Sigma-Delta (Σ-Δ) or Delta-Sigma (Δ-Σ)?

Editor's Notes from *Analog Dialogue* Vol. 24-2, 1990, by Dan Sheingold

This is not the most earth-shaking of controversies, and many readers may wonder what the fuss is all about—if they wonder at all. The issue is important to both editor and readers because of the need for consistency; we'd like to use the same name for the same thing whenever it appears.

But *which* name?

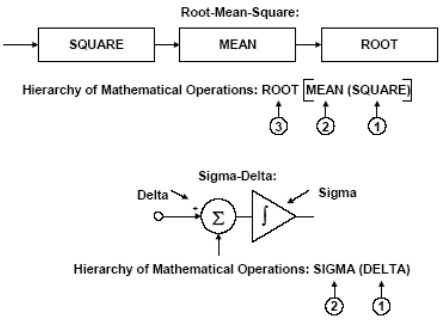
In the case of the modulation technique that led to a new oversampling A/D conversion mechanism, we chose *sigma-delta*. Here's why.

Ordinarily, when a new concept is named by its creators, the name sticks; it should not be changed unless it is erroneous or flies in the face of precedent. The seminal paper on this subject was published in 1962 (Reference), and its authors chose the name "**delta sigma** modulation," since it was based on *delta* modulation but included an integration (summation, hence Σ).

Delta-sigma was apparently unchallenged until the 1970s, when engineers at AT&T were publishing papers using the term *sigma-delta*. Why? According to Hauser (Reference), the precedent had been to name variants of delta modulation with adjectives preceding the word "delta." Since the form of modulation in question is a variant of delta modulation, the sigma, used as an adjective—so the argument went—should precede the delta.

Many engineers who came upon the scene subsequently used whatever term caught their fancy, often without knowing why. It was even possible to find *both* terms used interchangeably in the same paper. As matters stand today, sigma-delta is in widespread use, probably for the majority of citations. Would its adoption be an injustice to the inventors of the technique?

We think not. Like others, we believe that the name delta-sigma is a departure from precedent. Not just in the sense of grammar, but also in relation to the hierarchy of operations. Consider a block diagram for embodying an analog root-mean-square (finding the square root of the mean of a squared signal) computer. First the signal is squared, then it is integrated, and finally it is rooted (see Figure).



If we were to name the overall function after the causal order of operations, it would have to be called a "square mean root" function. But naming in order of the *hierarchy* of its mathematical operations gives us the familiar—and undisputed—name, *root mean square*. Consider now a block diagram for taking a difference (delta), and then integrating it (sigma).

Its **causal order** would give *delta-sigma*, but **in functional hierarchy** it is *sigma-delta*, since it computes the integral of a difference. We believe that the latter term is correct and follows precedent; and we have adopted it as our standard.

References

C. C. Cutler, "Transmission Systems Employing Quantization," **U.S. Patent 2,927,962**, filed April 26, 1954, issued March 8, 1960. *(a ground-breaking patent describing* oversampling and noise shaping *using first and second-order loops to increase effective resolution. The goal was transmission of oversampled noise shaped PCM data* without decimation, **not** a Nyquist-type ADC*).*

H. Inose, Y. Yasuda, and J. Murakami, "A Telemetering System by Code Modulation: Δ-Σ Modulation," **IRE Transactions on Space Electronics Telemetry**, Vol. SET-8, September 1962, pp. 204-209. Reprinted in N. S. Jayant, **Waveform Quantization and Coding**, IEEE Press and John Wiley, 1976, ISBN 0-471-01970-4. *(an elaboration on the 1-bit form of* Cutler's noise-shaping oversampling concept. ***This work coined the description of the architecture as 'delta-sigma modulation').***

D. J. Goodman, "The Application of Delta Modulation of Analog-to-PCM Encoding," **Bell System Technical Journal**, Vol. 48, February 1969, pp. 321-343. Reprinted in N. S. Jayant, **Waveform Quantization and Coding**, IEEE Press and John Wiley, 1976, ISBN 0-471-01970-4. *(the first description of using oversampling and noise shaping techniques* followed by digital filtering and decimation to produce a true **Nyquist-rate** ADC*).*

Max W. Hauser, "Principles of Oversampling A/D Conversion," **Journal Audio Engineering Society**, Vol. 39, No. 1/2, January/February 1991, pp. 3-26. *(one of the best tutorials and practical discussions of the sigma-delta ADC architecture and its history).*

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Chapter 2 in *Delta-Sigma Data Converters*
Edited by S. Norsworthy, R. Schreier, and G. Temes

Quantization Noise in $\Delta\Sigma$ A/D Converters

Robert M. Gray

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The name "Delta-Sigma" modulator was introduced by Inose and Yasuda in 1963 [38], who provided the first published description of its basic properties. The name was intended to reflect the fact that the system first took a difference (Delta) and then integrated (Sigma). The modern popularity of these systems, much of the original analysis, and the alternative name "Sigma-Delta" modulator is due to Candy and his colleagues [39, 40, 41, 42, 10]. The name $\Sigma\Delta$ reflects the fact that the system can also be represented as the cascade of an integrator (Sigma) and a Delta-modulator. In the author's opinion, this is a better name because the system does not really form a difference of successive samples of the input signal as suggested by the Delta Sigma name, it forms the difference between the input and a digital approximation of the previous input that is fed back. The name $\Delta\Sigma$ does not incorporate the key attribute of quantization in the system, the reverse order does. The author bows to the majority of coauthors, however, and adopts the older name.

[37] C. C. Cutler, "Transmission systems employing quantization," 1960. U. S. Patent No. 2,927,962.

[38] H. Inose and Y. Yasuda, "A unity bit coding method by negative feedback," *Proc. IEEE*, vol. 51, pp. 1524–1535, November 1963.

[39] J. C. Candy, "A use of limit cycle oscillations to obtain robust analog-to-digital converters," *IEEE Trans. Comm.*, vol. COM-22, pp. 298–305, March 1974.

[40] J. C. Candy, "A use of double integration in sigma delta modulation," *IEEE Trans. Comm.*, vol. COM-33, pp. 249–258, March 1985.

[41] J. C. Candy, "Decimation for sigma delta modulation," *IEEE Trans. Comm.*, vol. COM-34, pp. 72–76, January 1986.

[42] J. C. Candy, Y. C. Ching, and D. S. Alexander, "Using triangularly weighted interpolation to get 13-bit PCM from a sigma delta modulator," *IEEE Trans. Comm.*, pp. 1268–1275, November 1976.

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Max W. Hauser, "Principles of Oversampling A/D Conversion,"
Journal Audio Engineering Society, Vol. 39, No. 1/2, January/February 1991, p. 14

… The correct (and in initial years, the only) idiom is delta-sigma, coined and popularized by Inose and Yasuda. Candy, present at the origin of the Bell Labs variant "sigma-delta," has recently corroborated this [75], [76].

[75] M. W. Hauser, unpublished historical memoranda on delta-sigma modulation (1990)
[76] J. C. Candy, private communication (1990 Aug.)

http://archive.cs.uu.nl/pub/MIDI/DOC/oversampling

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From: prls!max@uwm.UUCP (Max Hauser)

  "A lot of things can be solved by the use of jargon - for example, the
 effort of thinking, or the danger of saying something that someone else
may not like.  You don't have to be clever, and you're always on the side
                                of whoever has the money or power ..."
                    -- Stanislav Andreski, author of "Social Sciences as Sorcery"
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"Delta-sigma" modulation and data conversion (the inventors' term) was ***unintentionally*** rechristened "sigma-delta" at the Bell Telephone Laboratories in 1963 and this reversal has propagated through many paper titles, so you will see both names in use.
No difference is intended.

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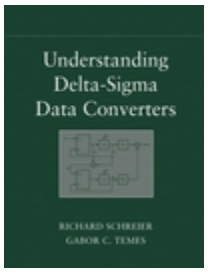

Understanding Delta-Sigma Data Converters
Richard Schreier, Gabor C. Temes
ISBN: 0-471-46585-2
Hardcover
464 pages
November 2004, Wiley-IEEE Press

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Although the basic idea of using feedback to improve the accuracy of data conversion has been around for about 50 years, the concept of noise shaping was probably first proposed (along with the name delta-sigma modulation) in 1962 by Inose et al.

It can be shown that the system of Fig. 1.4 can be obtained from that of Fig. 1.3 by cascading an integrator or summing block with the delta modulator. Hence, the structure of Fig. 1.4 came to be called a *sigma-delta* ($\Sigma\Delta$) *modulator*. Alternatively, one can observe the differencing at the input, followed by the summation in the loop filter, and hence call the structure a *delta-sigma* ($\Delta\Sigma$) *modulator*. Both terms have been used in the past to denote the first-order system of Fig. 1.4 with a single-bit quantizer. Other systems with higher-order loop filters, multi-bit quantizers, etc. are most properly called *noise-shaping modulators*, but it is common to extend the term $\Delta\Sigma$ modulator (or $\Sigma\Delta$ modulator) to these systems as well. This text follows the accepted usage.

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http://www.rmsinst.com/dt3.htm

SIGMA-DELTA A/D CONVERTERS - AUDIO AND MEDIUM BANDWIDTHS

Although now almost universally known as ***sigma-delta*** *converters*, these devices are still referred to by some manufacturers as ***delta-sigma*** *converters*. This term may be more appropriate in that, in its basic form, the converter consisted of a *delta modulator* (used for many years in telecommunications), where the input signal is added to the input of the integrator instead of its output (thus eliminating the need for an integrator at the receiver end) [1]. **Unfortunately, with time the words delta and sigma seem to have been interchanged arbitrarily.** Nowadays, a distinction seems to be made only when the relative position of the summing *(sigma)* block within the modulator is being emphasized. In addition to the above, **other terms** often found in the literature are *oversampling* and *noise shaping* ADCs.

The last two terms are in fact evocative of the two basic principles involved in the operation of sigma-delta ADCs - *oversampling and noise shaping*. The former spreads the quantization noise power over a bandwidth equal to the sampling frequency, which is much greater than the signal bandwidth. In the latter, the modulator behaves as a lowpass filter on the signal, and as a highpass filter on the noise, thus "shaping" the quantization noise so that most of the energy will be above the signal bandwidth. A digital lowpass filtering stage then greatly attenuates out-of-band quantization noise, and finally, downsampling brings the sampled signal to the Nyquist rate.

February 1996 (G. Noriega)

[1] J. Candy and G. Temes, "Oversampling methods for A/D and D/A conversion," in *Oversampling Delta-Sigma Data Converters*, pp.1-25, IEEE Press, 1992.

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http://www.beis.de/Elektronik/DeltaSigma/DeltaSigma.html

Delta Sigma Converters or Sigma Delta Converters?

Mankind does not seem to agree on one notation. Both notations are used equally often when you *search* via Google.

I decided to stay with that guy who told he is living in the Mississippi Delta, so deltas mean something to him - and for him only the Sigma River may have a Sigma Delta... good point.

Later I found that the *original* name "delta sigma" was coined by the inventors Inose and Yasuda and sigma delta is actually not correct. I was in luck..

June, 1st, 2005 (Uwe Beis)