \frac{\tau}{T_c} \right) \right)}{\pi \Delta f \tau \left( 1 - \frac{\tau}{T_c} \right)},
\]

where \( E_b = T_b A^2 / 2 \). This formula is a symmetrical function to \( \tau \), thus, when the sign of the delay is arbitrary, the expected value of the signal can be determined by the following expression:

\[
E \{ v_1(\tau) \} = 2E_b \left( 1 - \frac{|\tau|}{T_c} \right) \frac{\sin \left( \pi \Delta f |\tau| \left( 1 - \frac{|\tau|}{T_c} \right) \right)}{\pi \Delta f |\tau| \left( 1 - \frac{|\tau|}{T_c} \right)}.
\]
3.2. CORRELATION PEAK DETECTION

Then, let us focus on the second part of Eq. 3.3 that is a complex Gaussian distributed RV with zero expected value. Its deviation can be calculated as follows:

\[
E \left\{ A^2 \left| \int_{-T_c \tau}^{T_c \tau} n(t + \tau) g_{c1}^*(t) \, dt \right|^2 \right\} = E \left\{ A^2 \left( \int_{-T_c \tau}^{T_c \tau} n(t + \tau) g_{c1}^*(t) \, dt \right) \right\}
\]

\[
\left( \int_{-T_c \tau}^{T_c \tau} n^*(\sigma + \tau) g_{c1}(\sigma) \, d\sigma \right) = A^2 \int_{-T_c \tau}^{T_c \tau} \int_{-T_c \tau}^{T_c \tau} E \{ n(t + \tau) n^*(\sigma + \tau) \}
\]

\[
g_{c1}^*(t) g_{c1}(\sigma) \, dt d\sigma = A^2 \int_{-T_c \tau}^{T_c \tau} \int_{-T_c \tau}^{T_c \tau} 2N_0 \delta(t - \sigma) g_{c1}^*(t) g_{c1}(\sigma) \, dt d\sigma =
\]

\[
= A^2 2N_0 \int_{-T_c \tau}^{T_c \tau} |g_{c1}(t)|^2 \, dt = 2E_b 2N_0. \tag{3.4}
\]

Based on them, the output signal of the two correlators in Figure 3.3 can be expressed as

\[
z_1(0) = \left| 2E_b + \sqrt{2E_b n_0} \right|^2 = \left| 2E_b + \sqrt{2E_b n_{01}} + j \sqrt{2E_b n_{02}} \right|^2,
\]

\[
z_1(\tau) = \left| 2E_b \rho + \sqrt{2E_b n_{\tau}} \right|^2 = \left| 2E_b \rho + \sqrt{2E_b n_{\tau1}} + j \sqrt{2E_b n_{\tau2}} \right|^2, \tag{3.5}
\]

where \( n_0 \) and \( n_{\tau} \) are complex Gaussian distributed random variables with \( 2N_0 \) deviation, \( n_{01}, n_{02}, n_{\tau1} \) and \( n_{\tau2} \) are of these pairwise independent real and imaginary parts respectively with \( N_0 \) deviation. The correlation parameter between \( n_{01}, n_{\tau1} \) and between \( n_{02}, n_{\tau2} \) are the following:

\[
\rho = \left( 1 - \frac{|\tau|}{T_c} \right) \frac{\sin \left( \pi \Delta f |\tau| \left( 1 - \frac{|\tau|}{T_c} \right) \right)}{\pi \Delta f |\tau| \left( 1 - \frac{|\tau|}{T_c} \right)}, \tag{3.6}
\]
The correlation for \( n_{01} \) and \( n_{\tau 1} \) can be calculated as follows:

\[
\frac{E\left[n_{01}n_{\tau 1}^*\right]}{\sqrt{E\left[n_{01}^2n_{\tau 1}^2\right]}} = \frac{E\left[\frac{1}{\sqrt{T_c}} \int_{-T_c/2}^{T_c/2} n_1(t) g_{c1}^*(t) dt \right] \left[\frac{1}{\sqrt{T_c}} \int_{-T_c/2}^{T_c/2} n_1^*(t + \tau) g_{c1}(t) dt \right]}{N_0} = \\
= \frac{E\left[\frac{1}{T_c} \int_{-T_c/2}^{T_c/2} \int_{-T_c/2}^{T_c/2} n_1(t) n_1^*(\sigma + \tau) g_{c1}^*(t) g_{c1}(\sigma) \sigma d\sigma dt\right]}{N_0} = \\
= \frac{\int_{-T_c/2}^{T_c/2} \int_{-T_c/2}^{T_c/2} E[n_1(t) n_1^*(\sigma + \tau)] g_{c1}^*(t) g_{c1}(\sigma) \sigma d\sigma dt}{N_0 T_c} = \\
= \frac{1}{T_c} \int_{-T_c/2}^{T_c/2} \int_{-T_c/2}^{T_c/2} \delta(t - \sigma - \tau) g_{c1}^*(t) g_{c1}(\sigma) \sigma d\sigma dt = \\
= \frac{1}{T_c} \int_{-T_c/2}^{T_c/2} g_{c1}^*(\sigma + \tau) g_{c1}(\sigma) \sigma d\sigma = \left(1 - \frac{1}{\Delta f T_c} \left(1 - \frac{1}{\Delta f T_c} \right) \sin \left(\pi \Delta f \left(1 - \frac{1}{\Delta f T_c} \right)\right) \right) = \rho.
\]

The correlation function of the chirp modulated signal (for low \( \tau/T_c \)) is illustrated in Figure 3.4. It is clearly visible that the correlation quickly decreases as \( |\tau| \) increases.
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3.2.2.2 Wrong decision probability using noncoherent sliding-and-tracking correlators

In this subsection, the probability of the wrong decision is provided using the joint probability density function of the output signals of the sliding-and-tracking correlators.

**Theorem 3.1.** The wrong decision probability is determined by the following formula in case of noncoherent sliding-and-tracking correlators:

\[
\text{I} \{X_1 < X_2\} = \int_0^\infty \int_x^\infty f_{XX_2}(x, x_2) \, dx_2 \, dx = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{|\rho|^k}{(2l+1)!} (1 - \rho^2)^{\frac{1}{2}(l+1)} \, \gamma \, \exp(-\gamma) \left( \frac{(k+l)!}{(l!)^2} \right) \frac{1}{\sqrt{1-\rho^2}}, \tag{3.7}
\]

where \( \gamma \) is the SNR and \( \rho \) is the correlation parameter given by Eq. 3.6.

**Proof.** First, the joint probability density function of the output signals of Eq. 3.4 and Eq. 3.5 have to be determined. Let

\[
\begin{align*}
\text{Re} \{(v_1 (0))\} &= X = 2E_b + \sqrt{2E_b}n_{01}, \quad E(X) = 2E_b, \quad E \left( X - E(X)^2 \right) = 2E_b N_0 = \sigma^2, \\
\text{Im} \{(v_1 (0))\} &= Y = \sqrt{2E_b}n_{02}, \quad E(Y) = 0, \quad E \left( Y - E(Y)^2 \right) = 2E_b N_0 = \sigma^2, \\
\text{Re} \{(v_1 (\tau))\} &= V = 2E_b \rho + \sqrt{2E_b}n_{\tau 1}, \quad E(V) = 2E_b \rho, \quad E \left( V - E(V)^2 \right) = 2E_b N_0 = \sigma^2, \\
\text{Im} \{(v_1 (\tau))\} &= Z = \sqrt{2E_b}n_{\tau 2}, \quad E(Z) = 0, \quad E \left( Z - E(Z)^2 \right) = 2E_b N_0 = \sigma^2, \\
E(XY) &= 0, \quad E(VZ) = 0, \\
E((X - E(X))(V - E(V))) &= 2E_b N_0 \rho, \quad E((Y - E(Y))(Z - E(Z))) = 2E_b N_0 \rho. \tag{3.8}
\end{align*}
\]

The joint probability density function can be defined as follows \((T_c = 1 \text{ and } 2E_b = 1)\):

\[
f_{XVYZ}(x, y, v, z) = \left( \frac{1}{2\pi \sigma^2 (1 - \rho^2)} \right)^2 \exp \left( -\frac{1}{2\sigma^2 (1 - \rho^2)} \left( y^2 - 2\rho y z + z^2 \right) \right) \exp \left( -\frac{1}{2\sigma^2 (1 - \rho^2)} \left( (x - 1)^2 - 2\rho (x - 1)(v - \rho) + (v - \rho)^2 \right) \right).
\]

With the following transformations

\[
\begin{align*}
R_1 &= \sqrt{X^2 + Y^2}, \quad \Phi_1 = \text{arg} \left( \frac{Y}{X} \right), \quad X = R_1 \cos \left( \Phi_1 \right), \quad Y = R_1 \sin \left( \Phi_1 \right), \\
R_2 &= \sqrt{V^2 + Z^2}, \quad \Phi_1 = \text{arg} \left( \frac{Z}{V} \right), \quad V = R_2 \cos \left( \Phi_2 \right), \quad Z = R_2 \sin \left( \Phi_2 \right), \tag{3.9}
\end{align*}
\]
one can get

\[
\begin{align*}
    f_{R_1,R_2\Phi_1,\Phi_2}(r_1, r_2, \Phi_1, \Phi_2) & = r_1, r_2 \left( \frac{1}{2\pi \sigma^2 (1 - \rho^2)} \right)^2 \exp \left( - \frac{1}{2\sigma^2 (1 - \rho^2)} \left( (r_1 \cos \Phi_1 - 1)^2 - 2\rho (r_1 \cos \Phi_1 - 1) (r_2 \cos \Phi_2 - \rho) + (r_2 \cos \Phi_2 - \rho)^2 \right) \right) \\
    & \exp \left( - \frac{1}{2\sigma^2 (1 - \rho^2)} \left( (r_1 \sin \Phi_1)^2 - 2\rho (r_1 \sin \Phi_1) (r_2 \sin \Phi_2) + (r_2 \sin \Phi_2)^2 \right) \right) = \\
    & = r_1, r_2 \left( \frac{1}{2\pi \sigma^2 (1 - \rho^2)} \right)^2 \exp \left( - \frac{1}{2\sigma^2 (1 - \rho^2)} \left( r_1^2 + r_2^2 + (1 - \rho^2) \right) \right) \\
    & \exp \left( - \frac{1}{2\sigma^2 (1 - \rho^2)} \left( -2\rho r_1 r_2 \cos (\Phi_1 - \Phi_2) - 2r_1 (1 - \rho^2) \cos \Phi_1 \right) \right) .
\end{align*}
\]

After using the following transformations

\[ X_1 = \frac{X^2 + Y^2}{2\sigma^2(1 - \rho^2)}, \quad X_2 = \frac{V^2 + Z^2}{2\sigma^2(1 - \rho^2)}, \]

the joint probability density function of \( X_1, X_2, \Phi_1, \Phi_2 \) can be calculated as follows:

\[
    f_{X_1,X_2}\Phi_1,\Phi_2(x_1, x_2, \Phi_1, \Phi_2) = \left( \frac{1}{2\pi} \right)^2 (1 - \rho^2) \\
    \exp \left( - \left( x_1 + x_2 + \frac{1}{2\sigma^2} - 2\rho \sqrt{x_1 x_2} \cos (\Phi_1 - \Phi_2) - 2\sqrt{x_1 (1 - \rho^2)} \cos \Phi_1 \right) \right) .
\]

Based on this and after the integration of \( \Phi_1, \Phi_2 \), the joint probability density function of \( X_1, X_2 \), can be described with the following formula:

\[
    f_{X_1,X_2}(x_1, x_2) = (1 - \rho^2) \exp \left( - (x_1 + x_2 + \gamma) \right) I_0 \left( 2|\rho| \sqrt{x_1 x_2} \right) I_0 \left( 2\sqrt{\gamma (1 - \rho^2)} x_1 \right),
\]

where \( \gamma = E_b/N_0 = 1/2\sigma^2 \) is the signal-to-noise ratio.

The correlation peak is false detected, if the value of \( X_2 \) is bigger than \( X_1 \). The probability of this event is defined as follows:

\[
    \Pr \{ X_1 < X_2 \} = \int_0^\infty \int_{x_1}^\infty f_{X_1,X_2}(x_1, x_2) \, dx_2 \, dx_1 =
\]

\[
\int_0^\infty (1 - \rho^2) \exp \left( -x_1 \right) I_0 \left( 2\sqrt{\gamma (1 - \rho^2)} x_1 \right) \left( 2 \sqrt{\gamma (1 - \rho^2)} x_1 \right) I_0 \left( 2|\rho| \sqrt{x_1 x_2} \right) \, dx_2 \, dx_1 .
\]

(3.10)
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This formula can be calculated using [22]:

\[
\int_{x_1}^{\infty} \exp\left(- (x_2 + \gamma)\right) I_0 (2 |\rho| \sqrt{x_1 x_2}) \, dx_2 = \exp\left(- (x + \gamma)\right) \sum_{k=0}^{\infty} |\rho|^k I_k (2 |\rho| x).
\]

Then, one can get the following result:

\[
\Pr\{X_1 < X_2\} = \int_{0}^{\infty} \int_{x}^{\infty} f_{XX} (x, x_2) \, dx_2 \, dx = \\
\sum_{k=0}^{\infty} (1 - \rho^2) |\rho|^k \exp (-\gamma) \int_{0}^{\infty} \exp (-2x) I_0 \left(2 \sqrt{\gamma (1 - \rho^2)} x\right) I_k (2 |\rho| x) \, dx,
\]

where \( I_k(\cdot) \) is the k-th order modified Bessel function of the first kind. To simplify the previous equation, the Taylor series of the zero order Bessel function is applied as

\[
\int_{0}^{\infty} \exp (-2x) I_0 \left(2 \sqrt{\gamma (1 - \rho^2)} x\right) I_k (2 |\rho| x) \, dx = \\
\sum_{l=0}^{\infty} \frac{\left(\gamma (1 - \rho^2)\right)^l}{l! \Gamma(l + 1)} \int_{0}^{\infty} \exp (-2x) x^l I_k (2 |\rho| x) \, dx.
\]

Now, the following formula can be used from [23]:

\[
\int_{0}^{\infty} \exp \left(-t \frac{z}{\sqrt{z^2 - 1}}\right) I_\mu (t) t^\nu dt = \frac{\Gamma(\nu + \mu + 1)}{(z^2 - 1)^{-\frac{\nu}{2} - \frac{1}{2} \mu \nu}} P_{\nu}^{-\mu} (z), \quad \text{Re} \{\nu + \mu\} < 1,
\]

if \( t = 2 |\rho| x, \quad \nu = l, \quad \mu = k, \quad z = \frac{1}{\sqrt{1 - \rho^2}}, \)

where \( P_{\nu}^{-\mu}(\cdot) \) is the Legendre function of the third type with parameters \( \nu, \mu \). Based on this, one can get:

\[
\int_{0}^{\infty} \exp (-2x) x^l I_k (2 |\rho| x) \, dx = \int_{0}^{\infty} \exp \left(-t \frac{1}{|\rho|}\right) I_k (t) \left(\frac{t}{2 |\rho|}\right)^l dt = \\
\frac{1}{(2 |\rho|)^{l+1}} \frac{\Gamma(k + l + 1)}{\left(\frac{\rho^2}{1 - \rho^2}\right)^{-\frac{l}{2} (l+1)}} P_{l}^{-k} \left(\frac{1}{\sqrt{1 - \rho^2}}\right).
\]

Finally, the decision error probability is received after the substitution in Eq. 3.10.
3.2.2.3 Wrong decision probability using noncoherent sliding correlators

This subsection provides the derivation of the wrong decision probability in case of traditional sliding correlators.

**Theorem 3.2.** The probability of the wrong decision is determined by the following expression using noncoherent sliding correlators:

\[
P \{ X_1 < X_2 \} = \int_0^\infty \int_0^\infty f_{XX_2}(x,x_2)dx_2dx = \sum_{k=0}^\infty \exp \left(-\left(1 + \rho^2\right) \gamma \right)
\]

\[
\int_0^\infty \exp \left(-2x\right) \left| \rho \right| \sqrt{\frac{\gamma x}{2\pi}} I_0(2\sqrt{\gamma x}) I_k(2\left| \rho \right| \sqrt{\gamma x}) dx.
\]

**Proof.** If the measurements of the correlation values belonging to 0 and \( \tau \) delays are performed in non-overlapping time windows, then the notations Eq. 3.8 of the output signals given by Eq. 3.5 have to be modified according to the independency of the two complex Gauss distributed probability variables \( v_1(0) \) and \( v_1(\tau) \) as follows:

\[
\begin{align*}
\text{Re}\left\{ \langle v_1(0) \rangle \right\} &= X = 2E_b + \sqrt{2E_b n_0}, \quad E(X) = 2E_b, \quad E\left(\left(X - E(X)^2\right)\right) = 2E_b N_0 = \sigma^2, \\
\text{Im}\left\{ \langle v_1(0) \rangle \right\} &= Y = \sqrt{2E_b n_0}, \quad E(Y) = 0, \quad E\left(\left(Y - E(Y)^2\right)\right) = 2E_b N_0 = \sigma^2, \\
\text{Re}\left\{ \langle v_1(\tau) \rangle \right\} &= V = 2E_b \rho + \sqrt{2E_b n_{\tau 1}}, \quad E(V) = 2E_b \rho, \quad E\left(\left(V - E(V)^2\right)\right) = 2E_b N_0 = \sigma^2, \\
\text{Im}\left\{ \langle v_1(\tau) \rangle \right\} &= Z = \sqrt{2E_b n_{\tau 2}}, \quad E(Z) = 0, \quad E\left(\left(Z - E(Z)^2\right)\right) = 2E_b N_0 = \sigma^2,
\end{align*}
\]

\[
E(XY) = 0, \quad E(VZ) = 0, \\
E((X - E(X))(V - E(V))) = 0, \quad E((Y - E(Y))(Z - E(Z))) = 0.
\]

By using this, one can determine the four variable joint probability density function (if \( T_c = 1 \) and \( 2E_b = 1 \)):

\[
f_{X,Y,V,Z}(x,y,v,z) = \left(\frac{1}{2\pi\sigma^2}\right)^2 \exp \left(-\frac{1}{2\sigma^2} \left( (x-1)^2 + (v-\rho)^2 + y^2 + z^2 \right) \right).
\]

Applying the transformation given by Eq. 3.9, one can get the following four-dimension joint probability density function:

\[
f_{R_1,R_2,\Phi_1,\Phi_2}(r_1, r_2, \phi_1, \phi_2) = \exp \left(-\frac{1}{2\sigma^2} \left( (r_1 \cos \phi_1 - 1)^2 + (r_2 \cos \phi_2 - \rho)^2 \right) \right)
\]

\[
r_1, r_2 \left(\frac{1}{2\pi\sigma^2}\right)^2 \exp \left(-\frac{1}{2\sigma^2} \left( r_1 \sin \phi_1 \right)^2 + (r_2 \sin \phi_2)^2 \right) = r_1, r_2 \left(\frac{1}{2\pi\sigma^2}\right)^2
\]

\[
= \exp \left(-\frac{1}{2\sigma^2} \left( r_1^2 + r_2^2 + (1 + \rho^2) - 2r_1 \cos \phi_1 - 2\rho r_2 \cos \phi_2 \right) \right).
\]
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Then, by using the following transformations:

\[ X_1 = \frac{X^2 + Y^2}{2\sigma^2} = \frac{R_1^2}{2\sigma^2}, \quad X_2 = \frac{V^2 + Z^2}{2\sigma^2} = \frac{R_2^2}{2\sigma^2}, \]

the following joint density function can be calculated:

\[
\begin{align*}
& \quad f_{X_1, X_2; \Phi_1, \Phi_2}(x_1, x_2, \phi_1, \phi_2) = \left( \frac{1}{2\pi} \right)^2 \\
& \quad \quad \exp \left( - \left( x_1 + x_2 + \frac{1+\rho^2}{2\sigma^2} - 2\sqrt{\frac{x_1}{2\sigma^2}} \cos \phi_1 - 2\rho \sqrt{\frac{x_2}{2\sigma^2}} \cos \phi_2 \right) \right).
\end{align*}
\]

From this and after the integration of \( \phi_1 \) and \( \phi_2 \), the joint PDF of \( X_1 \) and \( X_2 \) variables is as follows:

\[ f_{X_1, X_2}(x_1, x_2) = \exp \left( - (x_1 + x_2 + (1 + \rho^2) \gamma) \right) I_0(2\sqrt{\gamma x_1}) I_0(2|\rho|\sqrt{\gamma x_2}). \]

Thus, the probability of the correlation peak’s false detection is given by the following formula:

\[
\Pr \{ \hat{X}_1 < \hat{X}_2 \} = \int_0^\infty \int_{x_1}^\infty f_{X_1, X_2}(x_1, x_2) \, dx_2 \, dx_1 = \\
\int_0^\infty \exp \left( - x_1 - (1 + \rho^2) \gamma \right) I_0(2\sqrt{\gamma x_1}) \int_{x_1}^\infty \exp \left( - x_2 \right) I_0(2|\rho|\sqrt{\gamma x_2}) \, dx_2 \, dx_1.
\]

The second integral of the previous equation can be calculated using [22] as follows:

\[
\int_x^\infty \exp \left( - x_2 \right) I_0(2|\rho|\sqrt{\gamma x_2}) \, dx_2 = \exp \left( -x \right) \sum_{k=0}^\infty \left( |\rho| \sqrt{\frac{\gamma}{x}} \right)^k I_k(2|\rho|\sqrt{\gamma x}).
\]

Therefore, the final form of the decision error probability can be received.

Now, both the sliding-and-tracking and the normal sliding correlators related expressions are available for noncoherent reception that will be used to compare their efficiency. The results will be presented after investigations belonging to the coherent case.

3.2.3 Statistical analysis assuming coherent reception

This section introduces the system model using coherent receiving, the joint PDF of the correlators’ output signals and the decision error probability for both sliding-and-tracking and sliding correlators.
3.2.3.1 Coherent synchronization system

It is assumed in the following that the phase position of the signal is known at the time of the symbol synchronization, therefore the low-pass equivalent of the input signals have only real values, and only the real part of the low-pass equivalent of the additive Gauss noise influences the correlators’ output signals.

The coherent synchronization system using training sequence is depicted in Figure 3.5. Similarly to the noncoherent reception, the aim of the calculation is the same, i.e. the receiver wants to detect the correlation function maximum at $\tau = 0$ position, thereby determine the maximum position. This means coherent measuring of the correlation function’s values that belong to 0 and $\tau$ delays, and then deciding on the greater of them. It is supposed that the transmitter sends a training sequence during the synchronization, e.g. it sends $g_{c1}(t)$ to the receiver, as long as the synchronization is not finished. Note that the synchronization is possible without training sequence in this case, too. Now, the statistical properties of this system are analysed in the two scenarios given in Figure 3.2a and Figure 3.2b.

The synchronous position can be determined by choosing the greater signal from the result of the two correlation calculation. Again, a null comparator is used for this purpose as it is shown in Figure 3.5.

The operation of the synchronizing system is given with the following equations:

$$z_1(\tau) = \int_{-T_c/2}^{T_c/2} r_{ekv}(t + \tau)g_{c1}^*(t)\,dt = \int_{-T_c/2}^{T_c/2} r_{ekv}(t + \tau)A\exp\left(-j2\pi\frac{\Delta f}{2T_c}t^2\right)\,dt,$$

where

$$r_{ekv}(t + \tau) = Ag_1(t + \tau) + Ag_1(t - T_c + \tau) + n(t + \tau),$$

and $z_1(\tau)$ is the output of the integrator at the end of the integration time, $n(t)$ is a Gauss distributed stochastic process with zero expected value and $N_0$ power density. The integration domains have $T_c/2$ offset for practical reasons.
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Now, the probability of \( z_1(0) < z_1(\tau) \) has to be calculated using a given signal-to-noise ratio \( (\gamma = E_b/N_0) \), i.e. according to the appropriate synchronous position, the \( \tau \) value is chosen based on the given measurement.

Using the simplification made in Eq. 3.3, the effect of the second part of Eq. 3.12 can be neglected because of the correlation properties of the chirp signal. Therefore, one can get the following formula:

\[
z_1(\tau) = A^2 \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} g_{c1}(t + \tau) g_{c1}^*(t) dt + A \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} n(t + \tau) g_{c1}^*(t) dt =
\]

\[
= A^2 \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} \exp\left(j2\pi\frac{\Delta f}{2T_c}(t + \tau)^2\right) \exp\left(-j2\pi\frac{\Delta f}{2T_c}t^2\right) dt +
\]

\[
+ A \int_{-\frac{T_c}{2}}^{\frac{T_c}{2}} n(t + \tau) \exp\left(-j2\pi\frac{\Delta f}{2T_c}t^2\right) dt.
\]

Based on this, the two correlation values are as

\[
z_1(0) = 2E_b + \sqrt{2E_b N_0},
\]

\[
z_1(\tau) = 2E_b \rho + \sqrt{2E_b N_\tau},
\]

where \( \rho \) is given in Eq. 3.6.

3.2.3.2 Wrong decision probability using coherent sliding-and-tracking correlators

In this subsection, the probability of the wrong decision is provided using the joint probability density function of the output signals of the sliding-and-tracking correlators.

**Theorem 3.3.** The wrong decision probability is determined by the following expression in case of coherent sliding-and-tracking correlators:

\[
\Pr\{X_1 < X_2\} = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{\gamma}{2}(1 - \rho)}\right).
\]

(3.14)

where \( \gamma \) is the SNR and \( \rho \) is the correlation parameter given by Eq. 3.6.

**Proof.** The calculation steps are similar to the noncoherent case. First, the joint PDF of \( z_1(0) \) and \( z_1(\tau) \) has to be determined that are defined by Eq. 3.13. To do that let us introduce the following notations:

\[
z_1(0) = X_1 = 2E_b + \sqrt{2E_b N_0}, \quad E(X_1) = 2E_b, \quad E\left((X_1 - E(X_1))^2\right) = 2E_b N_0 = \sigma^2,
\]

\[
z_1(\tau) = X_2 = 2E_b \rho + \sqrt{2E_b N_\tau}, \quad E(X_2) = 2E_b \rho, \quad E\left((X_2 - E(X_2))^2\right) = 2E_b N_0 = \sigma^2,
\]

\[
E\left((X_1 - E(X_1))(X_2 - E(X_2))\right) = 2E_b N_0 \rho.
\]

(3.15)
So, the bivariate joint probability density function can be expressed as follows (if $T_c = 1$ and $2E_b = 1$):

$$f_{X_1X_2}(x_1, x_2) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \exp \left( -\frac{1}{2\sigma^2(1-\rho^2)} \left( (x_1 - 1)^2 - 2\rho (x_1 - 1)(x_2 - \rho) + (x_2 - \rho)^2 \right) \right).$$

By applying the following transformations:

$$U = X_1 - 1, \quad V = \frac{X_2 - \rho - \rho (X_1 - 1)}{\sqrt{1-\rho^2}},$$

one is able to get:

$$f_{UV}(u,v) = \frac{1}{2\pi\sigma^2} \exp \left( -\frac{1}{2\sigma^2} (u^2 + v^2) \right).$$

The wrong decision probability can be determined as

$$\Pr\{X_1 < X_2\} = \Pr\left\{ U < V \sqrt{\frac{1+\rho}{1-\rho}} - 1 \right\}.$$

Therefore, it leads to the following equivalent expression:

$$\Pr\{X_1 < X_2\} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\gamma}{2}} (1-\rho) \right). \quad (3.16)$$

### 3.2.3.3 Wrong decision probability using coherent sliding correlators

**Theorem 3.4.** The wrong decision probability is determined by the following formula in case of coherent sliding correlators:

$$\Pr\{X_1 < X_2\} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\gamma}{2}} (1-\rho)^2 \right), \quad (3.17)$$

where $\gamma$ is the SNR and $\rho$ is the correlation parameter given by Eq. 3.6.
3.2. CORRELATION PEAK DETECTION

**Proof.** If the measurements of the correlation values belonging to 0 and \( \tau \) delays are performed in non-overlapping time windows, then the notations Eq. 3.15 of the output signals that are given by Eq. 3.13, have to be modified according to the independency of the two complex Gauss distributed probability variables \( v_1(0) \) and \( v_1(\tau) \) as follows:

\[
X_1 = 2E_b + \sqrt{2E_b}n_0, \quad E(X_1) = 2E_b, \quad E\left( (X_1 - E(X_1))^2 \right) = 2E_bN_0 = \sigma^2,
\]

\[
X_2 = 2E_b\rho + \sqrt{2E_b}n_\tau, \quad E(X_2) = 2E_b\rho, \quad E\left( (X_2 - E(X_2))^2 \right) = 2E_bN_0 = \sigma^2,
\]

\[
E((X_1 - E(X_1))(X_2 - E(X_2))) = 0.
\]

Therefore, the bivariate joint probability density function is as follows:

\[
f_{X_1,X_2}(x_1,x_2) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2\sigma^2} \left( (x_1 - 1)^2 + (x_2 - \rho)^2 \right) \right).
\]

Now, use the following transformations:

\[
U = X_1 - 1, \quad V = X_2 - \rho,
\]

one can get the following density function:

\[
f_{UV}(u,v) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2\sigma^2} \left( u^2 + v^2 \right) \right).
\]

Based on that, the wrong decision probability is

\[
\mathbb{P}\{X_1 < X_2\} = \mathbb{P}\{U < V - (1 - \rho)\}.
\]

Thus, the final expression of the theorem is received.

\[\blacksquare\]

So far, expressions are derived that describe the probabilities of wrong decisions for coherent and noncoherent reception assuming sliding-and-tracking and sliding correlators.

### 3.2.4 Numerical results

This subsection presents the numerical results. Each figure depicts the comparison of the sliding and the sliding-and-tracking correlators depending on different parameters. Figure 3.6 shows the decision error probability in function of the correlation parameter using noncoherent reception assuming low (Figure 3.6a) and high (Figure 3.6b) signal-to-noise ratios. Note that the probability interval, i.e. the Y axis is magnified in Figure 3.6a compared...
Figure 3.6: The decision error probability in function of the correlation parameter using noncoherent reception

to Figure 3.6b. It is clearly visible that the decision error probability is lower with sliding-and-tracking correlator than with simple sliding correlator. It should be noted, that there are some inaccuracy when $\rho$ is close to 1 due to complicated numerical calculations (see Eq. 3.7 and Eq. 3.11).

Figure 3.7 shows the decision error probability in function of the correlation parameter using coherent reception when the SNR is in the $[-15,0]$ dB (Figure 3.7a) and $[5,15]$ dB (Figure 3.7b) domains. In coherent case, the proposed sliding-and-tracking correlator achieve much lower error probability than the simple sliding correlator.

Figure 3.8 depicts the decision error probability depending on signal-to-noise ratio using noncoherent (Figure 3.8a) and coherent (Figure 3.8b) reception when $\rho = [0.1,0.9]$. It can be determined that the sliding-and-tracking correlator outperforms the sliding correlator in both coherent and noncoherent cases.

The results in (Figure 3.6 - Figure 3.8) basically tell that if there are some dependency between the two measurements, i.e. the noises are not statistically independent, then the sliding-and-tracking correlator is able to use this dependency more effectively to reduce the error probability of the wrong decision than the sliding correlator.

To summarize, a new, so-called sliding-and-tracking correlator are proposed and the statistical investigations of the error probability of correlation peak detection are provided in
3.2. **CORRELATION PEAK DETECTION**

![Graphs showing correlation peak detection](image)

(a) When $E_b/N_0 = [-15, 0]$ dB  
(b) When $E_b/N_0 = [5, 15]$ dB

Figure 3.7: The decision error probability in function of the correlation parameter using coherent reception

![Graphs showing signal-to-noise ratio](image)

(a) Using noncoherent reception  
(b) Using coherent reception

Figure 3.8: The decision error probability in function of signal-to-noise ratio in case of $\rho = [0.1, 0.9]$
AWGN channel in CSS based systems. The mathematical apparatus is able to cover all transmission schemes that have autocorrelation function with one well-determined peak, i.e. spectrum spread based systems like DS and FFH. The sliding-and-tracking correlator has the advantage of using the statistically not independent noises to reduce the error probability of finding the correlation peak. It is showed by numerical results that the sliding-and-tracking correlator outperforms the conventional sliding correlator in every investigated case. The presented formulas can be easily adapted for other spectrum spread based systems to achieve more accurate synchronization. Thus, emerging UWB and WLAN positioning systems can profit from these results that will be integral part of Industry 4.0 application areas, such as product tracking.

The next subsection arranges the presented theorems and defines the theses.

### 3.2.5 Theses related to the analysis of spread spectrum methods

**THESIS II.1.** [J2], [C2]  
I defined a new method for detecting the correlation peak in such radio communication systems that have autocorrelation function with one well-determined peak.

**THESIS II.2.** [J2], [C2]  
I introduced an analytical method for calculating the bit error rate of the correlation peak detecting mechanism provided in Thesis II.1.

Lists of theorems: Theorem 3.1, Theorem 3.2, Theorem 3.3, Theorem 3.4

### 3.3 Multi-user chirp spread spectrum technique

The last decade of telecommunication was about the increasing demand of higher data throughput and lower latency. However, as the focus changes from humans to machines, different requirements arise meaning that not every application needs high throughput. Reliable and robust communication is much important for these certain applications. Such possible applications are on the field of Industry 4.0 or smart agriculture, where reliable, but simple controlling of remote devices, automated vehicles or drones does not necessarily need high speed data connection. From a more general aspect the Internet of Thing is also a possible usage area of such Low-Power Wide-Area Networks, where similar requirements have to be fulfilled. For these applications, the wireless communication system can be based on spread spectrum techniques, which are quite simple and matured, but do not provide so high data transfer rates, than the nowadays popular Orthogonal Frequency-Division Multiplexing (OFDM) variants.
The spread spectrum techniques are known long ago and used by many different standards in vast of communication devices. Direct-Sequence Spread Spectrum and Frequency-Hopping Spread Spectrum techniques are used in numerous widely spread standards. The usage of DSSS includes the IEEE 802.11b standards, as the original Wi-Fi system; the IEEE 802.15.4 standard, which is the physical layer for ZigBee; the Code Division Multiple Access (CDMA) channel access method; and also it is integral part of different Global Navigation Satellite Systems (GNSS), like GPS, GLONASS and Galileo. FHSS can be familiar from Bluetooth, where its variant is used known as Adaptive Frequency-hopping spread spectrum.

Maybe, the Chirp Spread Spectrum based systems are not so well-known for the average user as other spread spectrum methods, but its benefits include high robustness against channel noise, interference and jamming. Furthermore, the resistance capability against the Doppler effect makes the CSS a very good candidate in such mobile communication environments, where support for terminals moving with high speeds and at long ranges is necessary. These reasons were led to standardize CSS as one of the supported physical layer of IEEE 802.15.4a for use in Low-Rate Wireless Personal Area Networks [48][49]. Its variants are also standardized in the last year for Long Range Wide Area Network, which is an emerging technology for LPWAN communication [50][51].

This section presents a new mobile communication system based on Chirp Spread Spectrum transmission. The downlink modulation scheme is extended with Pulse Position Modulation (PPM) to carry data for multiple mobile terminals simultaneously. The described novel mechanism ensures reliable and robust communication between the parties, especially for terminals moving with high speeds or at long range. Furthermore, the proposed system takes care of the uplink communication as well, where Closed-Loop Power Control is applied to handle the near-far problem and improve the performance of the system. Analytical investigations for downlink communication are described focusing on the instantaneous symbol error rate and average SER in Rayleigh fading channel. The results show that the proposed Multi-User Pulse Position based Chirp Spread Spectrum technique (shortly MU-PP-CSS) allows higher data rates that are used for the multi-user feature. In addition, numerical results are presented as well, and they point out the benefits of the applied CLPC mechanism.

The next subsection details the operation of the communication scheme as definitions. Then, its properties and performance are investigated using the results of Section 2.1 and Section 3.2. Finally, the results are presented and considerations regarding to the implementation of the proposed communication system are described.

### 3.3.1 System model

This subsection introduces the basic operation of the mobile communication system.
Definition 3.1. I developed a novel radio communication system that provides low-power wide area transmission for mobile terminals. It uses the MU-PP-CSS technique for downlink and PP-CSS for uplink communication, as well as it utilizes the measurement based closed loop power control mechanism to mitigate near-far problem.

Assume a communication system that provides wireless connectivity between a base station (BS) and mobile terminals (MT). The available bandwidth of the system is fully utilized by Chirp Spread Spectrum transmission, where the signals are modulated by chirp pulses, i.e. frequency varying sinusoidal pulses. To simplify the RF circuit design, time-division duplexity is applied in the scheme to separate the downlink and uplink communication. The downlink is based on the MU-PP-CSS technique, while the system applies PP-CSS on the uplink with measurement based CLPC method. The details and mathematical representations of these techniques are presented in the following subsections.

3.3.1.1 Downlink: MU-PP-CSS method

This subsection presents the elementary signals of the Multi-User Pulse-Position based Chirp Spread Spectrum technique.

Similarly to Eq. 3.1, the low-pass equivalent complex-valued representations of the elementary chirp signals can be described as

\[
\text{for } 0: \quad g_{c1}(t) = A \exp\left(j2\pi\frac{\Delta f}{2T_c}t^2\right), \quad t \in \left[\frac{-T_c}{2}, \frac{T_c}{2}\right)
\]

\[
\text{for } 1: \quad g_{c2}(t) = A \exp\left(-j2\pi\frac{\Delta f}{2T_c}t^2\right), \quad t \in \left[\frac{-T_c}{2}, \frac{T_c}{2}\right)
\]

(3.18)

where notations are also the same, i.e. \(A\) is the amplitude of the elementary signals, \(\Delta f\) is the chirp modulated signal’s frequency spreading domain and \(T_c\) is the frequency varying sinusoid signal’s symbol time. Figure 3.1 depicts the instantaneous frequency changing of the signals.

3.3.1.2 MU-PP-CSS variant A

This subsection describes the MU-PP-CSS variant A.

Definition 3.2. To extend the downlink chirp spread spectrum communication with pulse position modulation, framing is necessary in order to the terminals be able to distinguish the transmitted bits from the BS. Here, it is supposed that the elementary down-chirp signal is applied to indicate the starting of the frame, however, it is also possible to use the up-chirp signal for this purpose.
3.3. **MULTI-USER CHIRP SPREAD SPECTRUM TECHNIQUE**

Let \( g_{c0}(t) \) the elementary signal for frame synchronization and \( g_{c1}(t), g_{c2}(t), \ldots, g_{ci}(t), \ldots, g_{c2M}(t) \) the elementary signals for data symbols in the non-binary pulse position based chirp modulation system. The next expression describes the low-pass equivalent *complex-valued* representations of these signals:

\[
g_{c0}(t) = A \exp \left( -j2\pi \frac{\Delta f}{2T_c} (t - \tau_0)^2 \right), \quad (t - \tau_0) \in \left[ -\frac{T_c}{2}, \frac{T_c}{2} \right)
\]

\[
g_{ci}(t) = A \exp \left( j2\pi \frac{\Delta f}{2T_c} (t - \tau_i)^2 \right), \quad (t - \tau_i) \in \left[ -\frac{T_c}{2}, \frac{T_c}{2} \right)
\] (3.19)

where \( \tau_i \) is the delay related to the time position of each symbol, \( i = 1, \ldots, 2^M \) is the symbol number and \( 2^M + 1 \) is the overall number of the available symbols in the system. \( M \) is referred later as the data rate parameter. Other notations are identical to ones in Eq. 3.18.

The equation describes that the useful information carrying symbols are generated using proper time-shifted up-chirp signals. Due to the applied chirp modulation, this kind of time-shifted signals have the important attribute that in a time slot (with given delay) there is only one elementary signal available that has the appropriate even and odd auto- and cross-correlation according to the other symbols. With other words, CSS allows to use quasi-orthogonal signals with different, \( 2^M \) possible delays to transmit \( 2^M \) possible data symbols. These, well-distinguishable symbols can provide the simultaneous data transmission feature of the system.

![Figure 3.9: Model of the original CSS (A) and the extended MU-PP-CSS (B) based communication systems (variant A)](image-url)
The present variant, which denoted by $A$, is about that $2^M$ different data symbol are mapped to $M$-length binary words similarly to the Gray mapping in case of PSK modulations. The mapping itself is not detailed here. The principle is simply that the $M$-bit transmitted information using the $2^M$ different symbols are separated and assigned based on a predefined pattern to at most $M$ mobile terminals. One possible pattern is when a bit or a part of the transmitted $M$-length binary word is associated to a mobile terminal based on its sequence number or identifier. The operation of the traditional CSS based method and the proposed MU-PP-CSS (variant A) technique are illustrated in Figure 3.9.

For better understanding, let’s look the following example. Assume that the system configured with the data rate parameter $M = 4$ and there are the same number of mobile termi-
nals. If the BS sends the elementary symbol related to 0100, all the MTs will receive and demodulate it. Then the first, third and fourth terminal will get the 0 and the second the 1 binary information. Note, it is also possible to assign the 4-length binary word as the first MT gets the first 01 part and the second terminal the second 00 part of the transmitted word, when there are only two terminals in the system. The latter case is depicted in Figure 3.9 B. Of course, other allocation patterns can be applied for the separation. Just a reminder, to transmit \( M = 4 \) length binary words by the base station, \( 2^M = 16 \) different, time-shifted up-chirp signals are necessary (see Eq. 3.19).

The structure of the proposed non-coherent MU-PP-CSS variant A receiver using non-binary chirp modulated signals is illustrated in Figure 3.10.

### 3.3.1.3 MU-PP-CSS variant B

By extensively investigating the possibilities, this subsection presents a slightly different multi-user scheme.

**Definition 3.3.** Instead of mapping the time-shifted signals to binary words, which then are bitwise or partially assigned to the mobile terminals, the idea is to map the time-shifts directly to MTs. One \( \tau \) delay is required to be associated to a terminal, the up- and down-chirps related to the given delay as pairs carry the binary information for the MT. In other words, in this case the base station is able to send \( M \) number of different time-shifted up- and down-chirp signals, however, only one mobile terminal will receive one bit information depending on the \( \tau \)-shift. The exact value of the binary information is decided based on the direction of the given, transmitted signal. Framing is not necessary in this variant. The low-pass equivalent *complex-valued* representations of the signals of MU-PP-CSS variant B are given as

\[
\begin{align*}
g_{ci_1}(t) &= A \exp \left( -j 2\pi \frac{\Delta f}{2T_c} (t - \tau_i)^2 \right), \quad (t - \tau_0) \in \left[ -\frac{T_c}{2}, \frac{T_c}{2} \right] \\
g_{ci_2}(t) &= A \exp \left( j 2\pi \frac{\Delta f}{2T_c} (t - \tau_i)^2 \right), \quad (t - \tau_i) \in \left[ -\frac{T_c}{2}, \frac{T_c}{2} \right]
\end{align*}
\] (3.20)

where \( i = 1, \ldots, M \) is the symbol number and \( 2M \) is the overall number of the available symbols in variant B. Other notations are the same as in Eq. 3.19.

Let’s look another example. For the sake of simplicity, now assume two MTs in the system and the preconfigured data rate parameter is \( M = 2 \). In this case, there are \( M = 2 \) different, time-shifted up- and down-chirp signals to carry useful information. Let’s use the following mapping: \( \tau_1 \) delay (i.e. \( g_{c1_1} \) and \( g_{c1_2} \) elementary signals) is assigned to the
3.3.1.4 Uplink: PP-CSS using CLPC

This subsection describes the uplink communication of the proposed system including the proposed closed-loop power control mechanism.

Definition 3.4. In the proposed communication system, the same signals are used for the uplink transmission. However, the common channel is shared in time between the mobile...
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Figure 3.12: System model of the proposed measurement based feedback power control mechanism

Terminals in this case. Since the symbol synchronization is defined by the base station, each MT transmits in its own, dedicated time slot using pulse position based chirp spread spectrum modulation. The $i$–$th$ mobile terminal can send its binary information using the assigned elementary signals. According to MU-PP-CSS variant A, these signals can be the up-chirps starting with different time delays (Eq. 3.19) or they can be both up- and down-chirps with identical time-shift in case of variant B (Eq. 3.20). Hence, the overall throughput of MTs is $M$. Furthermore, measurement based feedback CLPC mechanism is applied in the uplink to overcome the near-far problem.

The signals of different mobile terminals can overlap at the receiver of the BS, however, the quasi-orthogonal property of the signals allows their efficient separation. It is important that the incoming signals are not fully orthogonal, thus they interfere with each other that can be amplified by the near-far problem. This means that the MTs usually are in different distances from the base station, therefore the wireless signals arrive with different amplitudes (and with different time delays) into the antenna of the BS during the uplink communication. Hence, a closer MT’s signal can suppress the farer one’s. To handle this issue and control the amplitude of the received signals, closed-loop power control mechanism is applied in the transmitter of MTs. Nevertheless, the system is insensitive to the estimation error of the amplitude’s level due to low odd and even cross-correlation between the signals (see next section), but assuming a noisy and fading channel, where the estimation is not perfect, the bit-error rate of the communicating system is strongly influenced. Therefore, the measurement based feedback power control method is applied on the uplink to ease the proper reception in the base station. This CLPC mechanism is detailed in Section 2.1, which contains its performance analysis using the non-coherent binary transmission as well. In addition, the investigations cover Rayleigh, Rice and Nakagami fading types, too. Here, its slightly changed version is used, but the presented results are still applicable in this case. The modified system model of the measurement based feedback CLPC mechanism is illustrated...
in Figure 3.12.

Similarly to Eq. 2.1, the low-pass equivalent complex-valued representation of the binary modulated signal $r^*$ without channel equalization at the base station’s front end during a slot is given as

$$r^* = \sqrt{E_s}g(b)z + n, \quad t \in (0, T_c],$$

where $E_s$ is the average symbol energy without fading and power control, $T_c$ is the symbol time of chirp signals, $b \in [0, 1]$ is the binary information, $\left\{g^{(b)}, \|g^{(b)}\|^2 = 1\right\}$ are the low-pass equivalent complex-valued representation of the elementary signals in non-coherent case $\langle g^{(0)}, g^{(1)} \rangle = 0$, $z$ is the complex fading channel parameter and $n$ is the low-pass equivalent complex-valued representation of the additive white Gaussian noise with $\mathbb{E}[n] = N_0$. As earlier, $z$ is a random variable that represents the instantaneous amplitude of the received binary signal ($\mathbb{E}[|z|^2] = 1$). The distribution of $z$ depends on the fading scenario, in the present case non-selective slow Rayleigh fading model is considered. It is supposed that the fading channel parameter is estimated in the transmitter of the mobile terminals by measuring an independent pilot channel, which has the same fading parameter as the communication channel. In this case the pilot and communication channels are the same ensuring that the fading is identical, however, the additive noise is assumed to be independent during the downlink and uplink communication. Furthermore, supposing MU-PP-CSS variant A, the down-chirp signal that is used for frame synchronization, is a good candidate to be the pilot signal for the proposed measurement based feedback power control mechanism.

By this obvious, but sophisticated way, all the derived expressions and results are directly applicable for the uplink of this communication scheme.

It is shown in the next section that the PP-CSS method is insensitive to the possible arrival time differences.

### 3.3.2 Correlation properties of elementary signals in PP-CSS technique

In this subsection the statistical investigation is introduced at the receiver side of the proposed PP-CSS communication system. It is shown that the mathematical apparatus of Section 3.2 can be reused to analyse the correlation properties of the elementary signals that are applied in the proposed communication system.

To simplify the analysis, the correlation of $g_{c1}$ (i.e. up-chirp signal related to $\tau_1$ delay) and $g_{c12}$ (i.e. down-chirp signal related to the identical delay) elementary signals will be analysed, because the other elementary signals can be derived from these by time shifts.

To detect the transmitted symbols in the receiver, the peak positions of the chirp signals related to the correlator’s output $z_{11}$ and $z_{12}$ has to be detected. Thus, the behaviour of
3.3. MULTI-USER CHIRP SPREAD SPECTRUM TECHNIQUE

The output pulses \( z_{11}(\tau) \) and \( z_{12}(\tau) \) of the correlator related to the up-chirp signal in the noncoherent binary chirp receiver

are relevant, when the signal is delayed according to the symbol synchrony. This behaviour is described by the formula below:

\[
\begin{align*}
    z_{11}(\tau) &= \left| \int_{-T_c/2}^{T_c/2 - \tau} Ae^{j\theta} g_{c1}(t + \tau) g_{c1}^*(t) dt + \int_{T_c/2 - \tau}^{T_c/2} Ae^{j\theta} g_{c1}(t + \tau) g_{c1}^*(t) dt \right|^2, \\
    z_{12}(\tau) &= \left| \int_{-T_c/2}^{T_c/2 - \tau} Ae^{j\theta} g_{c1}(t + \tau) g_{c1}^*(t) dt + \int_{T_c/2 - \tau}^{T_c/2} Ae^{j\theta} g_{c1}(t + \tau) g_{c1}^*(t) dt \right|^2,
\end{align*}
\]

where \( \tau \) is the delay, and where the output of the noncoherent detector related to the up-chirp signal is analysed twice, when the incoming signal contains two, identical up-chirps \( (z_{11}(\tau)) \) and when it includes one up-chirp and one down-chirp \( (z_{12}(\tau)) \) in \([-T_c/2, T_c/2]\) domain. The solution of these equation is given in Figure 3.13, when \( T_c = 1, \Delta f = 200, A = 1 \) and the symbol energy is \( E_s = 1/2 \). It is clearly visible, that the correlation peak can be detected very accurately allowing to determine the symbol synchrony of the chirp signal very accurately as well.

Besides, it is necessary to investigate the output of the other detector described by the

![Figure 3.13: The output pulses \( z_{11} \) and \( z_{12} \) of the correlator related to the up-chirp signal in the noncoherent binary chirp receiver](image-url)
In this case the output of the noncoherent detector related to the down-chirp signal is analysed twice, when the incoming signal contains two, identical up-chirps \( z_{21}(\tau) \) and when it includes one up-chirp and one down-chirp \( z_{22}(\tau) \) in \([-T_c/2, T_c/2]\) domain. The results are depicted in Figure 3.14 using the same values of the parameters.

Comparing Figure 3.13 and Figure 3.14, it is visible that the different chirp signals are almost orthogonal close to \( \tau = 0 \), therefore only the proper correlator produces useful output pulse during the sampling, while the output of the other correlator is almost zero.

As mentioned before, the base station is able to select from \( 2M \) different elementary signals to transmit during a \( T_c \) long slot. However, the BS radiates only one signal at a time, i.e. the number of transmitted information carrying bits is \( 1 + \log_2 M \) in a slot.
3.3. MULTI-USER CHIRP SPREAD SPECTRUM TECHNIQUE

Theorem 3.5. The PP-CSS transmission method uses quasi-orthogonal chirps as elementary signals that can be considered orthogonal practically.

Proof. In this case, the structure of optimal noncoherent receiver for CSS is essentially not differs from the general noncoherent receiver using orthogonal elementary signals. The only difference is that the integration required for decision have to be performed based on several $T_c$ long domains starting in different time instants. In addition, in non-binary case the index of the maximum of $2M$ decision signals ($z_{i1}$ and $z_{i2}$) have to be determined. This noncoherent receiver is illustrated in Figure 3.10, and its operation is represented by the following formulas:

$$z_{ik} = \left| \int_{-\frac{T_c}{2}+\tau_i}^{\frac{T_c}{2}+\tau_i} r_{ekv}(t) g^{*}_{ci_k}(t) dt \right|^2 \quad (3.21)$$

where $i = 1, \ldots, M$, $k = 1, 2$ means the up- and down-chirps, and the integration domains are shifted with $T_c/2$ due practical reasons. In noiseless case when $k = 1$ (up-chirp) the baseband equivalent on the receiver’s input in the $i$-th slot is:

$$r_{ekv}(t) = A \exp \left( 2j\pi \frac{\Delta f}{2T_c} (t - \tau_i) \right),$$

thus, after substituting in Eq. 3.21, one can get:

$$z_{i1} = \left| \int_{-\frac{T_c}{2}+\tau_i}^{\frac{T_c}{2}+\tau_i} r_{ekv}(t) g^{*}_{ci_1}(t) dt \right|^2 =$$

$$= A^4 \left| \int_{-\frac{T_c}{2}+\tau_i}^{\frac{T_c}{2}+\tau_i} \exp \left( 2j\pi \frac{\Delta f}{2T_c} (t - \tau_i) \right) \exp \left( -2j\pi \frac{\Delta f}{2T_c} (t - \tau_i) \right) dt \right|^2 = A^4 T_c^2 = 4E_s^2,$$

where $E_s$ is the symbol energy expressed as:

$$E_s = \frac{1}{2} A^2 \int_{-\frac{T_c}{2}+\tau_i}^{\frac{T_c}{2}+\tau_i} \left| g_{ci_k}(t) \right|^2 \text{ for } k = 1, 2.$$

Furthermore,

$$z_{i2} = \left| \int_{-\frac{T_c}{2}+\tau_i}^{\frac{T_c}{2}+\tau_i} r_{ekv}(t) g^{*}_{ci_2}(t) dt \right|^2 =$$

$$= A^4 \left| \int_{-\frac{T_c}{2}+\tau_i}^{\frac{T_c}{2}+\tau_i} \exp \left( 2j\pi \frac{\Delta f}{2T_c} (t - \tau_i) \right) \exp \left( 2j\pi \frac{\Delta f}{2T_c} (t - \tau_i) \right) dt \right|^2 \approx 0,$$
and for every \( i^* \) (if \( i \neq i^* \), \( k = 1, 2 \))
\[
z_{i^*_k} = \left| \int_{\frac{T_c}{2} + \tau^*_i}^{\frac{T_c}{2} + \tau^{*^*_i}} r_{ekv}(t) g_{ci^*_k}^*(t) dt \right|^2 = A^4 \left| \int_{\frac{T_c}{2} + \tau^*_i}^{\frac{T_c}{2} + \tau^{*^*_i}} \exp \left( 2j\pi \frac{\Delta f}{2T_c} (t - \tau^*_i)^2 \right) \exp \left( \pm 2j\pi \frac{\Delta f}{2T_c} (t - \tau^{*^*_i})^2 \right) dt \right|^2 \approx 0.
\]

This means that the output of the other detectors, which do not receive any useful signal, is approximately zero.

The following example justifies this hypothesis.

**Example 3.3.1.** Assume that \( T_c = 1, \Delta f = 200, A = 1, z_{i_1} = 4E_s^2 = 1 \), then
\[
z_{i_2} = A^4 \left| \int_{\frac{T_c}{2} + \tau^*_i}^{\frac{T_c}{2} + \tau^{*^*_i}} \exp \left( 4j\pi \frac{\Delta f}{2T_c} (t - \tau^*_i)^2 \right) dt \right|^2 = 2.39 \cdot 10^{-3} \ll 1
\]

and
\[
z_{i^*_k} = \left| \int_{\frac{T_c}{2} + \tau^*_i}^{\frac{T_c}{2} + \tau^{*^*_i}} r_{ekv}(t) g_{ci^*_k}^*(t) dt \right|^2 \approx 5 \cdot 10^{-3} \ll 1,
\]

when the difference of delays is above \( |\tau^*_i - \tau^{*^*_i}| \geq 0.03 \cdot T_c \).

Hence, the less the correlation of elementary signals are, the more the two messages can be distinguished from each other, i.e. the signals are almost orthogonal. By definition full orthogonality is only achieved, when the correlation equals exactly to zero. Nevertheless, in this case the correlation approximates it very closely.

Now, the behavior of the wireless channel will be taken into account, namely the disturbing effect of the random noise.

**Theorem 3.6.** The quasi-orthogonal property of the chirp signals in PP-CSS transmission technique is still valid, if additive white Gaussian noise is assumed in the communication channel.

**Proof.** The decision determining \( z_{i^*_k} \) variables in the noncoherent non-binary chirp receiver can be described by Eq. 3.21 as presented earlier. Suppose that the baseband equivalent of the input of the receiver is expressed as follows:
\[
r_{ekv}(t) = Ae^{j\theta} g_{ci^*_k}(t) + n_I(t) + jn_Q(t),
\]
where \( k = 1, 2 \), \( \theta \) is the unknown phase-shift of the radio channel, \( n_l(t) \) and \( n_Q(t) \) are the real and imaginary components of the Additive White Gaussian Noise (AWGN). \( n_l(t) \) and \( n_Q(t) \) are independent normal distributed stochastic processes, each has the power density \( N_0 \) and the autocorrelation function \( N_0 \delta(t) \). The attenuation of the channel is neglected, because it is already included in the Signal-to-Noise Ratio (SNR) during the calculation of the effective signal’s power.

\( z_{i1} \) and \( z_{i2} \) can be described in this case by the following expressions:

\[
\begin{align*}
z_{i1} &= \left| \int_{-\frac{T_c}{2} + \tau_i}^{\frac{T_c}{2} + \tau_i} r_{ekv}(t) g_{ci1}^*(t) dt \right|^2 = \left| \int_{-\frac{T_c}{2} + \tau_i}^{\frac{T_c}{2} + \tau_i} \left[ Ae^i\theta g_{ci1}(t) + n_{li}(t) + jn_{qi}(t) \right] g_{ci1}^*(t) dt \right|^2 = \\
&= \int_{-\frac{T_c}{2} + \tau_i}^{\frac{T_c}{2} + \tau_i} A^2 e^i\theta |g_{ci1}(t)|^2 dt + \int_{-\frac{T_c}{2} + \tau_i}^{\frac{T_c}{2} + \tau_i} n_{li}(t) g_{ci1}^*(t) dt + j \int_{-\frac{T_c}{2} + \tau_i}^{\frac{T_c}{2} + \tau_i} n_{qi}(t) g_{ci1}^*(t) dt \right|^2 = \\
&= |2e^{i\theta}E_s + n_{li} + jn_{qi}|^2,
\end{align*}
\]

and

\[
\begin{align*}
\approx n_{li} + jn_{qi}|^2,
\end{align*}
\]

where \( E_s \) is symbol energy as earlier, \( n_{li}, n_{li}, n_{qi}, n_{qi} \) and \( n_{qi} \) are almost independent Gaussian distributed probability variables with zero expected value and with \( 2E_s N_0 \) variation. The independency of noise component can be only fulfilled, if the \( g_{ci1}(t) \) and \( g_{ci2}(t) \) signal pair would be exactly orthogonal. Nevertheless, assuming independency is applicable in practical cases due to the small absolute value of the correlation. The differences of the correlators output are clearly visible.

Note that the peak related to \( \tau = 0 \) position follows Rice distribution in AWGN channel, which is illustrated in Figure 3.15. The Rice distribution is given by the following formula:

\[
F_r(y) = \exp\left(- (y + \gamma_0)\right) I_0\left(2\sqrt{y\gamma_0}\right),
\]
Figure 3.15: PDF of pulse’s peak value observable on the output of the receiver supposing different SNR values

\[ \gamma_0 = \frac{E_s}{N_0}, \quad Y = \frac{z_{l_0}(0)}{E_sN_0}. \]

It is also a consequence of the above described formulas that on the normalized output of every detector, which did not receive any useful signal, i.e. \( z_{l_2}(0) \) and \( z_{l_k}(0) \) for \( l = 1, \ldots, M \) and \( k = 1, 2 \), the probability density function follows Rayleigh distribution identically and independently:

\[ f_Y(y) = \exp(-y). \]

3.3.3 Performance analysis and numerical results

This section introduces the performance analysis of the proposed MU-PP-CSS and CLPC techniques. The investigations focus on instantaneous and average Symbol Error Ratio. In addition, theoretical results are presented regarding the efficiency of the measurement based feedback power control mechanism.
3.3.3.1 Performance and evaluation of downlink

To analyse the performance of MU-PP-CSS method properly, the joint PDF of $z_{i_k}, i = 1, 2, \ldots, M, k = 1, 2$ probability variables has to be determined. Furthermore, the probability of the wrong decision has to be investigated to calculate the exact symbol error ratio. This event formally happens if one from $z_{i_2}, z_{i_k}, i \neq i_*$ is higher than $z_{i_1}$, when $g_{ci_1}(t)$ (i.e. the $i$-th delayed, up-chirp symbol) was sent over the radio channel. Due to the non-coherent receiver structure of MU-PP-CSS, the following general expression for non-coherent reception in AWGN channel can be applied from [22]:

$$P_{SER_{DL}} = 2^{M-1} \sum_{m=1}^{2^{M}-1} (-1)^{m+1} \binom{2^{M}-1}{m} \frac{1}{m+1} \exp\left(-\frac{E_s}{N_0 m+1}\right),$$  \hspace{1cm} (3.22)

where $P_{SER_{DL}}$ is the symbol error rate for downlink. The SER for different data rates (i.e. for different $M$ parameters) are shown on Figure 3.16. The results for traditional non-coherent communication equals to the $M = 1$ case. It is clearly visible and also unequivocal that increasing binary data rate carried by one symbol requires higher SNR to achieve the same SER. However, the difference in SNR between $M = 1$ and $M = 4$ is at most about 5 dB around 0 dB and getting less in higher SNR domains. The calculation and the figure basically tell the price that have to be paid for the simultaneous transmission to more than one mobile terminal. This means that compared to the traditional CSS, to achieve the same SNR level and the same communication distance, the transmission power have to be slightly increased (by $2 - 5$ dB). The other option is to decrease the communication distance, while the same SNR and transmission power levels can be maintained. Reduction of the distance depends on the actual propagation attributes of the communication environment.

Beside the instantaneous SER, the average SER can be determined for slow, frequency non-selective Rayleigh fading channel by the following formula based on [22]:

$$\bar{P}_{SER_{DL}} = 2^{M-1} \sum_{m=1}^{2^{M}-1} (-1)^{m+1} \binom{2^{M}-1}{m} \frac{1}{m+1} \frac{1}{1 + \frac{m}{N_0 \bar{E}_s}},$$  \hspace{1cm} (3.23)

where $\bar{E}_s$ is the average symbol energy of the signal and $\bar{P}_{SER_{DL}}$ is the average SER for downlink. The average SER of the proposed MU-PP-CSS technique in Rayleigh fading channel using different data rates is depicted in Figure 3.17. Like on the previous figure, $M = 1$ means the traditional CSS based communication. Similar relations can be found in this case, however, the deviation between the data rates become constants in the higher SNR domains. The relevance of these results is to anticipate the performance of the MU-PP-CSS based system in such environment, where is no dominant propagation along a line of sight between the transmitter and receiver.
CHAPTER 3. ANALYSIS OF SPREAD SPECTRUM METHODS

Figure 3.16: SER of the traditional non-coherent systems ($M = 1$) and the MU-PP-CSS technique ($M \geq 2$)

Figure 3.17: Average SER of the traditional non-coherent systems ($M = 1$) and the MU-PP-CSS technique ($M \geq 2$) in Rayleigh fading channel
3.3. MULTI-USER CHIRP SPREAD SPECTRUM TECHNIQUE

3.3.3.2 Performance and evaluation of uplink

For the exact error analysis, the joint PDF of $z_{ik}, i = 1, 2, \ldots, M, k = 1, 2$ probability variables and the probability of the wrong decision have to be determined again. The same approach is used as in case of the downlink analysis. Thus, the solution of the instantaneous symbol error rate is the following:

$$P_{SER_{UL}} = \frac{1}{2} \exp \left( -\frac{E_s}{2N_0} \right),$$  \hspace{1cm} (3.24)

where $P_{SER_{UL}}$ is the symbol error rate for the uplink communication. Note, that this equation equals to Eq. 3.22 when $M = 1$.

Similarly, the average SER in slow, frequency non-selective Rayleigh fading channel is expressed as

$$\bar{P}_{SER_{UL}} = \frac{1}{2 + \frac{E_s}{N_0}},$$  \hspace{1cm} (3.25)

where $\bar{P}_{SER_{UL}}$ is the average SER of uplink. Both curves are visible in Figure 3.16 and Figure 3.17 in case of $M = 1$.

For extensive investigations, the influence of the quasi-orthogonality on the demodulator has to be analysed as well. In other words the operation of the receiver has to be investigated, when the correlation of the two elementary signals $g_{c_1}(t)$ and $g_{c_2}(t)$ is $\rho$. The solution of this problem is known from [22] for the channel without fading:

$$P_b = \left[ \sum_{k=0}^{\infty} \left( \frac{1 - \sqrt{1 - \rho^2}}{1 + \sqrt{1 - \rho^2}} \right)^k \times I_k \left( |\rho| \times \frac{E_b}{2N_0} \right) \right] \times \exp \left( -\frac{E_b}{2N_0} \right) \right],$$  \hspace{1cm} (3.26)

where $P_b$ is the bit error rate. The results are depicted in Figure 3.18 for different correlation parameters. Based on the figure the effect of correlation between the elementary signals can be neglected in the proposed system, if $|\tau| \geq 0.015 T_c$.

As mentioned, the original measurement based feedback power control mechanism from Section 2.1 is slightly changed and aligned to the nature of PP-CSS based communication system, but still, the results are directly applicable. Therefore, a brief summary is presented here focusing on the Rayleigh fading channel. However, Section 2.1 contains results related to other fading types, like Nakagami-m and Rician fadings.

The bit error rate of the proposed measurement based feedback power control mechanism
Figure 3.18: BER of non-coherent binary receiver depending on the SNR for different correlation parameters ($\rho$)

assuming Rayleigh fading channel is given as follow

$$
P_{b_{CLPC}}(\gamma) = \int_0^\infty \frac{1}{h(y)} \frac{\gamma_0}{1+\gamma_0} \exp \left( -\frac{\gamma_0}{1+\gamma_0} y \right) dy,
$$

(3.27)

where $P_{b_{CLPC}}(\gamma)$ is the average BER, $\gamma$ is the SNR of the channel, $\gamma_0$ is the SNR of the pilot signal, $y$ describes the stochastic process of the fading parameter estimation:

$$
Y = \left| z + \frac{\tilde{n}}{\sqrt{E_0}} \right|^2,
$$

and the $h(x)$ function introduces the threshold value related to the maximum transmission power of the mobile terminals similarly to Eq. 2.3:

$$
h(x) = \begin{cases} 
  c & \text{if } |x| \leq c \\
  x & \text{if } |x| > c
\end{cases}
$$

Basically, Eq. 3.27 represents the behaviour of the uplink communication taking into account the limitations of the mobile terminals regarding their transmission power and the
estimation error due to the imperfect reception of the pilot (i.e. the frame synchronizing down-chirp) signal. The numerical results are depicted in Figure 2.7 and Figure 2.3. The former figure illustrates the effect of changing the transmission power related threshold value in Rayleigh fading channel. The latter one shows the bit-error rate depending on different level of the pilot signal’s SNR. On both figures the theoretical BER of traditional, unequalised communication (that can be a binary CSS or a binary PPM based system) is depicted by blue colour. Compared to that systems the proposed CLPC mechanism is able to significantly increase the performance of the uplink communication. In some cases it can even exceed the performance of diversity usage as well. It is visible that the limits of mobile terminals and the quality of pilot signal, i.e. how effectively the MT can eliminate the distortions of the radio channel, have strong impact on the communication.

3.3.4 Discussion about the proposed transmission scheme

This section summarizes the pros and cons of the proposed system, and overviews some important practical aspects.

The realization of the described concept does not require complex signal processing compared to the traditional CSS based transmitter and receiver, while changing the chirp signal attributes proposed by other multiple access schemes might result in difficult transmitter and/or receiver structure. For effective communication effective management of the power amplifier is necessary especially in LPWAN systems. By investigating this issue, the MU-PP-CSS technique just slightly increases the peak-to-average power ratio (PAPR) at the base station side compared to the original CSS based communication. Nevertheless, the growth of PAPR is much smaller than in case of fully overlapping signals. Similarly, the transmission power limitation of the mobile terminals is introduced by the \( c \) parameter during the analysis of the CLPC mechanism. This parameter describes the natural behaviour that a transmitter is not able to compensate completely in every case the distortion of the radio channel. For example when very large or infinite amplitude compensation is required, then an MT can raise its transmission power only to its limit. These practical aspects are usually not considered by the related works.

Assuming the MU-PP-CSS variant A, the down-chirp signal has two purposes in the system. On the one hand, it provides the frame synchronization for the mobile terminals. On the other hand, it functions as pilot signal for the CLPC mechanism. Therefore it is a sophisticated solution to connect these two concepts in the proposed communication system.

The advantages of the proposed downlink communication are the following:

1. the base station is able to communicate with more than one mobile terminals simultaneously;
2. the transmitter sends basically only one elementary signal at a time, thus the transmission power only slightly grows as $M$ increases;

3. compared to the traditional CSS, the MU-PP-CSS technique does not require to significantly increase the transmission power to maintain the same coverage.

Contrarily, the drawbacks are as follows:

1. the elementary signals with different delays transmitted in neighbouring slots can overlap as mentioned (see Figure 3.9), due to this behaviour the PAPR of the resultant signal increases, i.e. the instantaneous power of the transmitter grows;

2. as visible from the receiver structures (Figure 3.10 and Figure 3.11) lots of correlators ($2M$ in case of MU-PP-CSS variant A and $2M$ in case of MU-PP-CSS variant B) are necessary in the receiver that significantly improves its complexity. However, applying matched filters instead of the correlators can simplify the structure. In case of variant A one filter is matched to the down-chirp and one to the up-chirps. The latter one is able to detect all the pulse-position based signals, i.e. all the time-shifted up-chirps, using a sliding time-window. Similarly, in case of variant B two matched filters are capable of handling the input signals.

3. the scalability of the proposed system is limited. In variant A, when a new mobile terminal needs to be added to the communication, the number of the necessary delayed up-chirp signals has to be doubled. In variant B, only one additional delay is required.

The benefits of the proposed uplink communication are the following:

1. every mobile terminal is able to communicate with the base station simultaneously;

2. the transmitter of every MT sends only one elementary signal at a time, this allows the transmission power be the same compared to the traditional chirp spread spectrum, i.e. the uplink communication range is not affected;

3. the system is barely sensitive to the fluctuations of time delay, i.e. to the synchronization errors (see Figure 3.13 and Figure 3.14). The base station can provide the time synchronization.

The disadvantage of the CSS-PPM based uplink communication is that transmission power control is required in the mobile terminals to tackle the near-far problem, however, this issue is handled by the proposed CLPC mechanism.

An additional remark is that in case of MU-PP-CSS variant B the downlink and the uplink channels are not symmetrical, since during one symbol time $1 + \log_2 M$ bit information
can be transmitted to the mobile terminals, while they can send $M$ data bits to the base station. Contrarily, the throughput of both the downlink and uplink channels are $M$ regarding variant A.

To summarize, in the present chapter, a novel transmission method, the pulse position based chirp spread spectrum was introduced. For downlink communication, two multi-user variants were presented based on this technique, while for uplink, beside the PP-CSS method, the measurement based feedback power control was proposed for use to handle the amplitude control of overlapping quasi-orthogonal chirp signals at the base station. In addition, analytical investigations were provided for the instantaneous and average symbol error rates, as well as for average bit-error rates assuming Rayleigh fading channel. The results show that efficient communication can be established between the base station and multiple mobile terminals using the proposed techniques.

The next subsection arranges the presented theorems and defines the theses.

### 3.3.5 Theses related to the multi-user chirp spread spectrum technique

**THESIS II.3. [C3]** I described a novel mobile communication system that can use two slightly different multi-user technique for downlink that combine pulse position modulation with chirp spread spectrum based mobile transmission scheme. Furthermore, they apply the CLPC method according to Thesis I.3. in uplink.

Lists of definitions and theorems: Definition 3.1, Definition 3.2, Definition 3.3, Definition 3.4, Theorem 3.5, Theorem 3.6

**THESIS II.4. [C3]** I provided analytical calculation method to determine the bit error rate of the multi-user schema described in Thesis II.3. assuming Rayleigh fading channel.

CHAPTER 3. ANALYSIS OF SPREAD SPECTRUM METHODS
Data and voice traffic increases continuously day by day, more and more percentage of that is handled by mobile equipment. The forecasts show significant growth of mobile subscribers thanks to connecting new markets and spread of smart applications using Machine-to-Machine (M2M) communication. To tackle this issue, new trends are emerging in next generation mobile networks. More precisely, bringing the base stations closer to users, providing better coverage and better QoS are quite new aspects in cellular networks. The so-called small cells or femtocells are low transmit power access points with relatively small coverage (∼30 meters) operating in licensed spectrum. A realization of the small cell concept is included in the 3GPP Long Term Evolution - Advanced (LTE-A) standard, namely in form of the Home evolved-NodeBs (HeNB) [52]. The HeNBs have the opportunity to provide sufficient signal strength to proximity users and better coverage inside the buildings along with cheap and easy installation. The femtocell user generated traffic is forwarded to the mobile operator via wired techniques (xDSL, cable etc.). The small cells have three different access modes to be in line with market demands. In open access mode, every potential user equipment (UE) can connect to femtocell, while in closed access mode just a group of users is allowed to join, i.e. the UE’s ID have to be on the femtocell’s so-called closed subscriber group (CSG) list. The UE also stores the CSG IDs of the allowed small cells. Finally, the hybrid access mode is the combination of the previous two sharing the radio resource between the CSG and non-CSG users. In this case the CSG users have the advantage in the service. The main drawback of using the small cells is the user installation, which means uncontrolled interference source and independent operation from the service providers. From another aspect, the operator can also install small cells to either indoor (e.g. offices) and outdoor (e.g. utility or lamp poles) locations. The benefit of the latter solution beyond from the aforementioned is the manageability by the service provider. The implementation of the small cell is supported by the LTE-A introducing a new entity (HeNB
In two-tier macrocell-small cell LTE-A, one key feature is the mobility management, i.e. the seamless switch from one base station to another, which is in the focus of this chapter. Many papers were published in the recent years proposing more and more sophisticated solutions of the handover (HO) decision procedure [53]–[59]. Numerous from these use different network provided information during the decision that are typically not included in the standard, e.g. Khalid et al. propose a mobility-based handover decision algorithm [53], which takes into account the speed value of user equipment and does not allow a high speed user to switch to a femtocell. In [57] the authors describe a policy-based decision procedure, where the decision is made on different policies, e.g. RSSI (Received Signal Strength Indicator), load information of cells, access type of HeNBs, etc. by the additional so-called Policy Function entity in Evolved Packet Core (EPC).

In this chapter a new handover decision algorithm for the two-tier macrocell-small cell LTE-A network is described. The algorithm uses positioning service and takes into account different network provided information, namely the reported RSSI values, the speed of the users and the actual load of the base stations. In addition, a sliding averaging window on the RSSI values is applied to enable the filtering of the instantaneous fluctuation of the radio channel and avoid unnecessary cell changing.

The chapter is organized as follows. In Section 4.1 the system architecture of two-tier LTE-A is described. The legacy and some related handover decision procedures along with the proposed novel algorithm are provided in Section 4.2. Section 4.3 presents the simulation environment and results of the performance evaluation. Finally, conclusions are drawn.

### 4.1 System architecture of the two-tier LTE/LTE-A

In this section the main small cell concept is introduced, moreover, the evolution of the LTE-A architecture completed with HeNBs are described. The two-tier architecture means that the network has two different layers, which are the followings: the layer of the eNodeBs (eNB) as the first, macrocell tier and the layer of the HeNBs as the second, femto or small cell tier. The introduction of small cells affects the operation and the architecture of LTE/LTE-A. In the earlier releases of the standard [52], the E-UTRAN has been completed with an additional entity named as HeNB Gateway (HeNB GW). The HeNB GW is optional, but if present, acts as an eNB to the Mobility Management Entity (MME) and appears as an MME to a HeNB. It serves as a concentrator for the control plane, thus it has the ability to support a large number of HeNBs in a scalable manner. The X2 interface provides the direct connection between the cells independently whether they are eNBs or HeNBs. The X2-based HO between small cells is not allowed only if access control at the MME is necessary, i.e. if
the UE handovers from a closed/hybrid access HeNB to a closed access HeNB with different CSG ID. The logical integration of the HeNB GW in the E-UTRAN is shown in Figure 4.1.

4.2 Handover decision algorithms

In this section the legacy HO decision mechanism is described, furthermore, some related works are introduced featuring their advantages and disadvantages. Finally, the novel HO decision algorithm is introduced.

The legacy handoff procedure follows the strongest cell (SC) decision policy. It begins with regular measurement reports from the UEs. Such report contains RSSI values and used by the serving (H)eNB to decide whether there is any other base stations in the UE’s proximity. If the reported RSSI is higher than the serving (H)eNB’s, the HO process is initiated to that cell. The main drawback of this solution is the decision on a single value, which can represent the instantaneous state of the channel. The SC decision worked in prior cellular mobile networks well (e.g. GSM, UMTS), however, in small cell environment it is not the best choice.
4.2.1 Mobility-based decision algorithms

During the mobility-based decision, usually a threshold of the mobile devices’ speed is taken into account [53]–[56]. If the UE moves with higher speed than this threshold, the HO is forbidden to a small cell. One can assume that a new handoff will be needed because of the small coverage area of the HeNBs, thus this solution has the benefit beyond the simplicity to avoid the too frequent, unnecessary handovers. However, relatively accurate positioning information is required at the base stations, which is typically not available in most active networks.

4.2.2 Policy-based decision algorithms

The policy-based decision can take into consideration several different parameters [57]–[59]. Bai et al. proposed an access mode, load level and RSSI based decision procedure [57]. A policy for decision can be made, e.g. on the ongoing traffic type (e.g. in [55]) or on a calculated cost-function (e.g. in [59]), too. These kinds of decision algorithms are more sophisticated and promise better solution than the legacy HO decision from different aspects, e.g. QoS, interference, energy consumption, etc.

4.2.3 Proposed decision algorithm

The main drawbacks of the aforementioned HO decision procedures are the oversimplified analytical and/or simulation model and the lack of investigation about their effect on QoS. Most works focus on the ratio of the appropriate handoffs, the signalling overhead, etc. During the recent years the 3GPP LTE-A standard has changed and the X2-based HO is allowed between eNBs and HeNBs. This means a slight signalling overhead reduction, i.e. it accelerates the HO procedure [60]. Therefore, firstly, the proposed algorithm focuses on guaranteeing the appropriate QoS and secondly, avoiding the unnecessary handovers.

It is assumed that the network can determine the geographic position and velocity of user equipment based on measuring radio signals. The standard gives the opportunity to do that providing different techniques [61] and protocol [62].

**Definition 4.1.** I developed a novel handover decision algorithm for two-tier LTE/LTE-A system that takes into consideration the velocity of the UE, the access mode of the small cells and received signal strength indicator extended by hysteresis margin. It is able to increase the overall system throughput and reduce the user latency compared to the legacy decision procedure.

The pseudo code of the proposed algorithm is described in Algorithm 1. The basic idea of the method is making a list of the possible target cells based on the reported RSSI values
4.2. HANDOVER DECISION ALGORITHMS

Algorithm 1 The hybrid handover decision algorithm

Require: RSSI values with (H)eNB IDs from the measurement report of the UE
1: Initialize the $HHM$, the speed threshold $v_{th}$ and the possible target (H)eNB list $L$ for the UE
2: Calculate/collection the speed of the UE $v$, the current serving (H)eNB RSSI $RSSI_{current}$ and the eNB ID with the highest RSSI $max eNB$
3: for all $i$ in $RSSI$ do
4: Update the sliding averaging window of the $i^{th}$ (H)eNB with $RSSI_i$
5: Calculate the new average $avg_i$
6: if $avg_i \geq RSSI_{current} + HHM$ then
7: Add $i$ to the end of $L$
8: end if
9: end for
10: if $v \geq v_{th}$ then
11: Initialize HO to $max eNB$
12: else
13: Sort the list by access mode, actual load and RSSI
14: repeat
15: Get the first (H)eNB $l_1$ from $L$
16: Initialize HO to $l_1$ from $L$
17: Erase $l_1$ from $L$
18: until HO is not successful or the list is not empty
19: end if

using the sliding averaging window and the Handover Hysteresis Margin (HHM) (see line 3-9). Then, this list is sorted by taking into account the access modes (in order of closed, hybrid, open access modes), the actual load (the lowest the best) and the RSSI of the target (H)eNBs (the highest the best) (line 13). Finally, the first base station from the list is selected as the target cell (line 15-16). If the HO is prevented by radio failure or the MME (e.g. the UE does not have an access to the given HeNB), the UE is directed to the next cell on the list (line 17-18). The speed values are used to decide whether the UE is allowed to handover to the target cell on the list or it has to switch to the macrocell with the highest RSSI value (line 10-11). This function ensures that a faster user cannot get stuck at a cell. Note that an overall eNB coverage is assumed here, but this is exactly the case in reality.

Higher QoS is expected, i.e. higher system throughput and lower user latency due to the load balancing effect caused by the higher priority of the actual load over the RSSI during the list sorting. One can assume a scenario, where two open access HeNBs are close
together. In case of the legacy algorithm, the closer cell will be chosen because of the Strongest Cell decision policy, hence, the balancing is not guaranteed. However, with the proposed algorithm the UE will switch to the small cell with less load, e.g. with less served mobile and will get more radio resource, therefore the QoS of the UE will be higher.

Note that a trade-off should be found between selecting the strongest cell, thus communicating on less resource with better transport format (i.e. with better modulation and coding) and selecting the cell with less load, i.e. having more resource but worse radio channel quality, hence worse transport format. Taking into account this consideration, the proposed algorithm will work better in those scenarios, where the small cells are placed randomly and the aforementioned case (two HeNBs are close together) can occur. Assuming the user installation of HeNBs, their locations can modelled as a random process.

4.3 Performance evaluation

In this section the simulation environment and results are presented.

4.3.1 Simulation environment

Extensive simulations were executed to analyse the performance of the proposed HO decision algorithm. The network layout contains 19 macrocells in classical hexagon form and \(\lambda = 72\) (\(\lambda\) is the parameter, i.e. the expected value of the Poisson distribution) small cells placed on a 500 \(\times\) 500 meters plain by spatial Poisson point process (SPPP). The SPPP provides more realistic topologies than the 3GPP recommended network scenario, where the HeNBs are distributed uniformly on a similar simulation plain [63]. The HeNBs are in open access mode and outdoor application of them is assumed. The path loss of the radio channel follows the Stanford University Interim (SUI) model with 10 dB shadowing. The movement of the UEs follows random waypoint mobility model that randomly chooses the destination of users on the simulation plain from the given velocity interval. If the destination is reached by a UE, then the model sets a new destination with new random velocity. Dense traffic is generated from different traffic types, i.e. web, VoIP, network games, VBR video, peer-to-peer traffic to observe the performance of a heavy loaded network. The length of the averaging sliding window is set to 3000 ms to eliminate the ping-pong effect. The simulated time is chosen to 30 minutes that is enough to generate evaluable output data, while the run time is still manageable (that is several hours depending on mostly the number of users). By using round robin, I wanted to eliminate the effects of the scheduling algorithm as much as possible. The value of the speed threshold has a major influence on the utilization of small cells. If this value is set too low, then lots of UEs are redirected to the macrocells during the
4.3. PERFORMANCE EVALUATION

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
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<td>Simulated time:</td>
<td>1800 sec</td>
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<tr>
<td>System nominal bandwidth:</td>
<td>20 MHz</td>
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<tr>
<td>Transmission power of eNBs:</td>
<td>20 W</td>
</tr>
<tr>
<td>Transmission power of HeNBs:</td>
<td>20 mW</td>
</tr>
<tr>
<td>Scheduling algorithm:</td>
<td>round robin</td>
</tr>
<tr>
<td>MIMO:</td>
<td>$2 \times 2$</td>
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<tr>
<td>Active users in the system:</td>
<td>100–250 UE</td>
</tr>
<tr>
<td>Speed of users ($v_{\min} - v_{\max}$):</td>
<td>1–20 m/s</td>
</tr>
<tr>
<td>Speed threshold:</td>
<td>5–20 m/s</td>
</tr>
<tr>
<td>Handover Hysteresis Margin:</td>
<td>3 dB</td>
</tr>
</tbody>
</table>

Table 4.1: Main simulation parameters

handover. The HHM basically tells how close a UE has to go to another BS, before considering it as a candidate cell. Other parameters are chosen by taking into account the proposed values from [63]. Table 4.1 shows the main simulation parameters.

For the simulation, a C++ based LTE/LTE-A simulation software is responsible that was developed during the years by several colleagues at the department including me. In the last period I took over the development tasks and implemented the proposed handover decision algorithm as well. The software is a complex, high-level, event-driven LTE/LTE-A network simulator that is composed of several modules like the user mobility module, traffic generator module, and the LTE/LTE-A core module. The latest one implements the core features of the physical (without the signal processing parts) and the second layer of LTE/LTE-A including the radio resource management, handover and data transmission functionalities. More details are available in [64]-[66].

4.3.2 Simulation results

The simulation results focus on two main QoS parameters: the overall system throughput and the average user latency. In addition, the number of handovers is investigated, too. Figure 4.2 depicts the percentage ratio of the overall system throughput between the legacy and the proposed algorithm against the number of active user equipment and grouped by the speed thresholds. It is visible that the overall system throughput gain is continuously decreases with the growing number of UEs. In case of 250 UE and without speed threshold the achieved overall system throughput by our algorithm is hardly more than the 90% of the legacy HO decision procedure. Nevertheless, the best performance is reached choosing the speed threshold to 5 m/s.
Figure 4.2: The percentage ratio of the overall system throughput between the legacy and the proposed algorithm vs the number of UEs

Figure 4.3 shows the percentage ratio of the average user latency between the legacy and the proposed algorithm in function of the number of active UEs and grouped by the speed thresholds. First of all, it is easy to see that the proposed algorithm reduces the average user latency in each scenario. Furthermore, the gain slightly increases with number of UEs. The highest benefit is achieved in $v_{th} = 5 \text{ m/s}$ case, when the average user latency with almost 16% less than using the legacy, SC decision policy. In addition, even without speed threshold, the proposed algorithm can decrease the latency.

Figure 4.4 illustrates the gain of handovers’ number against the number of UEs. A small reduction is visible with the growing number of UEs, however, the significant differences are between the speed thresholds. Choosing a high $v_{th}$ or without taking into account the speed values of the users, the number of handoffs can be more with the proposed algorithm than with the legacy procedure. However, with $v_{th} = 5 \text{ m/s}$ the number of HOs are less in every scenario about 4 – 6%.

One can see that the appropriate choice of the threshold parameter is very important. It has a significant effect on the performance of the network. The balancing effect through taking into account the actual load of the (H)eNBs and allowing the connection to small cells just for the slow speed users ensures the higher QoS. In addition, the proposed algorithm reduces the number of handovers in low speed threshold cases (5 – 10 m/s), therefore the signalling overhead also decreases.
4.3. PERFORMANCE EVALUATION

<table>
<thead>
<tr>
<th>Number of UEs</th>
<th>Legacy</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>84%</td>
<td>90%</td>
</tr>
<tr>
<td>150</td>
<td>86%</td>
<td>92%</td>
</tr>
<tr>
<td>200</td>
<td>88%</td>
<td>94%</td>
</tr>
<tr>
<td>250</td>
<td>90%</td>
<td>96%</td>
</tr>
</tbody>
</table>

Figure 4.3: The percentage ratio of the average user latency between the legacy and the proposed algorithm vs the number of UEs

<table>
<thead>
<tr>
<th>Number of UEs</th>
<th>Legacy</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>150</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>200</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>250</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Figure 4.4: The percentage ratio of the number of HOs between the legacy and the proposed algorithm vs the number of UEs
To summarize, a new handover decision algorithm was proposed using reported RSSI values, speed of the users and the actual load of base stations extended with a sliding averaging on RSSI. The algorithm’s performance is investigated from the QoS point of view by simulations. The results show higher throughput and lower delay in most cases with the appropriate choice of the speed threshold and enabling load balancing between the base stations by the new decision procedure.

There are several ways to improve the efficiency of the proposed algorithm: other decision criteria can be taken into account during the creation of the priority list and some parameters (e.g. length of the averaging window, HHM) should be automatically adjusted to the given scenario.

The next subsection arranges the presented theorem and algorithm into a thesis.

4.4 Theses related to the analysis of handover process in two-tier LTE/LTE-A

**THESIS III.1.** [J3], [C4], [C5] I defined a new hybrid handover decision algorithm for two-tier LTE/LTE-A network, and investigated its efficiency by simulations.

List of related definition and algorithm: Definition 4.1, Algorithm 1
The present dissertation describes all my research and scientific contribution from the recent years. I structured this thesis into three main chapters reflecting my three main research areas. First, the channel equalization related investigations are introduced. I presented a novel method for channel equalization, namely the measurement based feedback channel equalization that is considered as a closed loop power control mechanism. It compensates the amplitude fluctuation of the transmitted signal that is caused by the fading in the radio channel. The method assumes that the equalization is executed by the transmitter based on the measured and feedback information from the receiver. I introduced new formulas regarding to the bit error rate of the system taking into account different fading models, such as Rayleigh, Rician and Nakagami for both coherent and noncoherent receptions. The expressions might look difficult, but they are able to simplify the BER calculations, as well as the formulas are derived to closed form in some cases. It also has to be highlighted that in each case the maximum output power of the transmitter is considered as a limitation factor, since infinite compensation of the channel distortions is not possible in practice. Later, it is shown that this model and the related expressions are applicable for practical communication systems, i.e. the Pulse Position based Chirp Spread Spectrum scheme is capable of utilizing the presented CLPC mechanism.

In addition, investigations were carried out related to pilot signal assisted systems, in which the estimation error of the reference signals influences the symbol error rate of the whole system. Formulas are determined assuming BPSK and QPSK modulations that suffer from phase measurement error due to fading. Furthermore, the analysis is turned into a channel capacity optimization problem, where the trade-off has to be found between the number of pilot and data signals. Considering LTE/LTE-A network, the optimal number of reference symbols are determined taking into account the signal-to-noise ratio of the communication channel. The results could be used during the standardization of the LTE/LTE-A, however,
the 3GPP 5G New Radio can also profit from them, since it supports the flexible allocation of reference signals.

Then, the third chapter describes the spread spectrum techniques related investigations. First, a new synchronization method is presented that is based on the sliding-and-tracking correlators. The basic idea is that if two correlators are used in overlapping time window, then significant part of the noise will appear twice in the output of the correlators, and therefore it can be handled as non-independent random variables during the correlation calculations. The analysis covers both coherent and noncoherent receptions, and provides expressions related to the wrong decision probabilities.

The second section of the third chapter is about a novel communication scheme that uses pulse position modulation over chirp signals to enable simultaneous transmission for multiple mobile terminals. Two, slightly different variants are presented as the downlink communication, while the interesting part of the uplink is the earlier proposed CLPC mechanism that effectively manages the near-far problem. Formulas are derived regarding to the feasibility (quasi-orthogonality feature of the scheme), as well as the downlink and uplink performance.

Finally, the forth chapter introduces a novel handover decision algorithm for two-tier LTE/LTE-A system that takes into account the velocity of the user equipment, the load and the RSSI of the candidate cells, as well as their access mode. Therefore, the algorithm results a more balanced distribution of UEs between the base stations, while it provides higher system throughput and lower latency especially in such scenarios, where the small cells are deployed randomly.

5.1 Further Works

There are several ways to improve or extend the presented investigations. Regarding to the measurement based feedback channel equalization, expressions can be derived for more fading models like Weibull fading, as well as additional systems could be found that can apply the proposed model. Considering the analysis of the phase error, expressions can be determined for 8-PSK modulation. Furthermore, it should be investigated if the generalization of formulas is possible, thus one formula could cover all PSK based modulations. A further extension of the work is about the detailed investigation of 5G New Radio system, in which the reference signals are differentiated based on their purpose, and they are usable in a more scalable, flexible way. However, the standard itself does not determine any exact relationship between the number and type of signals and the channel conditions.

The new synchronization technique provided by the sliding-and-tracking correlators should be built and investigated in practice. So, the analytical results can be compared to
5.1. FURTHER WORKS

real measurements, and the accuracy of the expression can be analysed. Similarly, the pulse position based chirp spread spectrum could be implemented using software radio platforms, and then the performance related properties can be measured and investigated. As mentioned, CSS based communication is quite robust and reliable for long distances as well. Due to the proposed novel multi-user schema, a bit less transmission range is foreseen that could be properly determined with practical measurements. Since the chirp signal is insensitive to Doppler-effect, the mobile terminals can move with high speeds. The equalization of the uplink chirp signals are taken care by the proposed CLPC mechanism. As exploitation, patenting the PP-CSS should be considered.

Finally, a lot of knowledge were acquired regarding to the handover procedure in 4G LTE/LTE-A network. By introducing the concept of small cells in the 3GPP standards, a new situation was created in the network, which was investigated by many researchers including me. However, the home eNodeBs are not really spread, probably due to its drawbacks, and of course the mobile network operators are not really like to lose their influence over their network and especially over their licensed spectra. Nevertheless, the 5G New Radio has similar handover procedure, but the whole network concept is matured. This means that multi-tier and multi-RAT operations are natural, and including new types of end users (machines, vehicles) enables further investigations based on this knowledge.
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List of Publications

Published peer-reviewed journals


Conference articles


Other publications (not related to theorems)

[C6] Zoltán Jakó and Ádám Knapp, “Business Scenarios and Data Flow in NeMo Hyper-Network”, 2018 International Conference on Smart Systems and Technologies (SST), Osijek, 2018, pp. 139-144. DOI: 10.1109/SST.2018.8564701


