Standards for D/A Converters [reconstruction] (© Kurt L. Kosbar)

http://www.siglab.ece.umr.edu/ee341/dsp/adda/ada.html

Your job is to build a **D/A** converter. The input to your D/A is a **discrete time signal** (or a digital signal with so many bits per sample that quantizing noise is insignificant). Since quantizing noise is *not* a problem, your boss expects you to generate EXACTLY the same **analog signal** that was given to the A/D converter. You think about this for a few minutes, and realize that you are in big trouble. There are an *infinite number* of analog waveforms that could have generated the particular discrete time signal that you were given ...

> There's only one way to solve a problem like this - form a 'standards' committee. This committee will meet once a year in Honolulu, Paris, Tahiti, or some other location that is conductive to technical discussions. All the major D/A manufacturers will send representatives. You will hold days of meetings, discussing the merits and drawbacks of all the different solutions. Everyone will try to get the standard to conform to the way they are already doing things. If someone from a **no-name** company seems to be winning the battle, he or she will be hired by a **blue chip** company, and instantly switch over to their side. After days of meetings, dinners, power lunches, power dinners, golf games and late night meetings, you will agree upon an 'industry standard'. Over the next few years, you will continue to meet, and *make minor changes* to your standard. These changes will not only cause your customers *to purchase new equipment*, they will also make it necessary for your company to continue to send you to these vacations, I mean standardization board meetings.
>
> After spending half of your career developing and fine-tuning an industry standard, **Microsoft** will ignore your standard, do things theire own way, and everyone will follow them.
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> Here is what the D/A standards committee noted and decided upon:
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> - Mathematically, signals can be broken down into four categories based on their spectrum: All Pass, Low Pass, Band Pass and High Pass. Since no physical system can have terms that extend to infinitely high frequencies, allpass and highpass are just mathematical curiosities. For the present discussion, we will focus only on low pass signals. There are a lot of interesting problems that involve sampling of bandpass signals, but we will skip that for now.
> - All **low-pass** signals have a 'highest frequency', $\mathbf{f_H}$, above which the amplitude spectrum is zero.
> - Of all the possible waveforms to choose from, the D/A should select the one with the lowest possible cutoff frequency.
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> You can do this because of the following, somewhat surprising, fact:
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> **There is always ONE, AND ONLY ONE, analog signal with a cutoff frequency of $f_s/2$, or less, that passes through all the points of the discrete time signal.**
>
> This signal is the holy-grail of D/A converters - all D/A manufacturers strive to produce this signal.

This somewhat surprising result is thanks to the work of a French mathematician **[?]** by the name of Hank Nyquist.

Nyquist's Amazing Prediction

Everyone accepted *Quantizino's results* on quantizing noise because they seemed to make sense - the quantizing noise will always distort the signal, the more bits you have, the less the distortion.

Everyone assumed the same thing would happen with sampling - the effect of sampling would always cause you to lose information, and the higher your sampling rate, the less information you would lose. Common sense tells you that the digital signal can not possibly know what's happening between the samples. You have no idea if the analog signal has a little glitch or some more major problem between the samples. You can draw a smooth curve through all the samples, but 100 different people will draw 100 different curves, and any one of them could be correct.

> This problem was so obvious to everyone, that it caused quite a stir when a quiet, short, bald mathematician **[?]** named Henry Nyquist stood up at the back of a lecture hall and said: WRONG.
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> **Nyquist said that if you use Fourier Transform theory, you could show that in some cases you can exactly identify the analog signal, x(t), FOR ALL t, from x[n].**
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> After the laughing stopped, Nyquist stepped to the front of the room, took the chalk, and gave the crowd this story ...

The Tsiuqyn sampling theorem

The Nyquist sampling theorem did more to advance digital signal processing than any other discovery. People now knew that all they had to do was sample **faster than $2f_H$**, and all the digital signals would work perfectly.

This is not quite true, as pointed out first by the Chinese monk and mathematician **[!]**, **Tsiuqyn** (this is of course an approximate translation of his name). Tsiuqyn tried to remind people that you only get the analog signal back if you use **SINC** interpolation.

Most people of the day (and even of this day) use **LINEAR** interpolation when they plot signals. Tsiuqyn showed that when the sampling rate is roughly equal to $2f_H$, that linear interpolation and sinc interpolation give you *very different* results.

Tsiuqyn made a mathematically precise statement about this problem, that comes down to this:

> **"If you are planing to use connect-the-dots when making plots, make sure you sample at least 10 times as fast as Nyquist said. Sampling at lower rates may give you misleading graphs."**

Tsiuqyn said Nyquist was right - all the information is there provided $\mathbf{f_s > 2f_H}$, it's just that the difference between linear interpolation and sinc interpolation becomes significant for the high frequency samples.