INL Reconstruction of A/D Converters via Parametric Spectral Estimation

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***Abstract*—The work presented in this paper builds on previous research done by the authors in detailing a novel procedure for obtaining a very fast measurement of the integral nonlinearity of an analog-to-digital converter (ADC). The core of the method is the parametric spectral estimation of the ADC output; the static characteristic is subsequently reconstructed as a sum of Chebyshev polynomials, in accordance with a previously developed procedure. The method allows one to test an ADC with sinusoids of any reasonable amplitude (even a slight overdrive is allowed), frequency (no synchronization is needed), and phase (which is digitally compensated). This approach is less accurate than the histogram test but incomparably faster (about 8000 samples are sufficient regardless of the ADC resolution).**

***Index Terms*—Analog-to-digital conversion, Chebyshev functions, discrete Fourier transforms, dither techniques, nonlinearities.**

I. Introduction

The static error of an analog-to-digital converter (ADC) is commonly quantified by the integral nonlinearity (INL), and measured using the well-known statistical approach named *histogram* or *code density test* [1], [2]. This method measures the *average position of the threshold levels*, with a very high accuracy. This is essentially limited by the purity and synchronization accuracy of the sine wave deployed as well as the test time.

Test time is the first significant drawback of the histogram test. Due to its statistical nature, meaningful results are obtained only by acquiring some dozens of samples *per code bin*. As a consequence, the test of a high-resolution converter (16–20 bits) lasts many minutes or even many hours (at low sampling frequencies). The second drawback is that the test requires precise coherent sampling of the input sinusoid. Even if some tolerance is allowed for a given result accuracy [3], a good synchronization between the test signal and the sample clock is needed to avoid gross measurement errors.

The alternative procedure formerly developed by the authors [4]–[6] is based on the fast Fourier transform (FFT) of the ADC output having sinusoidal stimulus. The FFT values are used to reconstruct the INL as a sum of Chebyshev polynomials: we will therefore refer to this technique, as the *Chebyshev test*.

The Chebyshev test presents the following noteworthy features.

1) It is extremely fast, needing a few thousand of samples *regardless of the ADC resolution*.
2) The test accuracy depends on the smoothness of the ADC characteristic (this is the price for its speed).
3) The result can be employed for an easy *digital linearization*, especially in the case of *dithered ADC* [7].
4) Its performance critically depends on and demands *coherent sampling* of the input signal.

The last point is, of course, a drawback that makes the test less attractive. If a fast and simple measurement of the static error is required, it is desirable to eliminate problems of synchronization between the test signal and the sample clock.

In the following, a new version of the Chebyshev test is presented, in which the use of a *parametric* spectral estimation method, instead of the FFT, eliminates any special requirement on the test signal. It is shown in the following sections that the new test delivers, with very immediate and straightforward computations, acceptably accurate measurements of the static error, by simply using *a pure sine wave of any frequency or amplitude* compatible with the ADC under test.

II. Chebyshev Test Theory and Practice

First of all, we briefly recall the simple theory of the Chebyshev test. If we call $g(x)$ the characteristic of a static system (a nonlinear staircase function for an ADC), and we assume an input of the form

$$x(t) = V\cos 2\pi f_x t + C \tag{1}$$

the output of the system is then a signal with Fourier expansion

$$y(t) = g(x(t)) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n \cdot 2\pi f_x t) \tag{2}$$

(as the input of the static system is even, also the output is such and can be represented as a cosine series).

The combination of (1) and (2) gives, with simple algebraic manipulations, the formula

$$g(x) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n C_n\left(\frac{x-C}{V}\right). \tag{3}$$

The terms $C_n(\xi) = \cos(n \cdot \arccos(\xi))$ are the well-known *Chebyshev polynomials of the first kind*. Therefore, knowing

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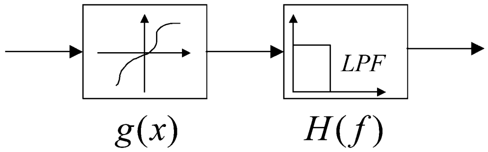

Fig. 1. Cascade of a high-order static characteristic $g(x)$ and an ideal low-pass filter. The result of a *truncated* Chebyshev test on $g(x)$ is the sinusoidal response of this system.

the Fourier expansion of the system output, in principle implies knowledge of the static characteristic $g(x)$ in polynomial form.

If $g(x)$ is a polynomial of finite degree (say, N_h), taking N_h+1 Fourier terms $c_0,\ldots,c_{N_h}$

$$h(x)=\frac{c_0}{2}+\sum_{n=1}^{N_h}c_nC_n\left(\frac{x-C}{V}\right) \tag{4}$$

gives an exact reconstruction of the characteristic, i.e., $h(x)=g(x)$. If $g(x)$ is, instead, not of finite degree (the real-world practical case), taking $h(x)$ as a measurement of $g(x)$ introduce a *truncation error* $e(x)=h(x)-g(x)$ that can be quantified in at least two different ways. It is easily demonstrated, indeed (see the Appendix), that

$$|e(x)|\leq\sum_{n=N_h+1}^{\infty}|c_n|=E_1(N_h) \tag{5}$$

$$\int_{-V+C}^{V+C}\frac{e^2(x)}{\pi\sqrt{V^2-(x-C)^2}}dx=\frac{1}{2}\sum_{n=N_h+1}^{\infty}c_n^2=E_2(N_h). \tag{6}$$

The quantities $E_1(N_h)$ and $E_2(N_h)$ are, respectively, the *maximum absolute value* and the *weighted sum squared value* of the truncation error introduced by an N_h-harmonic reconstruction of $g(x)$. As the weight function in the sum squared error (6) is the density function of the sinusoidal signal, $E_2(N_h)$ represents *the power at the output of a system with static characteristic* $e(x)$ when stimulated by a sinusoidal input (1). Equations (4) and (5) (which, of course, represent *uncertainties* in the measurement of $g(x)$ given by the Chebyshev test) can be evaluated with very good accuracy if one knows the omitted coefficients $c_{N_h+1},c_{N_h+2},\ldots$; otherwise, it is possible to estimate their magnitude using the "last coefficient rules of the thumb" reported in [8], which makes use only of the coefficient C_{N_h}.

A more qualitative and intuitive way of representing the truncation error is provided by Fig. 1. It is clear, indeed, that using (4), with a finite number of Fourier terms, is equivalent to measuring the sinusoidal response of the system as represented in Fig. 1, consisting of $g(x)$ cascaded with an ideal low-pass filter. The low-pass filter $H(f)$ limits the possibility of investigating $g(x)$ at the microscopic level, but does not destroy much information about the *metrological properties* of $g(x)$. If $g(x)$ is an ADC employed in a digital scope, the automatic measurements performed by the instrument usually entail, explicitly or implicitly, a more severe low-pass filtering than that represented by $H(f)$, for example waveform averaging to reduce noise. Therefore the result of the Chebyshev test, which leaves out only very small disturbances at very high frequency introduced by $g(x)$, maintains all the useful information about an ADC *as a measurement system for a dynamic signal*.

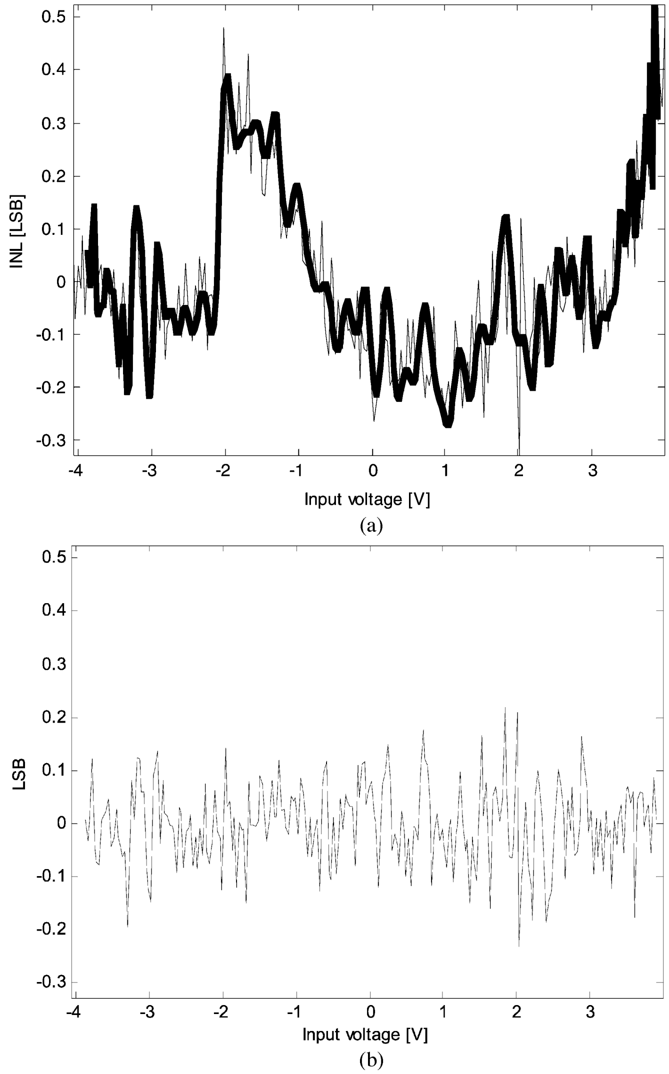

Fig. 2. (a) Comparison between the "true" INL (thin line) and the INL reconstructed by a 100-harmonic Chebyshev test (thick line). The harmonics come from a 2000-point FFT on a coherently sampled sine wave. (b) Plot of the difference between the INL estimates in (a).

The performance and the limit of the Chebyshev test are best illustrated by a practical example. Fig. 2(a) and (b) compares the INL of an 8-bit flash ADC with the reconstruction performed by a Chebyshev test. The "true" INL has been measured by a histogram test with about 1000 samples per code, while the reconstructed INL was obtained using the first 100 harmonics of an FFT of the ADC output. The FFT has been performed by averaging 4 records of only 2000 samples each, acquired with precise coherent sampling (each record was exactly 3 periods). The small difference between the curves is traceable to: 1) the intrinsic difference between INL and the *average static error* of the ADC, and 2) to the distortion components beyond the 100th harmonic, which are of very little relevance in most practical measurements. Effect 2) is made clear if one takes fewer harmonics in the reconstruction [Fig. 3(a) and (b)].

It is clear that considering fewer harmonics yields a smoother (and less accurate) reconstruction of the INL and of the average

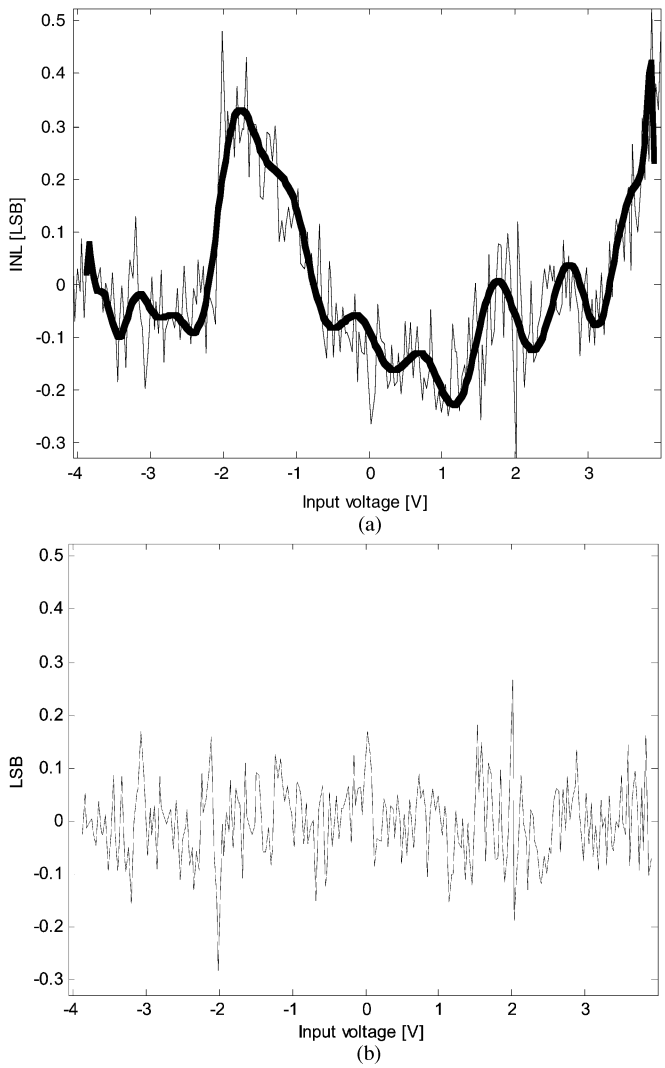

Fig. 3. (a) Same as Fig. 2, but with a 30-harmonic reconstruction. (b) Plot of the difference between the INL estimates in (a).

static error. To also give an idea of effect 1), Fig. 4 shows the "true" average static error, measured by comparing the input with the average of 10 000 output samples (the ADC input noise is $\sigma \cong 0.5$ LSB). It must be noted that, while the INL is useful for the ADC designer, as it gives information about the average thresholds of the circuit, the average static error (which is the quantity actually measured by the Chebyshev test) is the important one for ADC users, as it gives the true distortion introduced by the converter.

III. Parametric Spectral Analysis for the Chebyshev Test

Past experiments on actual ADCs have demonstrated that the Chebyshev test requires the correct estimation of a very large number of harmonics at the converter output (100 in the previous example). Performing the estimate via FFT is very fast and straightforward, but requires coherent sampling, with even more precision than the histogram test.

When the synchronization of the ADC sample clock is difficult to obtain, the FFT is no longer utilizable for the Chebyshev test. It is true that windows and spectral interpolation algorithms can help in determining harmonic amplitudes in noncoherent sampling conditions [9], [10], [11], but these algorithms are usually well-suited only for reconstructing a limited number of spectral terms with meaningful amplitude.

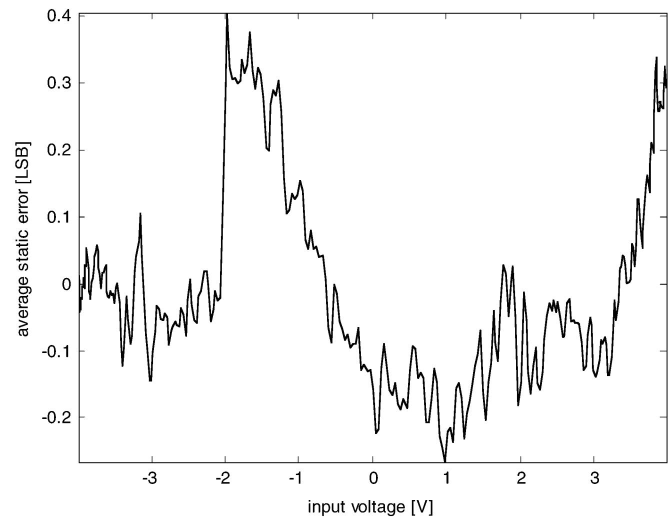

Fig. 4. "True" average static error of the considered ADC. This curve is smoother than the INL and is closer to the result of the Chebyshev test.

In order to perform the Chebyshev test without synchronization constraints it is preferable to use, instead, a parametric spectrum estimation method. We have chosen the *multiharmonic sine estimation* presented in [12], which is basically a generalization of the "semi-fixed" sine fitting methods discussed, for example, in [13] and [14]. All of these methods have already been successfully used in the ADC characterization field.

For the purpose of spectral analysis, the following *model* of the ADC output is assumed:

$$y(n) = \frac{a_0}{2} + \sum_{k=0}^{N_h} (a_k \cos k\Omega n + b_k \sin k\Omega n), \qquad n = 0, \ldots, N-1 \quad (7)$$

where $\Omega = 2\pi f_x / f_s$ is the digital angular frequency (f_s is the sampling frequency), N_h is the model order (the number of harmonics) and N the number of samples. We do not suppose the input sine wave to be sampled with exactly null phase, so (7) includes both cosine and sine terms. The Chebyshev test by multiharmonic sine estimation is arranged as follows.

1) An interpolated FFT (IFFT) is performed on the *first harmonic* of the ADC output $y(t)$ in order to obtain an estimate $\hat{\Omega}$ of the digital frequency Ω [10].
2) The matrix shown in (8) at the of the next page is constructed.
3) The following overdetermined linear system (N equations, $2N_h + 1$ unknowns) is solved:

$$H \cdot \hat{\theta} = y. \quad (9)$$

The obtained vector $\hat{\theta}$ is an estimate of

$$\theta = [a_0/2, a_1, \ldots, a_{N_h}, b_1, \ldots, b_{N_h}]^T. \quad (10)$$

4) The phase of $y(t)$ is estimated on the basis of the first harmonic

$$\hat{\varphi} = \arg(\hat{a}_1 - j\hat{b}_1). \tag{11}$$

5) The Chebyshev coefficients are estimated from the relationship

$$\hat{c}_n = \mathrm{Re}\{(\hat{a}_n - j\hat{b}_n)e^{-jn\hat{\varphi}}\} \tag{12}$$

which implements the correction of the nonzero phase of $y(t)$. In a "perfect" estimate with a perfectly static ADC the imaginary part of the term $(\hat{a}_n - j\hat{b}_n)e^{-jn\hat{\varphi}}$ is exactly zero.

A few remarks are needed about the algorithm. The most important is that it is a considerably simplified version of the one described in [12]: The original method includes, after the computation of $\hat{\theta}$, a suitable number of Gauss–Newton iterative steps to find the minimum mean-squared-error estimate of the digital frequency Ω. This version, instead, relies completely on the accuracy of the IFFT estimate of Ω. The algorithm could be made even simpler because, if the ADC is assumed to be *perfectly* static, the unknown b_k are not independent each other, as all the harmonics must be in phase, i.e.,

$$\begin{aligned} y(n) &= \frac{a_0}{2} + \sum_{k=0}^{N_h}(a_k \cos k\Omega n + b_k \sin k\Omega n) \\ &= \frac{a_0}{2} + \sum_{k=0}^{N_h} d_k \cos k(\Omega n + \varphi). \end{aligned} \tag{13}$$

Therefore, it is possible to reformulate the algorithm for estimating the coefficients d_k, starting from a prior estimation of φ (for example via IFFT). This halves the size of the linear system involved, but loses the possibility of verifying that the harmonics are *actually* in phase, i.e., that the ADC is a static system at the test signal frequency. If this is not true, the amplitudes of the coefficients b_k give information about the deviation of the converter from the static behavior.

A final remark regards the *model order selection*, which is thoroughly discussed in [12]. The experimental results presented in Section IV show that, *for the purpose of INL reconstruction*, a careful selection of the model order is not needed.

IV. Experimental Results

Since the theoretical properties of the described spectral estimation method have been deeply studied, there is no point in examining its performance with simulated signals. It is instead interesting to try it with actual ADCs, comparing the reconstructed INL with the "true" one (obtainable with the histogram test), having at hand also the reconstruction provided by the FFT method.

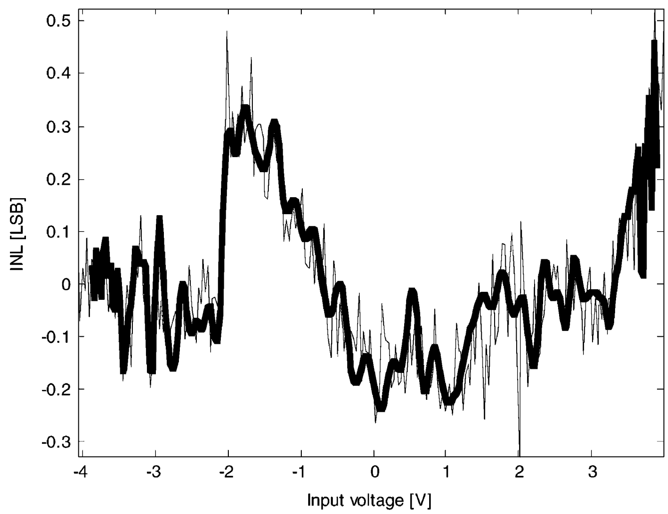

Fig. 5. Comparison between the "true" INL (thin line) and the INL reconstructed by a 100-harmonic Chebyshev test (thick line). The harmonics come from the parametric spectral analysis of a 2000-point *incoherently* sampled sine wave.

One should not take for granted that the parametric method will perform as well as the FFT. First of all, in the past the method has been used to estimate *a small number of harmonics* (<10) of a *considerably distorted signal* [12]. We have seen that a satisfactory INL reconstruction requires, on the contrary, the correct estimation of *dozens of very small harmonics*. Besides, our simplified procedure does not include a special criterion for selecting the model order, which is instead usually a requirement for parametric spectral methods. It is clearly impractical to try and establish the precise order of a signal with hundreds of very small components. Finally, a key point is that our procedure avoids the considerable computational burden of the Gauss-Newton iterative steps, at the expense of a worse frequency estimation.

A first set of experiments have been performed with the same 8-bit flash ADC considered in Section II. The analyzed waveform has been obtained, again, by averaging four records of 2000 samples, but this time we have totally removed the synchronization between the sample clock and the signal generator (each record was actually made of about 3.1 periods). Fig. 5 shows that the INL obtained with a 100-harmonic reconstruction is very similar to that obtained via FFT (Fig. 2), and is a good approximation to the true one. This result demonstrates that, for the sake of INL reconstruction, a careful determination of the fundamental frequency via the Gauss–Newton iterative steps is not needed. The digital frequency estimate described in [10], coupled with the linear system (9) delivers excellent results.

$$H = \begin{bmatrix} 1 & \cos\hat{\Omega} & \cos 2\hat{\Omega} & \cdots & \cos N_h\hat{\Omega} & \sin\hat{\Omega} & \sin 2\hat{\Omega} & \cdots & \sin N_h\hat{\Omega} \\ 1 & \cos 2\hat{\Omega} & \cos 4\hat{\Omega} & \cdots & \cos N_h 2\hat{\Omega} & \sin 2\hat{\Omega} & \sin 4\hat{\Omega} & \cdots & \sin N_h 2\hat{\Omega} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos N\hat{\Omega} & \cos 2N\hat{\Omega} & \cdots & \cos N_h N\hat{\Omega} & \sin N\hat{\Omega} & \sin 2N\hat{\Omega} & \cdots & \sin N_h N\hat{\Omega} \end{bmatrix}. \tag{8}$$

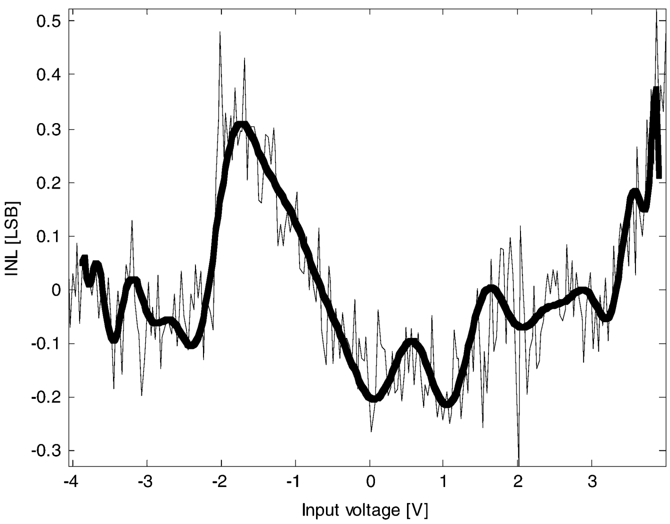

Fig. 6. Same as Fig. 5, but with a 30-harmonic reconstruction.

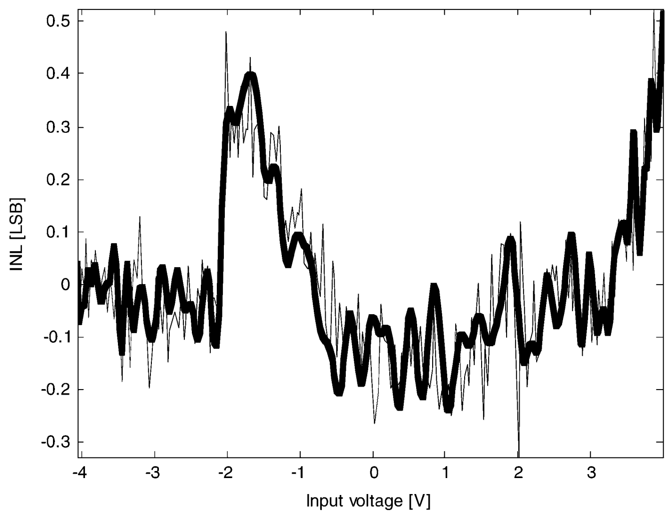

Fig. 7. Same as Fig. 5, but with a slightly overdriving sine wave at the input. The whole of the full-scale range is tested.

If one wants to approximate the INL with a polynomial of a lower order, e.g., with a 30th degree polynomial like in Fig. 3, the parametric spectral analysis offers two possibilities. The first, and more rigorous approach, consists in performing the analysis with a high-order model of the ADC output (say, a 100th-order model), and then using only the first 30 harmonics for the reconstruction. The second approach is quicker and more direct: obtaining the required 30 harmonics by using a 30th-order model of the ADC output. Even if the latter approach seems not advisable (because usually a reduced order model implies larger errors in the final estimates of the harmonics), Fig. 6 shows that in the Chebyshev test of nonlinearity it is perfectly feasible. The result is indeed very close to that of Fig. 3: or, in other words, the result given by a 30th order model is nearly the same obtained by selecting the first 30 harmonics of an FFT, i.e., the best 30th-order polynomial approximation to the INL.

As a final result concerning this 8-bit ADC, we present the performance of the test with *a slightly overdriving input.* An undesirable feature of the frequency-domain test methods for ADCs is that usually one cannot stimulate exactly the full-scale range. On the contrary, the histogram test (and also the modified time-domain analysis proposed in [14]) allows one to employ a saturating sine wave, so testing the whole characteristic. Fig. 7 shows the INL reconstruction of the parametric method with an 8.2 V peak-to-peak input sine wave (the full-scale range is 8 V). It is clearly shown that, even if additional harmonics due to the saturation are obviously present at the ADC output, the INL reconstruction is still accurate, *provided a high enough signal model order is selected*. It can be shown that, when working with overdriving sinusoidal inputs, a low order model leads to wrong reconstructions of the INL (contrary to the FFT method).

We have tried the Chebyshev test via parametric spectral analysis also on a second ADC with a much larger number of codes, a 16-bit sigma-delta converter embedded in a PC sound card. The test is particularly attractive for this kind of converters: The high resolution and the low sampling rate makes the histogram test a lengthy work, and the usually inaccurate sample clock frequency makes very difficult to obtain the exact coherent sampling required by the FFT analysis. In Fig. 8(a) and (b), the result

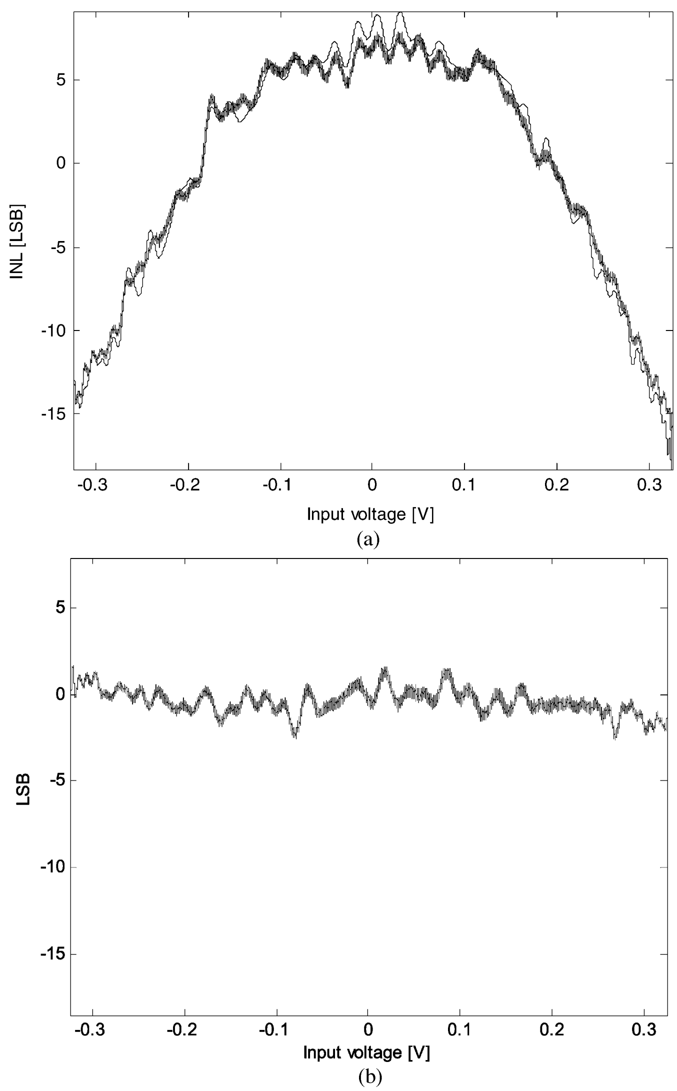

Fig. 8. (a) Comparison between the INL measured via histogram test (thick line) and the INL reconstructed by a 100-harmonic Chebyshev test (thin line) for a 16-bit sigma-delta converter embedded in a PC sound card. The harmonics come from the parametric spectral analysis of a 1024-point *incoherently* sampled sine wave. (b) Plot of the difference between the INL estimates in (a).

of the histogram test, performed with more than 8 million samples, is compared with the INL derived by the Chebyshev test on an incoherently sampled sine wave of about five periods and 1024 samples. It is impressive to see how good the result of the Chebyshev test is, especially if one considers that it uses only one sample per 64 code bins. It is also clear that, by employing the *Chebyshev linearization* discussed in [6], the big nonlinearities of the converter can be nearly nullified, so transforming an inexpensive digitizer into a highly linear and accurate measurement device.

V. Conclusion

In previous works the authors have obtained a polynomial approximation of the INL of an ADC, by determining via FFT the Fourier series of the output when the input is a sine wave. A drawback of the FFT method is that it requires very accurate coherent sampling, i.e., synchronization between the test sine wave and the sampling circuit.

In this paper, the authors have analyzed the performance of a simplified *parametric* spectral estimation method, instead of the FFT, in reconstructing the INL. The experiments have shown the following remarkable facts.

1) The parametric method is able to reconstruct accurately as many as 100 output harmonics of the ADC under test, obtaining an INL reconstruction as accurate as that provided by the FFT.
2) The time-consuming iterative search of the signal frequency is not needed in practice—a standard IFFT provides a sufficient estimate for INL reconstruction purposes.
3) A precise selection of the model order is not needed—a lower order means simply a smoother approximation of the INL.
4) The INL reconstruction is not affected by *a slight overdrive* of the ADC, and therefore the whole full-scale range can be easily tested (this is also true, however, for the FFT method).

In short, many typical drawbacks of the parametric spectral estimation methods disappear in this particular application: no need for time-consuming iterations, or of a prior knowledge of the signal spectrum. On the contrary, the method is computationally simplified and hence rapidly delivers a useful approximation of the INL, using a few thousand samples of a sine wave (regardless of the ADC resolution), and without needing any synchronization between the test signal and the sample clock.

Appendix
Demonstration of Formulas (5) and (6)

Deriving (5) is straightforward if one considers that the Chebyshev polynomials $C_n(x)$ are bounded by one

$$|C_n\left(\frac{x-C}{V}\right)| \le 1, \quad \forall x \in [-V+C, V+C]. \tag{14}$$

From (14), it is readily obtained that

$$\begin{aligned}|e(x)| &= \left|\sum_{n=N_h+1}^{\infty} c_n C_n\left(\frac{x-C}{V}\right)\right| \\ &\le \sum_{n=N_h+1}^{\infty} |c_n| \left|C_n\left(\frac{x-C}{V}\right)\right| \\ &\le \sum_{n=N_h+1}^{\infty} |c_n| = E_1(N_h).\end{aligned} \tag{15}$$

Equation (6) is instead derived by considering the orthogonality property of the Chebyshev polynomials:

$$\int_{-1}^{1} \frac{C_m(x)C_n(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0, & \text{for } m \ne n \\ \pi/2, & \text{for } m = n > 0 \\ \pi, & \text{for } m = n = 0 \end{cases}. \tag{16}$$

Using this property and the variable change $y = (x-C)/V$, we have at once

$$\begin{aligned}\int_{-V+C}^{V+C} \frac{e^2(x)}{\pi\sqrt{V^2-(x-C)^2}} dx &= \int_{-1}^{1} \frac{\sum_{n=N_h+1}^{\infty} c_n^2 C_n^2(y)}{\pi\sqrt{1-y^2}} dy \\ &= \frac{1}{2} \sum_{n=N_h+1}^{\infty} \frac{c_n^2}{\pi} \int_{-1}^{1} \frac{C_n^2(y)}{\sqrt{1-y^2}} dy \\ &= \frac{1}{2} \sum_{n=N_h+1}^{\infty} c_n^2 = E_2(N_h).\end{aligned} \tag{17}$$

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