Putting Undersampling to Work

Aliasing

To avoid aliasing, we know from the Nyquist criterion that we must digitize a signal at a sampling rate of at least twice the bandwidth of the signal. For 'baseband' signals with frequency components starting at DC and extending up to some maximum frequency, this means the sampling rate must be at least twice this maximum frequency. In this case, the bandwidth and the maximum frequency are the same.

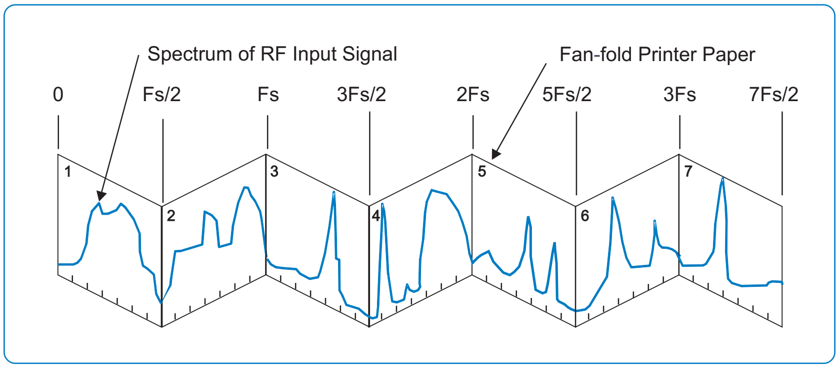

Figure 1. Fan-fold paper showing the spectrum of a RF input signal.

Bandpass Signals

Let's consider 'bandpass' signals like the IF (intermediate frequency) output from a standard communications receiver with a 70 MHz center frequency and a 5 MHz bandwidth, for example.

Does this mean that we can digitize this 70 MHz bandpass signal with an A/D operating at twice the bandwidth, or just above 10 MHz? If so, this can be a big benefit compared to baseband sampling, where we would otherwise need to use an A/D converter with a sampling rate of about 150 MHz. This is the basic mission of 'undersampling'.

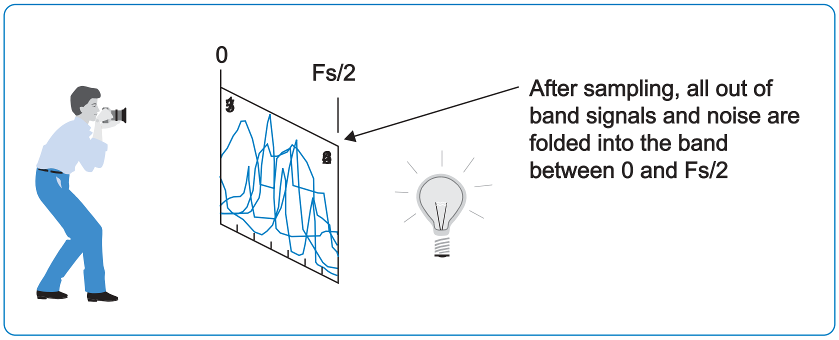

Figure 2. Looking through the collapsed stack reveals the resulting spectrum.

Undersampling

In order to apply undersampling successfully, a careful frequency plan must be developed. One tried and true technique is the 'fan-fold' paper method. Start with a small stack of semitransparent fan-fold computer printer paper, or the imaginary equivalent. Holding the paper with the folds in the vertical direction, plot the frequency axis from left to right along the bottom edge with the inward creases at multiples of the A/D sampling frequency, Fs, and the outward creases at odd multiples of Fs/2, as shown in Figure 1.

The vertical axis is used to plot the spectral amplitude of the signal, such as wideband RF signal shown. In order to see what happens after sampling, simply collapse the stack of fanfold paper, hold it up to a light and look through the stack. You'll see the spectra from all sheets superimposed on top of each other, which represents the exact frequency content in the A/D output samples. As shown in Figure 2, signals on all of the sheets above Fs/2 are effectively "folded" down into sheet 1 between 0 Hz and Fs/2.

For the signals on every odd numbered sheet, the effect is a frequency translation by a multiple of Fs. For the signals on even numbered sheets, there is a reversal of the frequency axis on that sheet, followed by a translation by an odd multiple of Fs/2. Again, this is much easier to follow by visualizing the fan-fold model.

Choose the Sampling Rate

We can take advantage of this model for undersampling bandpass signals by a careful choice of the sampling rate based on the frequencies present in the band. Suppose all of the frequencies in the bandpass signal fall on a single sheet of fan-fold paper as shown in Figure 3.

In this case, after sampling, all of the signal energy on sheet 5 will fold down onto sheet 1 and be represented in the output sample stream as if it were a baseband signal between 0 and Fs/2. As shown in Figure 4, the undersampling process results in a downward frequency translation by 2·Fs, with no spectral reversal. If the bandpass spectrum of the input signal rested entirely on sheet 4 instead, the frequency axis would be reversed and then translated down by 3·Fs/2.

The key rule for undersampling using the fan-fold model is simple: choose the sampling rate, Fs, so that the entire band of the bandpass signal falls on a single sheet. Depending on the odd or even number of that sheet, the frequency axis of the resulting sampled spectrum will either be normal or reversed, respectively.

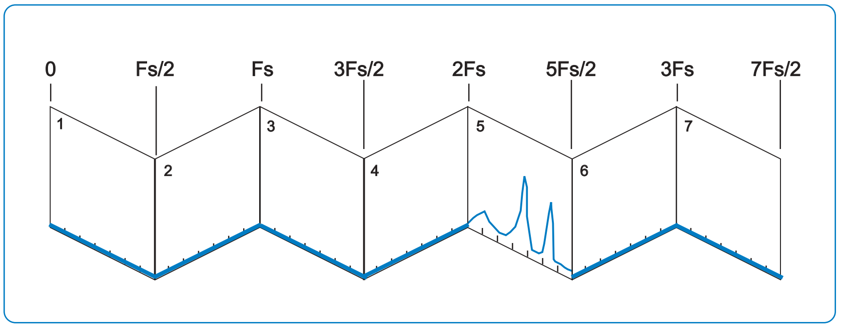

Figure 3. Fan-fold paper showing the spectrum of a bandpass signal such a an IF output.

There are usually several different sample clock frequencies that will work for undersampling. While this model can show all of the correct frequency plans, the best choice will usually be determined by several other important practical considerations:

- Some A/D converters are specifically characterized for undersampling applications, while others are designed only for baseband sampling. Check the manufacturer's specifications carefully.
- The analog signal path of the A/D converter must handle the input frequencies of the bandpass signal with minimum distortion and noise. In this case, a transformer-coupled input stage is often the best answer.
- The quality of the sample-and-hold amplifier at the front end of the A/D converter becomes more critical at higher bandpass input frequencies. Often, an additional external high-performance sample-and-hold will be necessary.
- Any out of band signals or noise must be kept to a minimum because they will fold down into the output spectrum, exactly as shown in Figure 1. An additional input bandpass filter can help reduce these components.
- Jitter and phase noise of the sample clock signal can seriously degrade undersampling performance. Use of a high-quality crystal oscillator with simple, direct connections to the A/D converter is extremely helpful.

Undersampling can be an extremely valuable tool for software radio applications, but care must be taken to ensure good performance. ❑

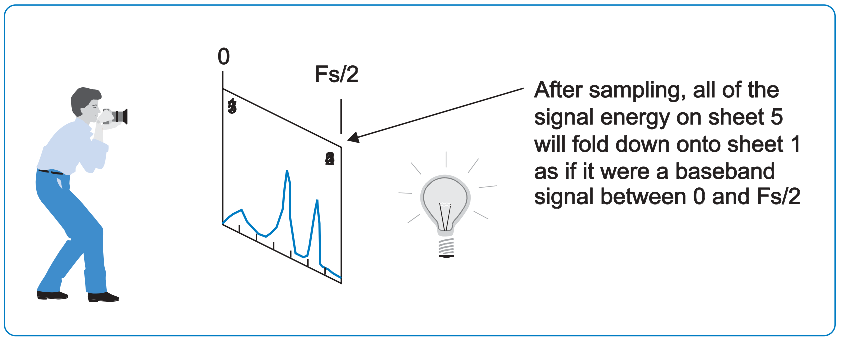

Figure 4. The proper choice of sampling frequency translates the spectrum of the bandpass signal down to baseband.

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