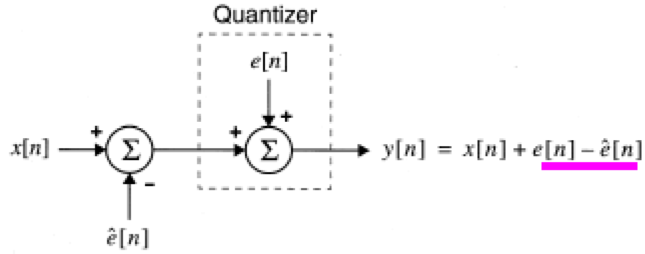
Basic Theory of Operation

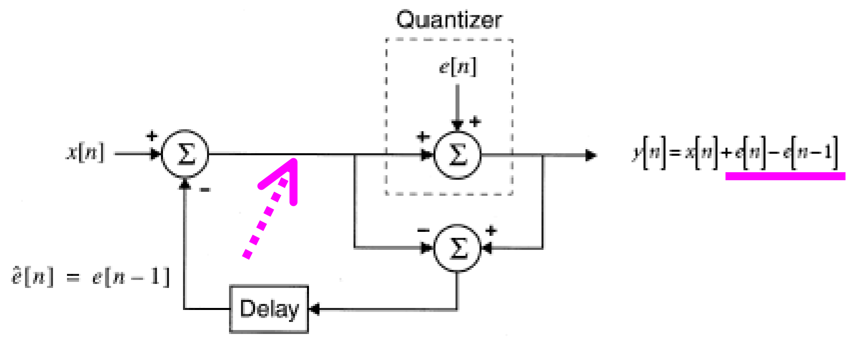
In <u>Nyquist rate</u> converters, the error is **reduced** by using a large number of small steps in the quantizer characteristic.

In <u>oversampled data converter</u>s, specifically sigma-delta modulators, the error is **corrected** by estimating the error in advance and subtracting it from the input.

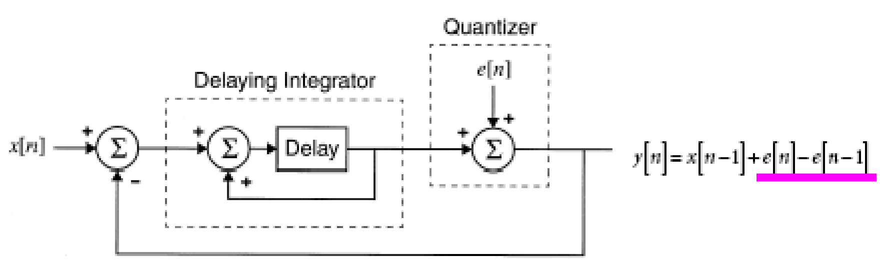


However, since the error is not known until it is made, e[n] *is not known when* ê[n] *is needed.* Therefore, some means must be found to estimate the error.

In the case of **sigma-delta** converters, the error can be estimated by exploiting some knowledge of the frequency domain behavior of the input signal. Specifically, *it is <u>assumed</u> that the signal is changing very slowly from sample to sample*, or equivalently, its bandwidth is much less than the sampling rate. For exceedingly slow signals, a **first-order estimate of the error** to be committed in quantization can be formed. The first-order estimate of the current error *e* [n] is *simply* the previous error *e* [n – 1].



With a few straightforward steps, the system can be transformed into that of Fig below, where the delay element is now immersed in an integrator feedback loop.



Fattaruso, J.W., Williams III, L.A.
"Oversampled Analog-to-Digital and Digital-to-Analog Converters"
The VLSI Handbook.
Ed. Wai-Kai Chen
Boca Raton: CRC Press LLC, 2000

DSM topológia variációk

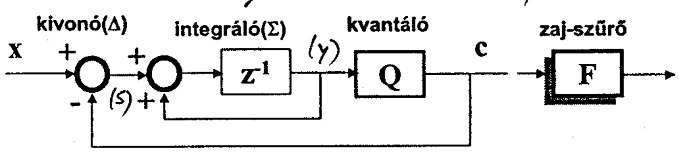
Zajformálás (zaj-differenciálás) és szűrés

LP (Low Pass) MOD1:

spektrum szeparálás

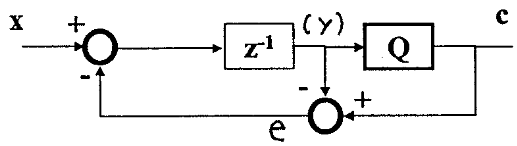
Visszacsatolással (tracking loop) átlagosan zérus értékű az (ábrán diszkrét-idejű) **integráló** kimenete, ez minimalizálja a kisfrekvenciás *differenciát* ***x*** és ***c*** között.
Tehát a **nagy mintagyakoriságú** numerikus mintasorozat (***c***: data stream) lokális "átlag"értéke pontosan követi a bemenetet (a késleltetési operátor $\Delta t = 1/f_s$)

"jel"visszacsatolás $(s+y)\cdot z^{-1} = y \rightarrow y = \frac{z^{-1}}{1-z^{-1}}\cdot s$



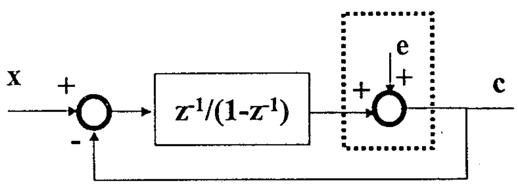
Q-modell

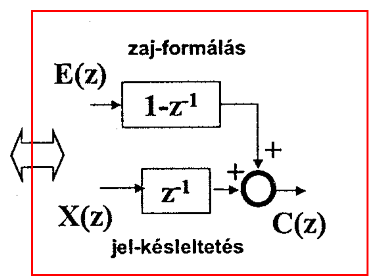
ekvivalens topológia: **"hiba"**visszacsatolás



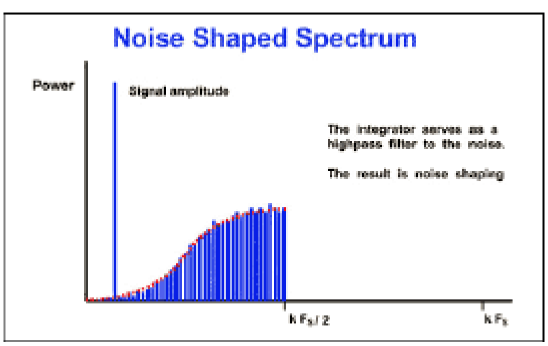
$$c = (x-e)\cdot z^{-1} + e$$
$$= x\cdot z^{-1} + (1-z^{-1})\cdot e$$

linearizált modell (a kvantálási hiba "fehér zaj")

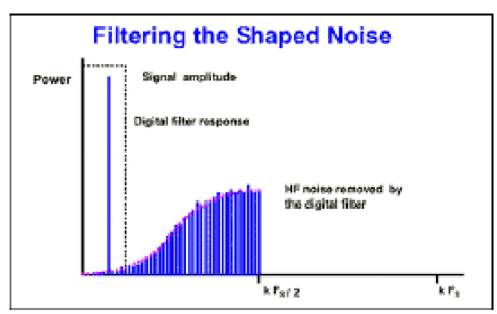




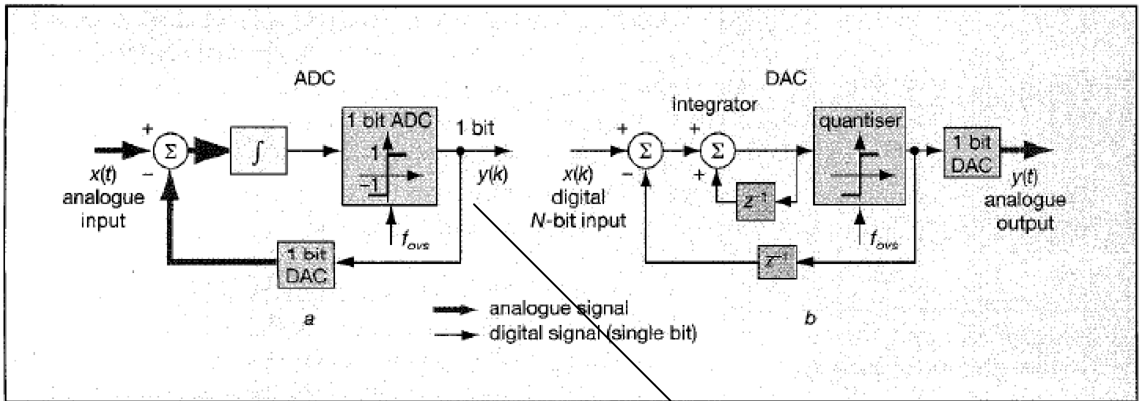
A jel és kvantálási-zaj spektrum szeparálódik (!), a keskenysávú *jelre* a transzfer függvény: z^{-1} (ez csak késleltetés), míg a szélessávú *zajt formáló* transzfer függvény: $1 - z^{-1}$ (ez diszkrét idejű **elsőrendű** *differenciáló* → frekvencia szelektív: nagyfrekvenciás kiemelő, *felüláteresztő* szűrő)



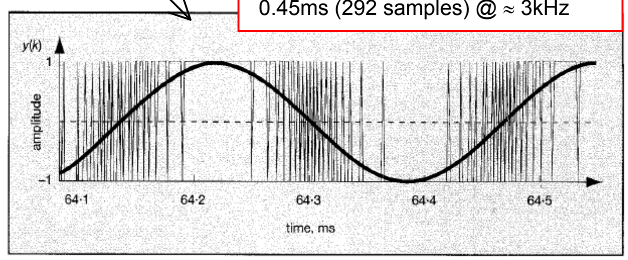
Affect of the integrator in the sigma-delta modulator



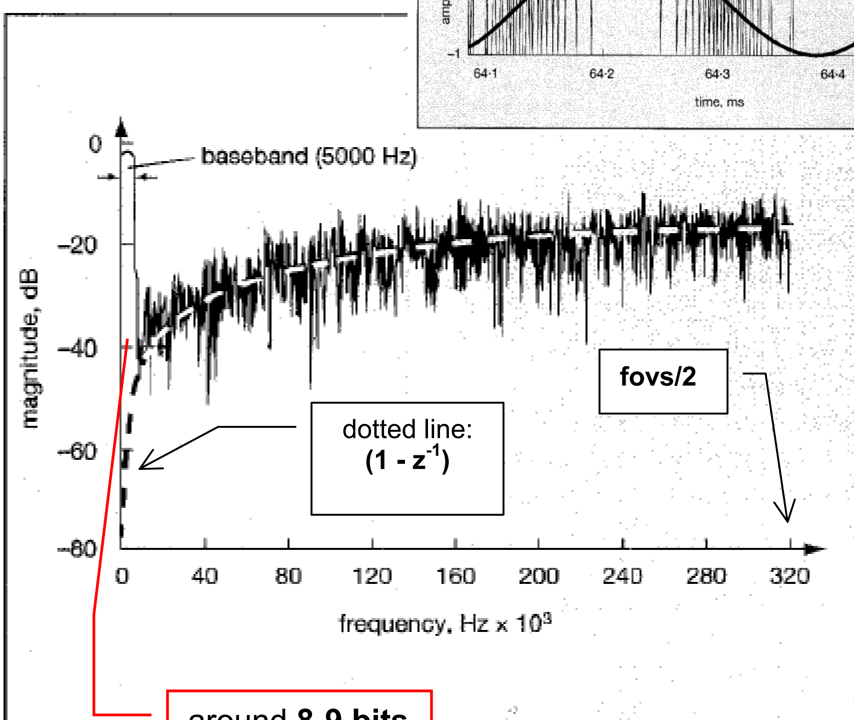
LP MOD1:



0.45ms (292 samples) @ ≈ 3kHz



In: linear **chirp** (100Hz to 5000Hz)
fovs: 0.64 MHz; **M = 64 (L = 1, n = 1)**
FFT: 64K (65 536 data points, ≈ 0.1s)

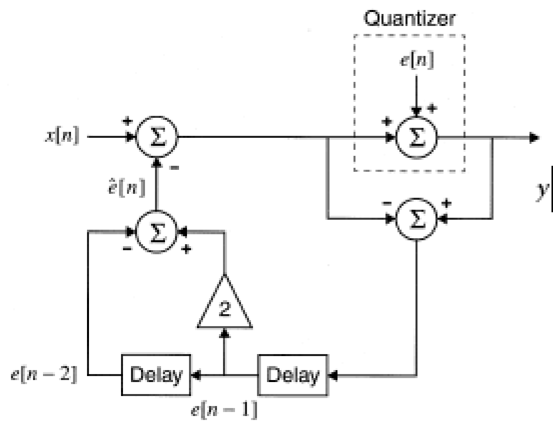


around **8-9 bits** by dig LPF (decimation)

A **second-order estimate of *e*[*n*]** may be formed by assuming that the error *e*[*n*] varies <u>linearly</u> with time. In this case, an *estimate* of the current error *e*[*n*] may be computed by changing the previous error *e*[*n*– 1] by an amount equal to the change between *e*[*n*– 2] and *e*[*n*– 1]. The second order *error* estimate is thus

$$\hat{e}[n] = e[n-1] + \left(e[n-1] - e[n-2]\right) = 2e[n-1] - e[n-2]$$

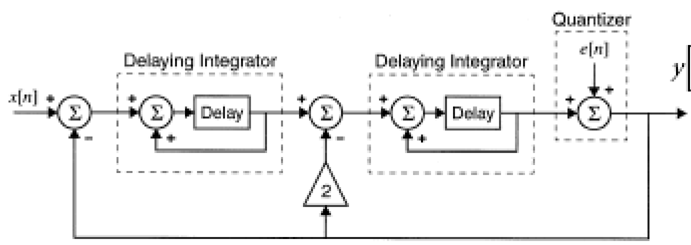
and the **second-order modulator** is the following



$$y[n] = x[n] + e[n] - 2e[n-1] + e[n-2]$$

Second-order error estimation.

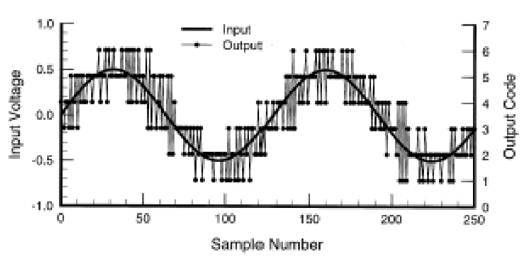
After a number of steps, that the modulator can be transformed into a modulator in which the feedback loop delays are again immersed in practical *integrator* blocks



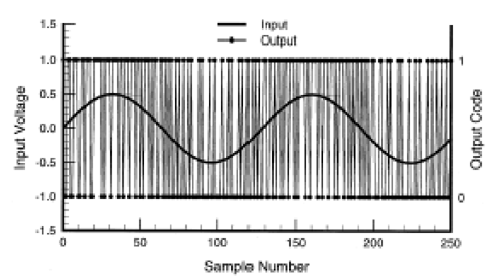
$$y[n] = x[n-2] + e[n] - 2e[n-1] + e[n-2]$$

Second-order equivalent modulator.

Fig's show the simulated output of the modulator when fed with a simple *sinusoidal input*



eight-level quantizer



two-level quantizer

Fattaruso, J.W., Williams III, L.A.
"Oversampled Analog-to-Digital and Digital-to-Analog Converters"
The VLSI Handbook.
Ed. Wai-Kai Chen
Boca Raton: CRC Press LLC, 2000

PCM vs. 2nd order DSM

Uniform Quantizer:

48-Msample/s input sequence, consisting of a 48-kHz **sinusoid** with small amplitude [plus a small amount of white noise such that the input signal-to-noise ratio (SNR) is 100 dB]. Fig. 2(a) shows the power spectral density (PSD) plot of the resulting quantizer output sequence, and Fig. 2(b) shows a time-domain plot of the quantizer output sequence over two periods of the sinusoid

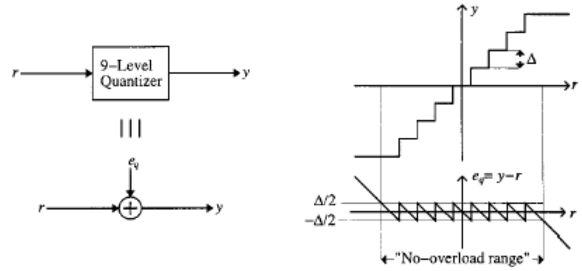


Fig. 1. Ideal nine-level uniform quantizer.

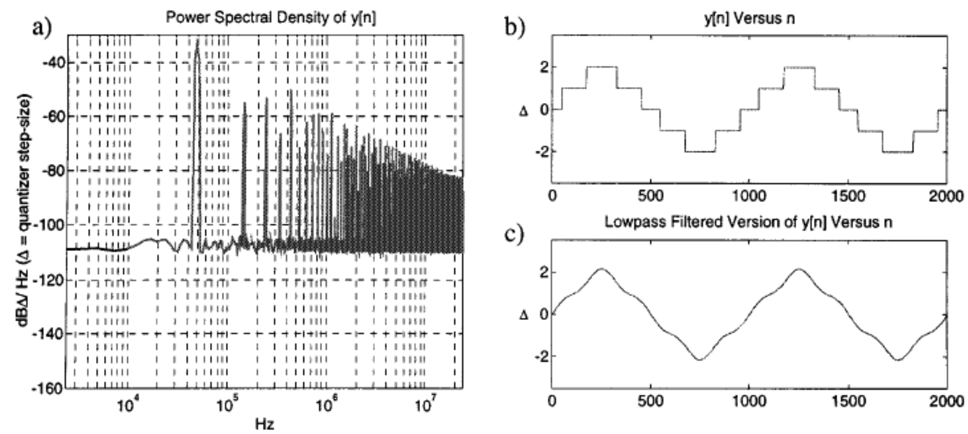


Fig. 2. (a) PSD plot of the quantizer output in decibels, relative to the quantization step-size, per hertz. (b) Time-domain plot of the quantizer output. (c) Time-domain plot of the quantizer output filtered by a sharp lowpass filter with a cutoff frequency of 500 kHz.

ΔΣ Modulator:

Same 48-Msample/s input sequence considered above is applied to the input of the modulator, and the discrete-time integrators in the modulator are clocked at 48 MHz. Fig. 4(a) shows the PSD plot of the resulting modulator output sequence, , and Fig. 4(b) shows a time-domain plot of over two periods of the sinusoid

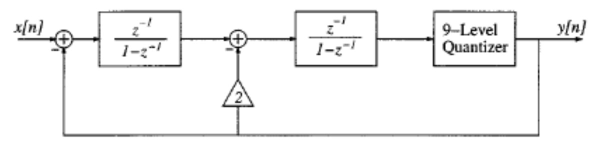


Fig. 3. Second-order ΣΔ modulator architecture.

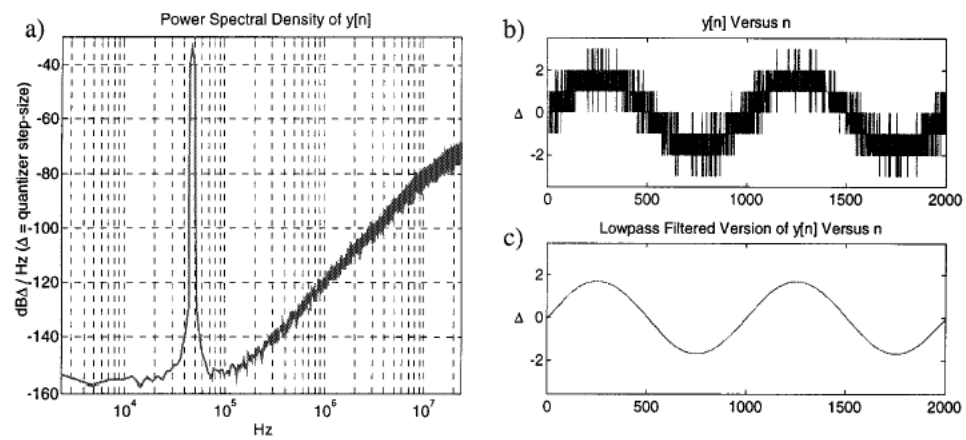


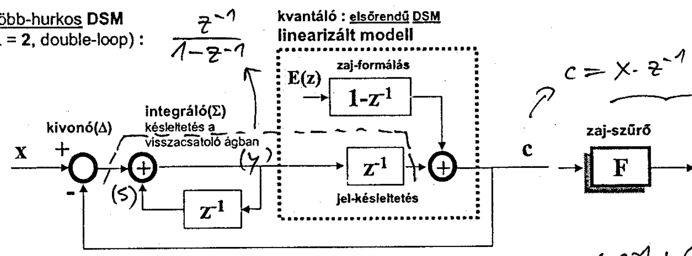
Fig. 4. (a) PSD plot of the ΔΣ modulator output in decibels, relative to the quantization step-size, per hertz. (b) Time-domain plot of the ΔΣ modulator output. (c) Time-domain plot of the ΔΣ modulator output filtered by a sharp lowpass filter with a cutoff frequency of 500 kHz.

Two important **differences** with respect to the uniform quantization example shown in Fig. 2 are apparent: (1) the quantization noise PSD is significantly **attenuated** at low frequencies, and (2) **no** spurious tones are visible anywhere in the discrete-time spectrum.

LP MOD2:

2.3 A linearizált kvantáló modellt felhasználva, igazoljuk, hogy másodrendű (L = 2) zajformáló az alábbi *több-hurkos* **(multi-loop)** illetve *kaszkád* **(MASH)** átalakító alapváltozat

Több-hurkos **DSM** (L = **2**, double-loop) :



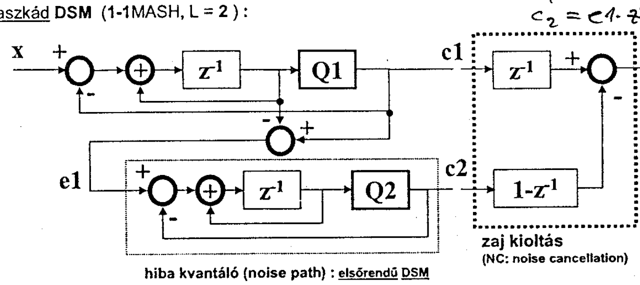
$s + y \cdot z^{-1} = y$

$y = \frac{1}{1-z^{-1}} \cdot s$

? $c = x \cdot z^{-1} + (1-z^{-1})^2 \cdot e$

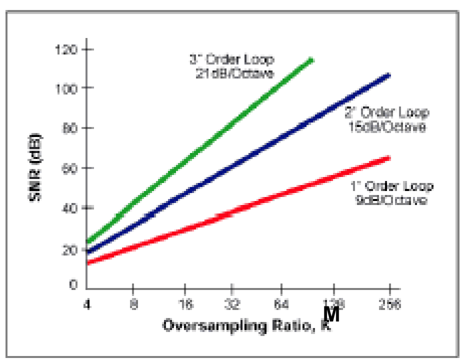
Kaszkád **DSM** (1-1MASH, L = **2**) :

$\left.\begin{array}{l} c_1 = x \cdot z^{-1} + (1-z^{-1}) \cdot e1 \\ c_2 = e1 \cdot z^{-1} + (1-z^{-1}) \cdot e2 \end{array}\right\}$



$c = x \cdot z^{-2} +$
$\cancel{+ (1-z^{-1}) \cdot z^{-1} \cdot e1} -$
$- \cancel{z^{-1} (1-z^{-1}) \cdot e_1} -$
$- (1-z^{-1})^2 \cdot e2$
$= x \cdot z^{-2} - (1-z^{-1})^2 \cdot e2$

A kaszkád DSM kimenete mindig multi-bites (még akkor is, ha a kvantálók 1 bitesek), miért?



Relationship between order of sigma-delta modulator and the amount of oversampling necessary to achieve a particular SNR

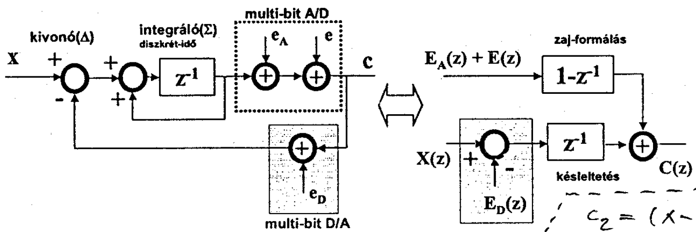
ANALOG MULTIBIT LP MOD1

2.6 Multi-bites (n > 1), **analóg DSM.**

(a) Igazoljuk az ekvivalens topológiát (L = 1)**:** a visszacsatoló ágban lévő D/A átalakító e_D torzítására **nincs** zajformálás (a hasznos sávba eső zajt csak a túlmintavételezés csökkenti)! Ezért igen *nagy linearitású* (kis transzfer torzítású) D/A átalakítót kell használni.

Ráadásul, csak egy órajelnyi késleltetés lehet a hurokban (loop rate = 1 clock latency), ami igen gyors (*egy* lépéses: word-at-a-time = **flash**) A/D átalakítót kíván. A modell számba veszi az A/D realizálási hibáját is: plusz e_A torzítás

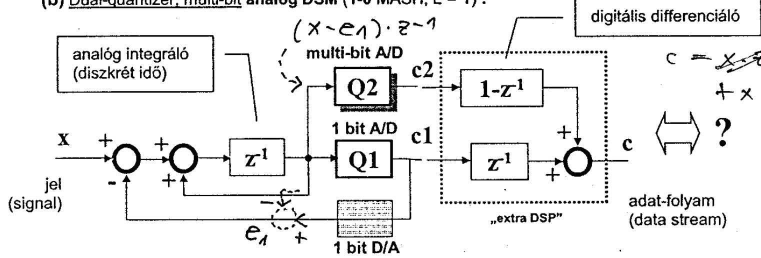
(a) Multi-bit **analog DSM** (L = **1**) **:**



$c_2 = (x-e_1)\cdot z^{-1} + e_2$

$c_1 = x\cdot z^{-1} + (1-z^{-1})\cdot e_1$

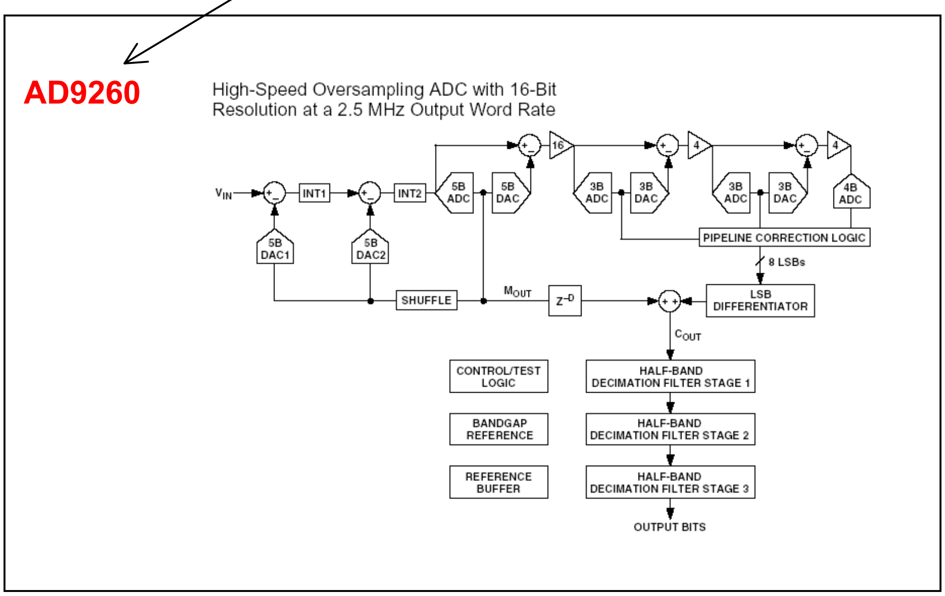
(b) Dual-quantizer, multi-bit **analog DSM** (**1-0** MASH, L = **1**) **:**



$c = \cancel{x\cdot z^{-2}} + \cancel{(1-z^{-1})\cdot z^{-1}\cdot e_1}$
$+ x\cdot z^{-1}(1-\cancel{z^{-1}}) -$
$- \cancel{z^{-1}(1-z^{-1})\cdot e_1} +$
$+ (1-z^{-1})\cdot e_2 =$
$= x\cdot z^{-1} + (1-z^{-1})\cdot e_2$

(b) Egy trükk a probléma megoldására: **1-0** MASH *két kvantálós* architektúra [T. Leslie, B. Singh 1990]; a hatásos kvantáló (Q2) felbontása és a visszacsatolás (Q1) felbontása szétválik („multi-bites" kvantálás *kontra* „1 bites" visszacsatolás). Adjuk meg az ekvivalens topológiát (linearizált kvantáló modellek, hibamentes 1 bites D/A).

A multi-bites A/D a hurkon kívül van, így **kell** külső szűrő (a hiba korrekcióhoz); de pl. *több* lépéses, párhuzamos műveletvégzésű is lehet a konverter bit-kereső algoritmusa (**pipelining**), amelynek terjedési késleltetése kompenzálható. (A módszer **kombinálja** az OSADC és a Nyquist-rate technikákat, lásd **8.** példa.)

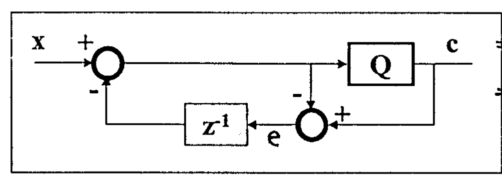


DIGITAL LP MOD1 and MOD2

2.9 A DSM eljárás egyik alapforrása a *"hiba"visszacsatolás* topológia (self-dithering, deterministic dither). *Analóg* DSM esetén *nem* praktikus az elrendezés, mert a visszacsatoló ágban végzett művelet hibájára nincs zajformálás (lásd **2.6** feladat). Ez nem hátrány **digitális DSM** esetén (sőt, pl. egyszerű szóhossz csonkítással realizálható a kvantálás és hiba-képzés, lásd 31. oldal)

"Hiba"visszacsatolás (L = **1**, késleltetés a visszacsatoló ágban)

Q modell



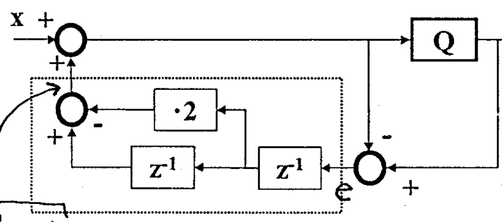
$= x - e\cdot z^{-1} + \hat{e}$

$= \underbrace{x + (1 - z^{-1})\cdot e}$

[C. Cutler 1954/1960, U.S. Patent]

Másodrendű **digitális DSM** (**"LSB(error)-feedback"**, L = **2**) **:**

Q modell



numerikus minta (data)

$(1 - z^{-1})^2$

$(z^{-2} - 2\cdot z^{-1} + 1) - 1 = H(z) - 1$

$c = x + [H(z) - 1]\cdot e + \hat{e}$

$= \underbrace{x + H(z)\cdot e}$

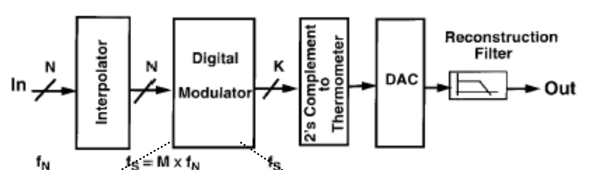
adat-folyam (data[bit] stream)

Ha L > 1 az igény zajformálásra, a késleltetést "predikciós" szűrő helyettesíti (amelynek átvitele: *H(z) - 1*, ahol *H(z)* a kívánt zaj-formáló transzfer függvény).

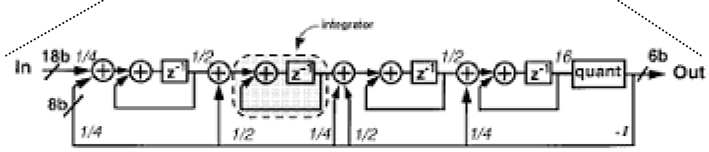
Igazoljuk, hogy a vázolt topológia *másodrendű* zajformáló. Megjegyzés: a szorzás (**·2**) = shift.

A (N =)14-bit, (f_N =)10 MSPS DAC Using Multi-bit Delta-Sigma Modulation

M = 12 **L = 4** **K (= n) = 6**



FALAKSHAHI et.al.: 14-BIT D/A CONVERTER USING MULTIBIT Delta-Sigma MODULATION
IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 34, NO. 5, MAY 1999



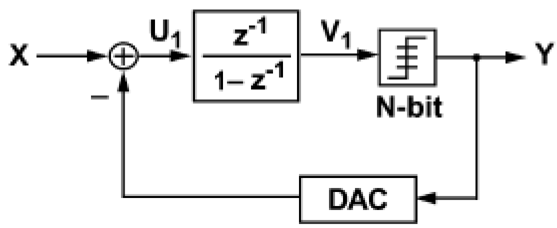
IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. [illegible], NO. [illegible], SEPTEMBER [illegible]

A Low-Voltage Low-Power Sigma-Delta Modulator for Broadband Analog-to-Digital Conversion

KiYoung Nam, *Student Member, IEEE*, Sang-Min Lee, *Student Member, IEEE*, David K. Su, *Senior Member, IEEE*, and Bruce A. Wooley, *Fellow, IEEE*

… a feedforward architecture

The input and output of the integrator, U_1 and V_1, each have a component that depends on the modulator input:

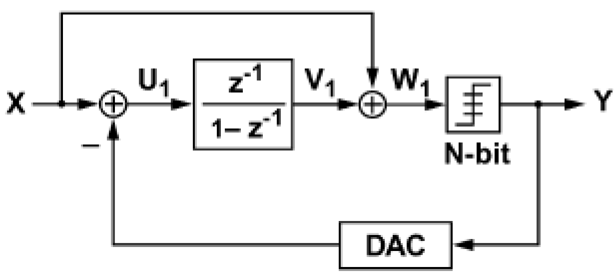


$$Y = z^{-1} \cdot X + (1 - z^{-1}) \cdot E_N$$
$$U_1 = (1 - z^{-1}) \cdot X - (1 - z^{-1}) \cdot E_N$$
$$V_1 = z^{-1} \cdot X - z^{-1} \cdot E_N$$

Conventional first-order $\Sigma\Delta$ modulator.

This *input dependence results in* large signals for U_1 and V_1 when the input is large (→ *op amp* clipping, harmonic distortion …)

The **input-feedforward path** in modulator is a method of relaxing the requirements on analog blocks:



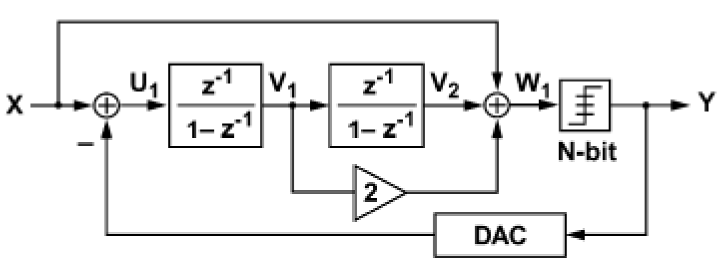
$$Y = X + (1 - z^{-1}) \cdot E_N$$
$$U_1 = -(1 - z^{-1}) \cdot E_N$$
$$V_1 = -z^{-1} \cdot E_N$$
$$W_1 = X - z^{-1} \cdot E_N.$$

First-order reduced integrator swing range (RISR) $\Sigma\Delta$ modulator.

The input and output of the integrator, U_1 and V_1 *no longer depend* on the modulator input X. The removal of the input signal component *reduces* the swing at the internal nodes of the modulator which relaxes the headroom requirements, and allows for more efficient *op amp* architectures to be used.

[However, the input-feedforward path presents a couple of *complications*, namely the increased loading the input has to drive, the analog adder at the quantizer input …]

The approach can be expanded to a second-order noise-differencing modulator:



$$Y = X + (1 - z^{-1})^2 \cdot E_N$$
$$U_1 = -(1 - z^{-1})^2 \cdot E_N$$
$$V_1 = -z^{-1}(1 - z^{-1}) \cdot E_N$$
$$V_2 = -z^{-2} \cdot E_N$$
$$W_1 = X + z^{-1}(z^{-1} - 2) \cdot E_N.$$