Random Decimation in Anti-Aliasing

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Purpose:

To present a "one-button" experiment for exploring a technique, used in all of the AGILENT 546xx series oscilloscopes, which is remarkably effective against aliasing.

Description:

Aliasing is a potential problem in the DSO (Digitizing Storage Oscilloscope): the undersampled, high frequency component assumes the alias (or false identity) of a spurious, low frequency component. Luckily, there are techniques to anti-aliasing; one of them is the random decimation. The trick is simple and effective: regular aliasing is replaced by random noise. This experiment demonstrates the phenomena in a very simple way.

Equipment:

- Agilent 546xx-series Oscilloscope with FFT option
- Agilent 33120A Function/ARB Generator

Background and simulation:

Nyquist zones, aliasing

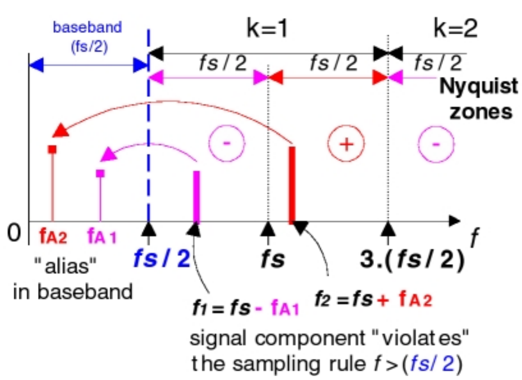
If a sine wave component with frequency f is uniformly sampled with rate f_s , where $f > f_s/2$, i.e.

$$f = \mathbf{k} \cdot f_s \pm \mathbf{f_A} \quad \underline{\text{and}} \quad \mathbf{f_A} < f_s/2, \quad \mathbf{k} = 1,2,...$$

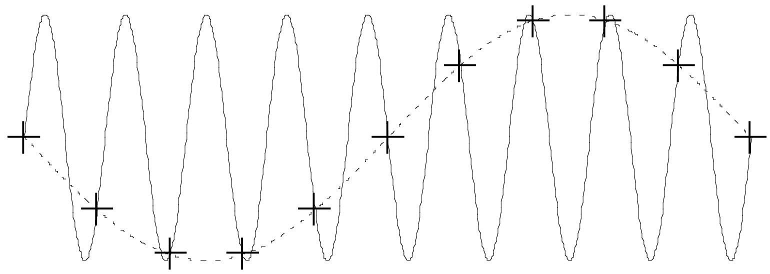
then an aliased sine wave of frequency $\mathbf{f_A}$ will occur:

$$\sin\left[2 \cdot \pi \cdot (\mathbf{k} \cdot f_s \pm \mathbf{f_A}) \cdot i \cdot \left(\frac{1}{f_s}\right)\right] = \sin\left(\pm 2 \cdot \pi \cdot \frac{\mathbf{f_A}}{f_s} \cdot i\right)$$

the numeric frequency of the sampled values is independent of **k**. The figure illustrates the *frequency domain* approach to aliasing



Note: the sin(-x) = -sin(x) *phase change* doesn't appear in the amplitude spectrum. The mechanism becomes obvious when you look at the undersampling process in the *time domain*:



Here, sampling $f_s = 10$ [$\rightarrow f_s/2 = 5$] with an input $f = 9$ (=10-1) produces an alias $\mathbf{f_A=1}$ and we get a phase-change.

The sweep speed we use determines the sample rate we get.
The *actual* sample rate of DSO is determined by the time base setting "s/DIV" (there are 10 major DIVisions on the horizontal time axis) and the acquisition memory depth R (the record length):

$$f_s = \frac{R}{10 \cdot [s / DIV]}$$

If $f_s < f_{smax}$, the maximum *real-time* sample rate, we need to prune (*decimate*) the sampled data down to the memory size. [**Note:** if $f_s > f_{smax}$ we need to *interpolate*, or
– in the case of *repetitive* signal – use *ETS* = Equivalent Time Sampling.]
As we reduce the horizontal sweep speed to view a more extended time segment of the signal, decimating will occur and the scope becomes susceptible to aliasing.

Random decimation (see *References*)
Simple decimation (= *regular* sampling) is the most common decimation technique in DSO's and it is exceptionally prone to aliasing. However, the discarded samples can be used to prevent the *display* of aliased waveform. The random sample selection prevents an alias (or *beat*) frequency from developing by converting low frequency "spurs" to noise. (For masochists only – A. V. Balakrishnan: *On the Problem of Time Jitter in Sampling*, IRE Trans. on Info. Theory, April 1962.)
With random decimation (= *stochastic* sampling) the resulting display is a fuzzy band much like what would be seen on an analog scope. However, this technique should be turned off for analysis that requires samples at exact regular time intervals. One of those cases is the FFT. (***Wow…***we can use just this simple case to demonstrate the effect of the anti-aliasing techniques.)

What about our eyes?
They have a finite number of photo receptors but we do not see aliasing effects. Why?The non-uniform distribution of the receptors is a way for the visual system to cope with aliasing. [See R.L. Cook: *Stochastic Sampling and Distributed Ray Tracing*, **in** "Glassner (Ed.): An Introduction to Ray Tracing", Academic Press, 1989. Note: also, here you find a simple intuitive explanation of Balakrishnan`s results.]

References:

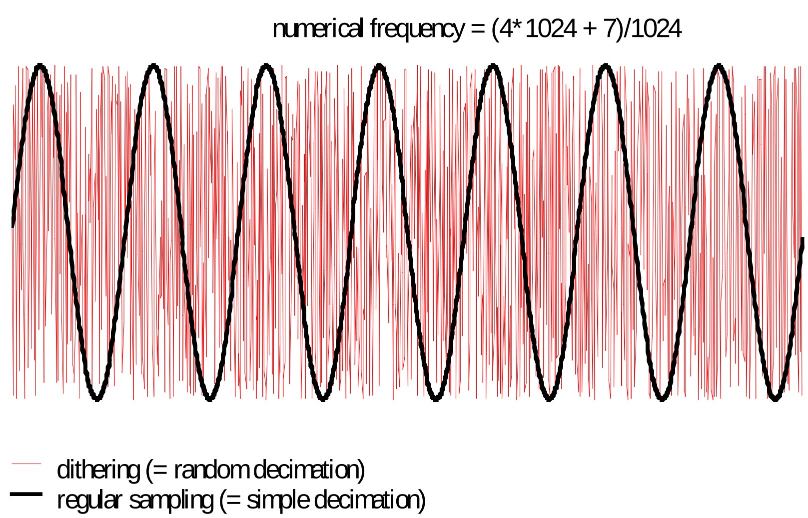
How to prevent your scope from aliasing
in "Scope measurements hints"
http://www.educatorscorner.com/tools/lectures/appnotes/scopehints/index.shtml

The decimation problem
in “Holcomb *at all*: Design of a Mixed-Signal Oscilloscope”
HP Journal, April 1997

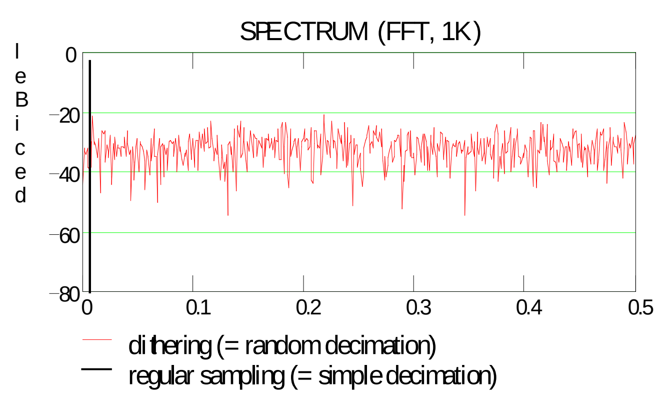
Intra-acquisition dithering (= random decimation)
in “D.E.Toeppen: Acquisition Clock Dithering in Digital Oscilloscope”
HP Journal, April 1997
US-patent 5115189, “Anti-aliasing dithering method and apparatus for low-frequency signal sampling”, HP Co., 1992

Simulation:

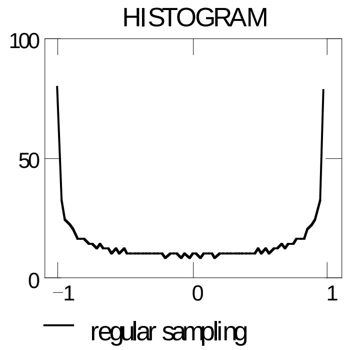
With yours favorite math software, you can easily visualize that aliasing (a highly objectionable artifact) can be replaced with noise (an artifact that our visual system tolerates very well). Letting **k** = 4, $\mathbf{f_A}$ = 7, the record length 1K and for stochastic sampling, *dither* the regular sample time points by a uniform distribution.
With stochastic sampling, the display in the *time domain* is a fuzzy band, without any signs of an aliased waveform.

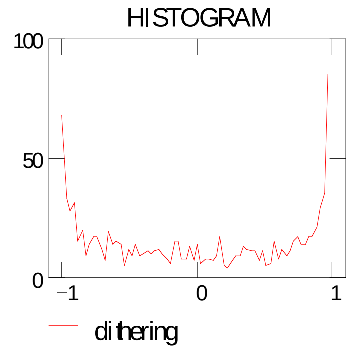


Because of the interference (beating effect) between the sampling and the signal frequency, the aliasing results in a single line (located at 7/1024) in the *frequency domain*. On the other hand, the spectrum due to stochastic sampling is considered as noise.



It is interesting to note, however, that the symptom of the sinusoid is retained in the *statistical domain*:





"One touch" experiment:

Preset your instruments

Agilent 33120A settings:

Set the function generator to generate a **1.0007 MHz sine** wave with 0.1Vp-p amplitude and connect the output to CH1 of the scope.

Agilent 546xx settings:

(1) Touch AUTOSCALE on the scope [you will see 200ns/ time base] and measure the frequency (Measure: TIME hardkey, Time measurement softkey: FREQ.

(2) Set the time base (TIME/DIV) to **1ms/** .
[You will see a fuzzy band and the display will read: "Freq(1) not found".]

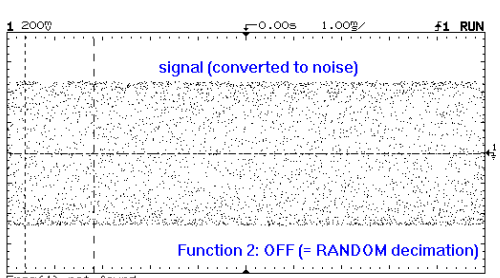
(3) Configure FFT:

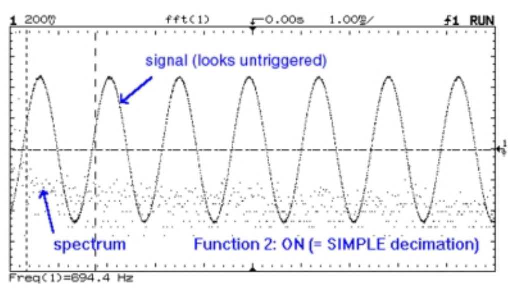
- ± key
- Function 2 (**Off**): MENU
- Operation: FFT
- Operand: 1
- PREVIOUS MENU

(4) Set Function 2: **ON**
[You will see an untriggered sine wave and "Freq(1) ≈ 700 Hz" information. *Fine tune* the frequency of function generator, and you will see – according to the *beating* effect – an untriggered sine consisting of 7 cycles. Note: with my equipment it was 1.0006852 MHz.]

Put your finger on it!

Toggle OFF/ON the Function 2 softkey (**OFF** = random decimation, and **ON** = simple decimation)





Exercises (What do you predict? Explain the results.)

1. *Touch* STOP on scope (and scope Display: Vectors On)
2. Run mode, Function 2: ON and *touch* **SQUARE** WAVE on generator

(**Hint:** this is good news. In *controlled* aliasing, the signal preserves its spectral order as the sampling process relocates – aliases – the component. This concept is referred as *subsampling* and gives similar result to down converting the signal to baseband ... but, this is another story.)

3. Run mode, Function 2: ON and *touch* AUTOSTORE on scope

Play “what if” games with other waveforms and settings

Conclusions:

- As you improve the resolution of FFT by slowing down the horizontal sweep speed, spectral lines may appear in places where no frequency components exist.
- The statistically rigorous requirements of the FFT preclude the use of dithering as an anti-aliasing technique.