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Advanced A/D and D/A Conversion Techniques and Their Applications
&
7th European Workshop on ADC Modelling and Testing

Differential Non-linearity, Integral Non-linearity, and Signal to Noise Ratio of an Analog to Digital Converter

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4. A simple Analytic Model on INL and ~~SNR~~

SFDR (THD)

In this section, an analytical model for the INL is proposed. A group of cubic curves are considered to represent the transfer functions of ADC. The function of the curve is given by $y = ax^3 + bx^2 + cx + d$. Using the property that the transfer function of an ADC has a full scale range of 2, three points contained in the curve are: $(-1, -1)$, $(0, 0)$ and $(1, 1)$. Hence, the function is $y = ax^3 + (1-a)x$. By changing the value of the coefficient 'a', we can obtain a group of curves with different features.

Cubic BOW (S curve)

endpoint INL (offset and gain NULLIFIED)

According to the definition of INL, it is the maximum deviation of the curve from the line, $y=x$. The deviation between the curve and the line, $y=x$, can be represented by $D(x) = x - (ax^3 + (1-a)x) = ax - ax^3$, and a local maximum happens when $D'(x) = 0$, i.e., $x = 3^{-1/2}$. Therefore, at that point we have $\mathrm{INL} = a\,(3^{-1/2} - 3^{-3/2})$.

@ **n-bit** ADC:

$$\max INL_{LSB} = \frac{a \cdot \frac{2}{3 \cdot \sqrt{3}}}{(2/2^n)}$$

Suppose a sine wave, $x = \sin(\omega t)$, is digitized by an ADC with the transfer function $y = ax^3 + (1-a)x$, then the output is

$$\begin{aligned} y &= a\sin^3(\omega t) + (1-a)\sin(\omega t) \\ &= (1-0.25a)\sin(\omega t) - 0.25a\sin(3\omega t) \qquad (2) \end{aligned}$$

From Equation 2, the SNR of output is

SFDR$_{dB}$

$$\begin{aligned} SNR &= 20\log_{10}[(4-a)/a] \\ &= 20\log_{10}(1/HD3) \\ &\approx 20\log_{10}(2^n / \max INL_{LSB}) \end{aligned}$$

$$HD3 = \frac{a}{4-a} \approx \frac{a}{4} \qquad a \ll 1$$

3rd order Harmonic Distortion

Example: **n** = 10 bits, **maxINL_{LSB}** = 1 → **$SFDR_{dB}$** ≈ 60.

Note: $SQNR_{dB} = 6.02 \cdot n + 1.76 \approx 6 \cdot n + 1.8$

@ **2^m** point *FFT*: $SNR_{dB} = NoiseFloor_{dB} - 3 \cdot (m-1)$

INLvsSFDR.mcd
(Mathcad)

Nº of bits: $n := 10$ $\Delta := \frac{2}{2^n}$

cubic BOW (S curve):

$a := 0.0051$ $y(x) := (1-a)\cdot x + a\cdot x^3$ $INL(x) := \frac{y(x)-x}{\Delta}$ $maxINL := \frac{2\cdot a}{3\cdot\sqrt{3}}\cdot\frac{1}{\Delta}$

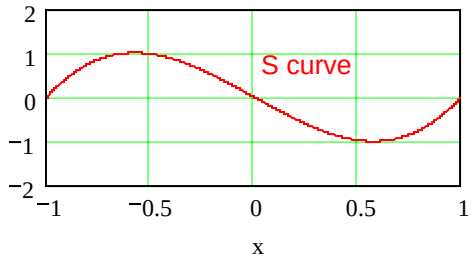
sine input, 4K FFT: $m := 12$ $S := 2^m$ $i := 0..S-1$

maxINL= 1.005

$L := 201$ $x_i := \sin\left(2\cdot\pi\cdot\frac{L}{S}\cdot i\right)$

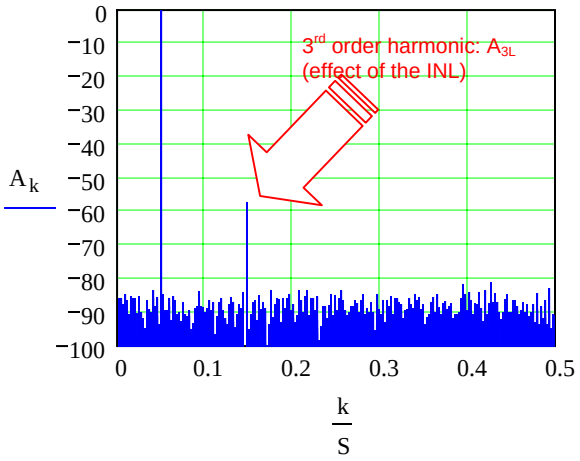
$Qy_i := \Delta\cdot\text{round}\left(\frac{y(x_i)}{\Delta}\right)$

$c := FFT(Qy)$



$M := last(c)$ $k := 0..M$ $dBV_k := 20\cdot\log\left(\sqrt{2}\cdot|c_k| + 10^{-10}\right)$

$A_k := dBV_k - dBV_L$ $A_L = 0$ $A_{3\cdot L} = -57.734$



THD:

$THD := 20\cdot\log\left(\frac{a}{4-a}\right)$

THD = –57.879

SFDR:

$SFDR := 20\cdot\log\left(\frac{2^n}{maxINL}\right)$

SFDR = 60.162

Noise Floor: $NFl := \frac{1}{M-2}\cdot\left[\left[\sum_{k=1}^{M-1} 2\cdot\left(|c_k|\right)^2\right] - 2\cdot\left[\left(|c_L|\right)^2 + \left(|c_{3\cdot L}|\right)^2\right] + \left(|c_M|\right)^2\right]$

$NoiseFloor := 10\cdot\log(NFl) - dBV_L$ NoiseFloor = –95.122

SNR:

$SNR := -NoiseFloor - 3(m-1)$

SNR = 62.122

SQNR:

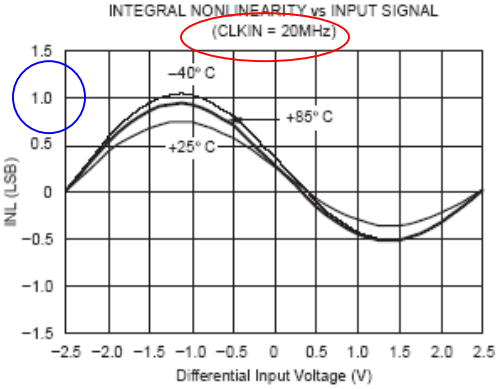
$SQNR := 6.02\cdot n + 1.76$

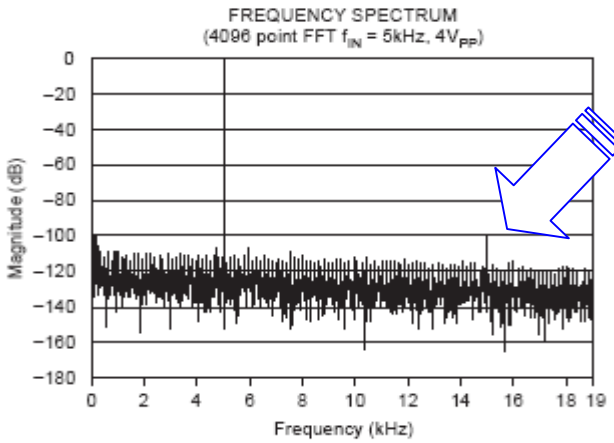
SQNR = 61.96

ADS1204: 16-bit resolution, 14-bit linearity

TYPICAL CHARACTERISTICS

AV_{DD} = 5V, BV_{DD} = 3V, CH x+ = +0.5V to +4.5V, CH x− = +2.5V, REFIN = external, CLKSEL = 0, and 16-bit $Sinc^3$ filter, with OSR = 256, unless otherwise noted.





dominant 3rd harmonic (HD3)

ELECTRICAL CHARACTERISTICS

Over recommended operating free-air temperature range at −40°C to +85°C, AV_{DD} = 5V, BV_{DD} = 3V, CH x+ = 0.5V to 4.5V, CH x− = 2.5V, REFIN = REFOUT = internal +2.5V, CLKIN = 20MHz, and 16-bit $Sinc^3$ filter with decimation by 256, unless otherwise noted.

PARAMETER		TEST CONDITIONS	ADS1204I MIN	ADS1204I TYP(1)	ADS1204I MAX	UNITS
Resolution			16			Bits
DC Accuracy						
INL	Integral linearity error(2)			±1	±3	LSB
				±0.001	±0.005	% FSR
	Integral linearity match				6	LSB
					0.009	% FSR

ELECTRICAL CHARACTERISTICS

Over recommended operating free-air temperature range at −40°C to +85°C, AV_{DD} = 5V, BV_{DD} = 3V, CH x+ = 0.5V to 4.5V, CH x− = 2.5V, REFIN = REFOUT = internal +2.5V, CLKIN = 20MHz, and 16-bit $Sinc^3$ filter with decimation by 256, unless otherwise noted.

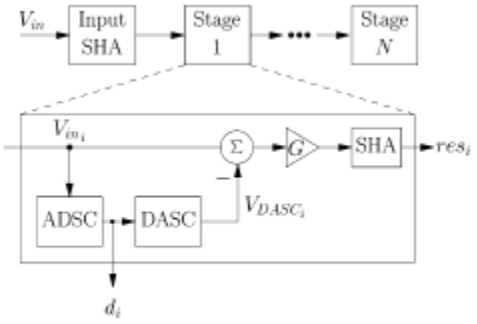
PARAMETER		TEST CONDITIONS	ADS1204I MIN	ADS1204I TYP(1)	ADS1204I MAX	UNITS
Resolution			16			Bits
AC Accuracy						
THD	Total harmonic distortion	V_{IN} = ±2Vpp at 5kHz		−96	−88	dB
SFDR	Spurious-free dynamic range	V_{IN} = ±2Vpp at 5kHz	92	100		dB
SNR	Signal-to-noise ratio	V_{IN} = ±2Vpp at 5kHz	86	89		dB
SINAD	Signal-to-noise + distortion	V_{IN} = ±2Vpp at 5kHz	85	89		dB
	Channel-to-channel isolation(3)	V_{IN} = ±2Vpp at 50kHz		85		dB
ENOB	Effective number of bits		14	14.5		Bits

$$20 \cdot \log\left(\frac{2^{14}}{1}\right) = 84.288$$

$$20 \cdot \log\left(\frac{2^{16}}{1}\right) = 96.33$$

A 12-bit 80-MSample/s Pipelined ADC With Bootstrapped Digital Calibration

Carl R. Grace, Paul J. Hurst, *Fellow, IEEE*, and Stephen H. Lewis, *Fellow, IEEE*



Block diagram of pipelined ADC.

Each stage except the last contains an analog-to-digital subconverter (ADSC), a digital-to-analog subconverter (DASC), an analog subtracter, and a SHA with gain G.

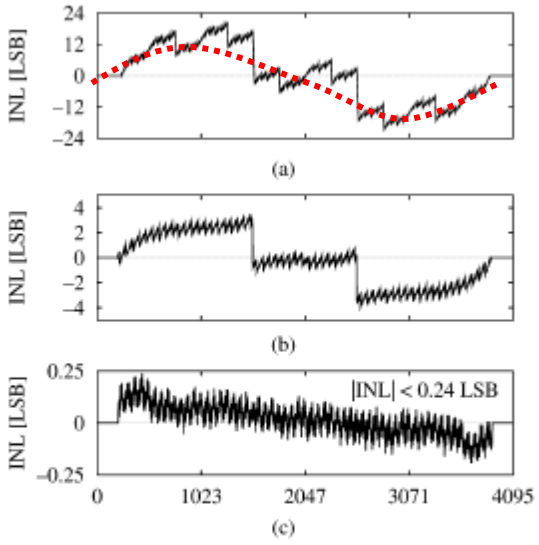


Fig. 14. INL with foreground calibration: (a) with no calibration, (b) with constant-gain calibration only, and (c) with full calibration. Sampling rate is 80 MS/s.

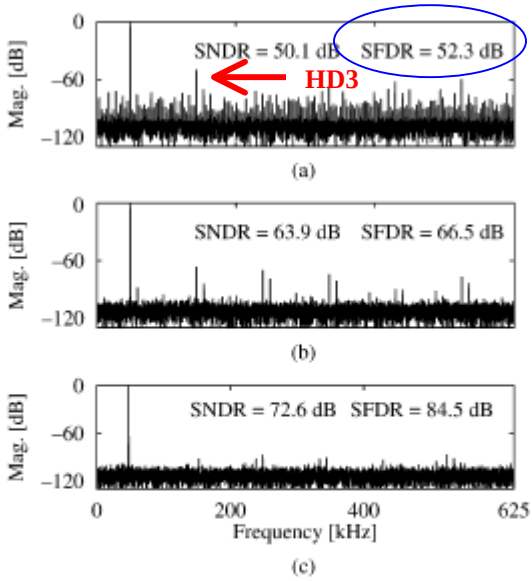


Fig. 15. ADC output spectra with foreground calibration: (a) with no calibration, (b) with constant-gain calibration only, and (c) with full calibration. Sampling rate is 80 MS/s. Output downsampled by factor of 64.

$$20 \cdot \log\left(\frac{2^{12}}{13}\right) = 49.97$$