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LETTER

Bandpass Sampling Algorithm with Normal and Inverse Placements for Multiple RF Signals

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SUMMARY Bandpass sampling algorithm is effectively adopted to obtain the digital signal with significantly reduced sampling rate for a single radio frequency(RF) signal. In order to apply the concept to multiple RF signals, we propose bandpass sampling algorithms with the normal and the inverse placements since we are interested in uniform order of the spectrum in digital domain after bandpass sampling. In addition, we verify the propose algorithms with generalized equation forms for the multiple RF signals.
key words: *bandpass sampling, multiple RF signals, joint intersection range, normal/inverse placement*

1. Introduction

In communication signal processing, the bandpass sampling algorithm is well developed for a single RF signal to be down converted with significantly reduced sampling rate. Recently, for multiple RF signals, several algorithms have been introduced to arrange each spectrum properly in a necessary order [1]–[4]. The approach proposed in [1] determines the lower bound of the sampling frequency by the condition that the minimum sampling frequency is larger than the sum of the bandwidth of the multiple RF signals, which is computationally complex because constraints on the sampling frequency should be inspected from the minimum sampling rate up to the Nyquist rate. In [2], the sampling range is obtained from the combination of the valid sampling ranges with respect to the each RF signal. This approach is also computationally complex since the valid ranges are found by using the traditional sampling range introduced in [3]. In addition, there is proposed an algorithm with simple formulas for two RF signals in [4], which can reduce the computational complexity to a certain extent.

In this paper, we extend the results in [4], and suggest bandpass sampling algorithms with significantly reduced necessary conditions for the multiple RF signals. Specially, to simplify the necessary conditions and easily manipulate sampled signals, we consider the spectrum arrangement with the normal and the inverse placements although there exist numerous spectrum arrangements for a given number of RF signals more than three [5].

To explain the spectrum arrangements after sampling, consider a set of N RF signals with the i th RF signal, $x_i(t)$, where $i = 1, 2, \cdots, N$, which N is the number of RF signals. Those spectrums are depicted in Fig. 1, with carrier frequency(f_{Ci}), upper cutoff frequency(f_{Ui}), lower cutoff frequency(f_{Li}), and bandwidth(B_i), where it is assumed that $f_{Ui} \leqq f_{Li+1}$. The spectrums of the RF signals are assumed to satisfy the following boundary condition:

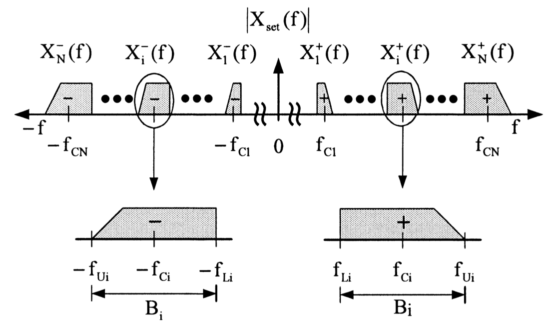

Fig. 1 Spectrums of a set of N RF signals. Parameters are depicted with carrier frequency (f_{Ci}), upper cutoff frequency (f_{Ui}), lower cutoff frequency (f_{Li}), and bandwidth (B_i) of RF signal.

$$X_i(f) = 0, \quad |f| \geqq f_{Ui}, \text{ or } |f| \leqq f_{Li}, \; i = 1, 2, \cdots, N. \quad (1)$$

When the N RF signals are sampled at every T_S second, i.e., the sampling frequency $f_S = 1/T_S$, the continuous-time Fourier transform of the sampled signals is expressed as follows:

$$\bar{X}_S(f) = \frac{1}{T_S} \sum_{i=1}^{N} \sum_{n=-\infty}^{\infty} X_i(f - nf_S), \quad (2)$$

where N is the number of RF signals [6].

2. Bandpass Sampling for Multiple RF Signals

2.1 Spectrum Arrangement with Normal Placement

The sampling algorithm with the normal placement down converts the RF band of signals into the baseband without changing the order of each RF. In order to recover baseband signals with the normal placement without aliasing in the interval $[0, f_S/2]$, let us define the weight factor, W_i, as

$$W_i = \frac{\sum_{k=1}^{i} B_k}{\sum_{k=1}^{N} B_k}, \quad (3)$$

where $i = 1, 2, \cdots, N$, and $W_0 = 0$. By using this weight

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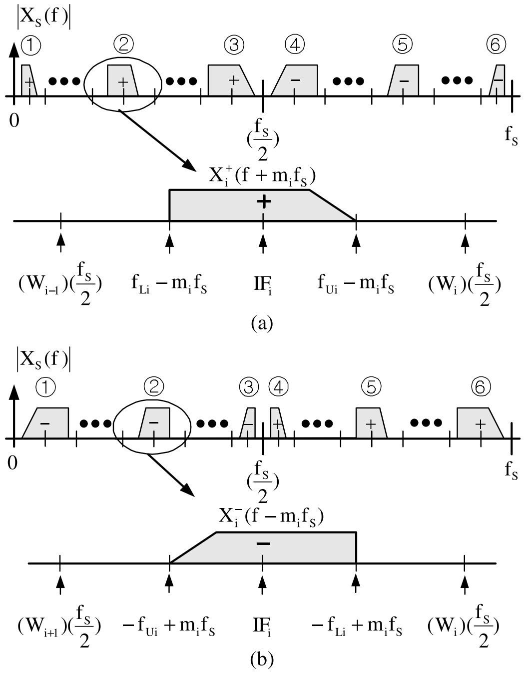

No.	Normal Placement	Inverse Placement
①	$X_1^+(f+m_1f_S)$	$X_N^-(f-m_Nf_S)$
②	$X_i^+(f+m_if_S)$	$X_i^-(f-m_if_S)$
③	$X_N^+(f+m_Nf_S)$	$X_1^-(f-m_1f_S)$
④	$X_N^-(f-(m_N+)f_S)$	$X_1^+(f+(m_1-1)f_S)$
⑤	$X_i^-(f-(m_i+1)f_S)$	$X_i^+(f+(m_i-1)f_S)$
⑥	$X_1^-(f-(m_1+1)f_S)$	$X_N^+(f+(m_N-)f_S)$
IF_i	$f_{Ci}-m_if_S$	$-f_{Ci}+m_if_S$

(c)

Fig. 2 Spectrum description after bandpass sampling. (a) Normal placement, (b)Inverse placement, (c) Spectrum location. All RF bands of signals are down converted into the interval, proportionally to each bandwidth of the RF signal.

factor, we divide the interval $[0, f_S/2]$ into N compartments, proportionally to each bandwidth of the RF signal, as shown in Fig. 2(a). Further, this figure shows that the i th RF spectrum left shifted to the m_i times f_S from RF, $X_i^+(f+m_if_S)$, should be confined into the interval $[(W_{i-1})(f_S/2), (W_i)(f_S/2)]$ not to make an aliasing effect. Subsequently, to satisfy conditions that $f_{Li} - m_if_S \geqq (W_{i-1})(f_S/2)$ and $f_{Ui} - m_if_S \leqq W_i(f_S/2)$, we can derive the i th RF sampling range, f_{SRi}, as follows:

$$\frac{2f_{Ui}}{W_i + 2m_i} \leqq f_{SRi}(m_i) \leqq \frac{2f_{Li}}{W_{i-1} + 2m_i}, \tag{4}$$

where $i = 1, 2, \cdots, N$, and $m_i = 1, 2, \cdots, max_i$. Regarding max_i, the frequency shift coefficient, m_i, can be maximized when the upper bound of (4) is equal to its lower bound. Thus, the maximum frequency shift coefficient, max_i, is given by the expression,

$$max_i = \lfloor (f_{Li}W_i - f_{Ui}W_{i-1})/(2B_i) \rfloor, \tag{5}$$

where $\lfloor x \rfloor$ stands for the largest integer not bigger than x.

For the case when $N = 1$, (4) provides a set of f_{SRi}, $f_{SR1} = \bigcup_{m_1=1}^{max_1} f_{SR1}(m_1)$, where RF sampling ranges of $f_{SR1}(m_1 = 1)$, $f_{SR1}(m_1 = 2)$, $\cdots$, and $f_{SR1}(m_1 = max_1)$ are exclusive. In order to arrange all spectrums of N RF signals into the corresponding compartments as shown in Fig. 2(a), we need to find the sampling range satisfying the constrains for N different RF signals, and this sampling range corresponds to the joint intersection range among $f_{SR1}, f_{SR2}, \cdots$, and f_{SRN}, as the approach in [2]. The joint intersection range, JIR, is represented as

$$JIR = \bigcap_{i=1}^{N} \left\{ \bigcup_{m_i=1}^{max_i} f_{SRi}(m_i) \right\}. \tag{6}$$

In practical situation, we have to check the validity of the JIR. From the valid JIR, we select one appropriate range as the available sampling range, $ASR(m_1, m_2, \cdots, m_N)$, where $m_1, m_2, \cdots$, and m_N are determined in this process. Subsequently, the lower and the upper bounds of $ASR(m_1, m_2, \cdots, m_N)$ become as

$$max\left\{ \frac{2f_{Ui}}{W_i + 2m_i} \,\middle|\, i = 1, 2, \cdots, N \right\} \leqq ASR \leqq min\left\{ \frac{2f_{Li}}{w_{i-1} + 2m_i} \,\middle|\, i = 1, 2, \cdots, N \right\}, \tag{7}$$

for the corresponding $m_1, m_2, \cdots$, and m_N. The middle point of ASR is chosen as the sampling frequency, f_S. The intermediate frequency is given by the expression,

$$f_{IFi} = f_{Ci} - m_if_S. \tag{8}$$

On the basis of the analysis, the flowchart of the bandpass sampling algorithm is shown in Fig. 4. In order to demonstrate the usage of proposed algorithms, we consider the RF signals with such parameters as $f_{L1} = 864.2$ MHz, $f_{U1} = 864.4$ MHz, $f_{L2} = 890.2$ MHz, $f_{U2} = 890.4$ MHz, $f_{L3} = 935.6$ MHz, and $f_{U3} = 935.8$ MHz, which correspond uplink signal of CT-2, down link and uplink signals of GSM-800, respectively. From the flow chart, the parameters are calculated as $W_1 = 1/3$, $W_2 = 2/3$, $W_3 = 1$, $max_1 = 720$, $max_2 = 741$, and $max_3 = 779$. The first sampling range[MHz] is obtained as $5186.4/(6m_1 + 1) \leqq f_{SR1}(m_1) \leqq 5185.2/(6m_1)$, $m_1 = 1, 2, \cdots, 720$. Then, the second and the third sampling ranges[MHz] are also obtained as $5342.4/(6m_2 + 2) \leqq f_{SR2}(m_2) \leqq 5341.2/(6m_2 + 1)$, $m_2 = 1, 2, \cdots, 741$, and $5614.4/(6m_3 + 3) \leqq f_{SR3}(m_3) \leqq 5613.6/(6m_3 + 2)$, $m_3 = 1, 2, \cdots, 779$, respectively.

The joint intersection ranges of 23 are found by checking the validity as shown in Table 1. Further, these ranges can be selected as ASR. Assuming that the interested sampling range is around 20 MHz, we can select the available sampling range around 20 MHz as shown in Table 1, where $JIRs$ of 3 exist when $[m_1, m_2, m_3] = [39, 40, 42]$, $[40, 41, 43]$, and $[41, 42, 44]$, respectively. One of the three $JIRs$ is chosen as $21.541935 \leqq ASR(40, 41, 43) \leqq 21.590769$ [MHz], when $21.520332 \leqq f_{SR1}(m_1 = 40) \leqq 21.605000$, $21.541935 \leqq f_{SR2}(m_2 = 41) \leqq 21.624291$, and

Table 1 Joint intersection ranges for the normal placement.

No.	Joint Intersection Ranges(JIRs)	Sampling FRQ.(Hz)
1	$398953846 \leq JIR(m_1 = 2, m_2 = 2, m_3 = 2) \leq 400971429$	$f_s = 399962637$
⋮	⋮	⋮
6	$22076033 \leq JIR(m_1 = 39, m_2 = 40, m_3 = 42) \leq 22100787$	$f_s = 22088410$
(7)	$21541935 \leq JIR(m_1 = 40, m_2 = 41, m_3 = 43) \leq 21590769$	(f_s) $= 21566352$
8	$21033071 \leq JIR(m_1 = 41, m_2 = 42, m_3 = 44) \leq 21078049$	$f_s = 21055560$
⋮	⋮	⋮
23	$2138971 \leq JIR(m_1 = 404, m_2 = 416, m_3 = 437) \leq 2139047$	$f_s = 2139009$

No.	m_1	$f_{SR1}(m_1)$ Hz		m_2	$f_{SR2}(m_2)$ Hz		m_3	$f_{SR3}(m_3)$ Hz		Valid JIR
	35	24580095	24691429	36	24506422	24613825	38	24306494	24406957	No
	36	23900461	24005556	37	23850000	23951570	39	23691139	23786441	No
	37	23257399	23356757	38	23227826	23324017	40	23106173	23196694	No
	38	22648035	22742105	39	22637288	22728511	41	22549398	22635484	No
6	39	22069787	22158974	40	22076033	22162656	42	22018824	22100787	Yes
(7)	40	21520332	21605000	41	21541935	21624291	43	21512644	21590769	Yes
8	41	20997571	21078049	42	21033071	21111462	44	21029213	21103759	Yes
	42	20499605	20576190	43	20547692	20622394	45	20567033	20638235	No
	43	20024710	20097674	44	20084211	20155472	46	20124731	20192806	No
	44	19571321	19640909	45	19641176	19709225	47	19701053	19766197	No

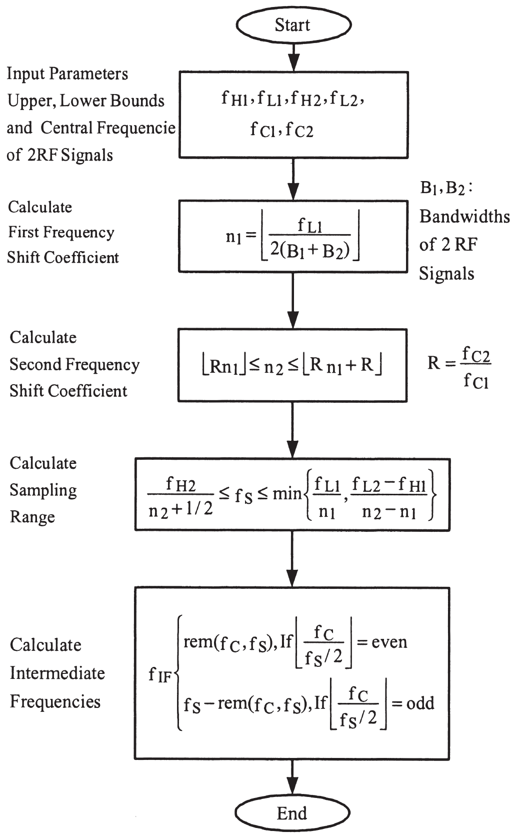

Fig. 3 Bandpass sampling algorithm with normal placement for 2RF signal, introduced in previous work.

$21.511264 \leqq f_{SR3}(m_3 = 43) \leqq 21.590769$. From the middle point of the ASR, the available sampling frequency becomes $f_S = 21.566352$ [MHz]. The corresponding intermediate frequencies in digital domain become $IF_1 = 1.645920$, $IF_2 = 6.079568$, and $IF_3 = 8.346864$ [MHz].

In addition, the proposed algorithm is consistent with the results of [5] for one RF signal, and those of [4] for two RF signals as shown in Fig. 3, with respect to the normal placement. Further, when we search an available sampling range for the normal and inverse placements, we may encounter some cases that the valid JIR doesn't exist, which implies that there may exist several kinds of spectral placements depending on the number of RF signals [4].

2.2 Spectrum Arrangement with Inverse Placement

Regarding the sampling algorithm with the inverse placement, the spectrums, $X_1^-(f)$, $X_2^-(f), \cdots$, $X_N^-(f)$, should be properly arranged without aliasing within the interval $[0, f_S/2]$, as shown in Fig. 2, (b), where the weight factor, W_i, is defined as

$$W_i = 1 - \frac{\sum_{k=1}^{i-1} B_k}{\sum_{k=1}^{N} B_k}, \; W_1 = 1, \tag{9}$$

where $i = N, N-1, \cdots, 2$, and $W_i = 0$, when $i > N$. Similar to the normal placement, the generalized sampling range and frequency shift coefficient for the i th RF signal are expressed by

$$\frac{2f_{Ui}}{2m_i - W_{i+1}} \leqq f_{SRi}(m_i) \leqq \frac{2f_{Li}}{2m_i - W_i}, \tag{10}$$

where $i = N, N-1, \cdots, 1$, and $m_i = 1, 2, \cdots, max_i$, which $max_i = \lfloor (f_{Ui}W_i - f_{Li}W_{i+1})/(2B_i) \rfloor$. Further, the joint intersection range, JIR, is expressed as

$$JIR = \bigcap_{i=1}^{N} \left\{ \bigcup_{m_i=1}^{max_i} f_{SRi}(m_i) \right\}. \tag{11}$$

From the joint intersection ranges, we can select one appropriate range as an available sampling range, ASR. Subsequently, the lower and the upper bounds of $ASR(m_1, m_2, \cdots, m_N)$ become as

$$max\left\{ \frac{2f_{Ui}}{2m_i - W_{i+1}} \,\middle|\, i = 1, 2, \cdots, N \right\} \leqq ASR$$
$$\leqq min\left\{ \frac{2f_{Li}}{2m_i - W_i} \,\middle|\, i = 1, 2, \cdots, N \right\}. \tag{12}$$

The middle point of ASR is chosen as the sampling frequency, f_S. In addition, the intermediate frequency is obtained by

$$f_{IFi} = -f_{Ci} + m_i f_S. \tag{13}$$

On the basis of the analysis, the bandpass sampling algorithm with the inverse placement can be extended straightforwardly by using the flowchart shown in Fig. 4. The algorithm with inverse placement is also consistent with the results of [5] for one RF signal, and those of [4] for two RF signals.

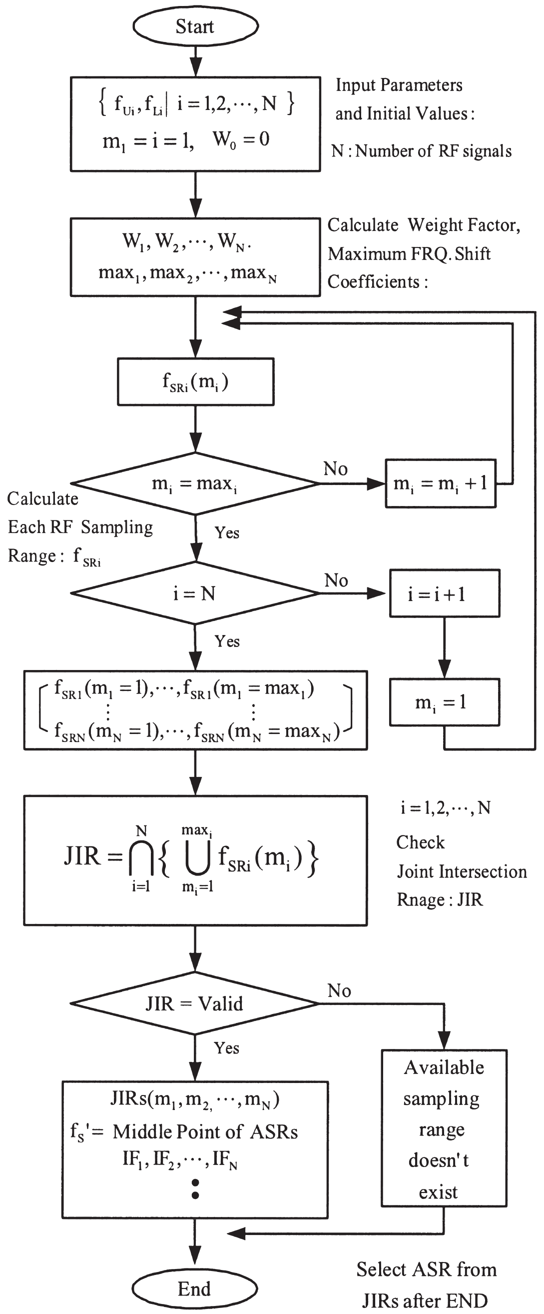

Fig. 4 Bandpass sampling algorithm with normal placement for *N* RF signals, proposed in this paper.

3. Conclusions

In practical case, it is not easy to apply the bandpass sampling algorithm to the multiple RF signals more than three because available sampling ranges reduce significantly. In order to overcome this weak point, we propose the bandpass sampling algorithms with generalized equation forms. Specially, even though the number of RF signals increases, we can apply the proposed algorithm to the multiple RF signals without any modification of the related equations. In addition, after bandpass sampling, the spectrums of the RF signals are inversely or normally placed with respect to the order of the carrier frequency of each RF signal without any aliasing, which is convenient for digital signal processing.

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