A 3.3-V 12-b 50-MS/s A/D Converter in 0.6-μm CMOS with over 80-dB SFDR

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II. SFDR: Fundamentals

An n-bit ideal quantizer exhibits a sawtooth error characteristic. With the FS input amplitude normalized to one (FS/2 = 1), the periodic error function is parameterized with its frequency $\omega_x = 2\pi/\text{LSB} = 2^n\pi$, where the least significant bit (LSB) represents the quantization step. The error distorts an FS input sinewave and creates wideband harmonics, with significant spectral energy up to the order of ω_x. The Fourier series of either the quantized sinewave or the periodic sawtooth error function lead to closed-form expressions for the harmonics [8]–[11] and plots of distortion spectra as shown in Fig. 1(a). The plots for various n show that the largest harmonic is located near ω_x, and is about $9n$ dB below the fundamental, that is

$$k_{\max} \approx \omega_x = 2^n\pi \tag{1}$$

and

$$\text{SFDR} \approx 9n - c \quad \text{(dB)} \tag{2}$$

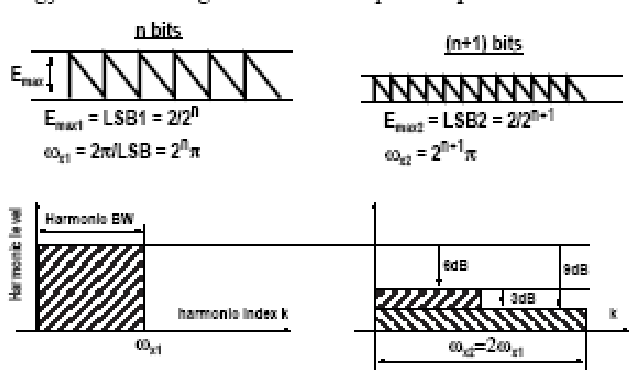
@ highly oversampled case (!!)

where $k_{\max}$ is the index of the largest harmonic and the offset c ranges from 0 for low resolutions to 6 for high resolutions [10].

Though strict validation of these empirical equations is mathematically interesting, it gives better insight to derive (2) from energy conservation. As n increases by one, the quantization error is halved in amplitude, and the total error energy $\text{LSB}^2/12$, which is *asymptotically independent* of the signal distribution [12]–[15], decreases by 6 dB. This leads to the signal-to-noise-and-distortion ratio (SNDR)

$$\text{SQNR}_{\max}\ (=\text{SINAD}) = \text{SNDR} = 6n + 1.76 \quad \text{(dB)}. \tag{3}$$

Also, now the error sawtooth at double the frequency produces twice as many important harmonics, so the overall spur level must go down by an additional 3 dB to keep the total harmonic energy unchanged, resulting in the $9n$ term in (2). This suggests that the key to high SFDR should be to spread a given error energy across as large a number of spurs as possible.



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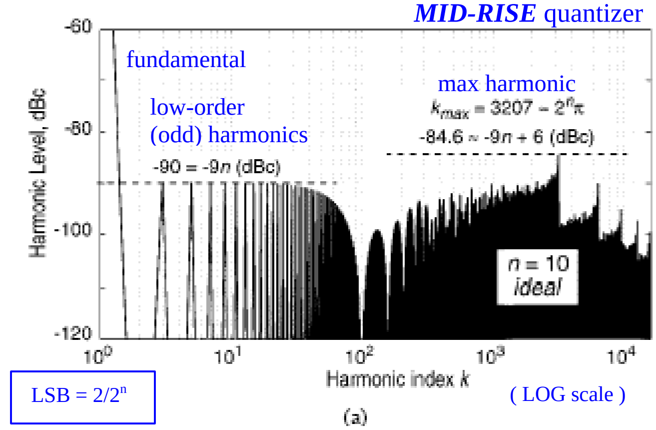
MID-RISE quantizer



(a)

Fig. 1. Spectra of quantized sinewave. (a) Ideal 10-b quantizer.

Unlike the asymptotically constant quantization error energy, spurs are sensitive to the waveform being quantized. For example, an ideal quantization of an FS sawtooth input signal leads to a sawtooth error waveform which contains spurs much higher than what is given by (2); a tiny deviation (~LSB/4) from FS amplitude may null some low-order harmonics. The largest spur rises by 3 dB for each 6-dB reduction in input amplitude, because the input signal sees half of the error sawtooth spreading the same amount of spur energy. This contradicts the "common sense" that nonlinearity improves as signal decreases in power.

As a result, small-signal SFDR gets worse and a few bits of margin is required over what (2) specifies to determine the resolution. In addition, the spurs can be drastically affected by the noise (dither) at the input and the sampling frequency. Those details are discussed in [11].

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Spectra of quantized sinusoid
in **highly oversampled** case

aquant2.mcd
(*Mathcad*)

No. of samples: $S := 2^{16}$ $\quad i := 0..S-1$

No. of bits: $n := 10$ $\quad \Delta := \dfrac{2}{2^n}$

Prediction of max spur **location**:
(harmonic number, @ L = 1)

$F := 2^n \cdot \pi$

Prediction of max spur **level (dBc)**:

$SFDR := -9 \cdot n + 6$

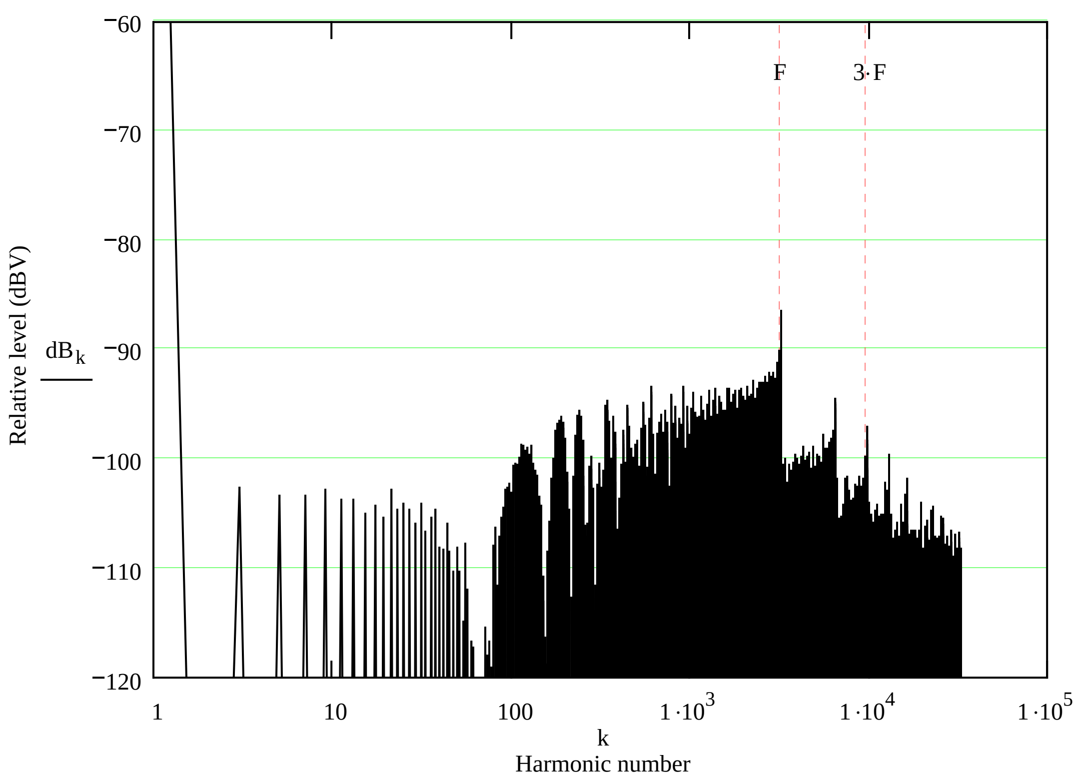
Signal (single-tone, fundamental):

location	sampling	quantization (**midtread**)
$L := 1$	$x_i := 1 \cdot \sin\left(2 \cdot \pi \cdot \dfrac{L}{S} \cdot i + 0\right)$	$Qx_i := \Delta \cdot \mathrm{round}\left(\dfrac{x_i}{\Delta}\right)$

Spectrum:

$c := FFT(Qx)$ $\quad M := \mathrm{last}(c)$ $\quad k := 0..M$ $\quad dB_k := 20 \cdot \log\left(\sqrt{2} \cdot |c_k| + 10^{-10}\right)$

$M = 32768$ $\quad dB_L = -3.01$



Level of **max spur**: $\mathrm{reverse}(\mathrm{sort}(dB))_1 - dB_L = -83.355$

prediction: $SFDR = -84$

Spectral Spurs due to Quantization in Nyquist ADCs

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There is some prior literature on this subject; however, none of it satisfactorily addresses the problem posed above. Most previous studies deal with the statistical and asymptotic aspects of the quantization errors [10]–[14], without much attention to the spectra of quantized *periodic* signals. The few exceptions [15]–[19] involve intensive mathematics, at times even ingenious technique, but are of little value to the circuit designer of practical integrated ADCs. In this paper, we couple analysis with simulation of ADC spectra on MATLAB, which leads to ready visualization of the spectral signatures associated with quantization imperfections. Then aided by an understanding of the underlying physical phenomena, we seek practical solutions.

In this spirit, we offer an intuitive yet quantitative understanding of an ADC's spectral properties.

To begin with, Section II analyzes the spectra of ideally quantized sinewaves and considers the signal dependence of the spectra. Sinewave inputs to the ADCs are given special attention, not only because they are the basis of all periodic signals, but because full-scale (FS) sinewaves are used in the definition of SFDR, a critical specification for radio receiver applications

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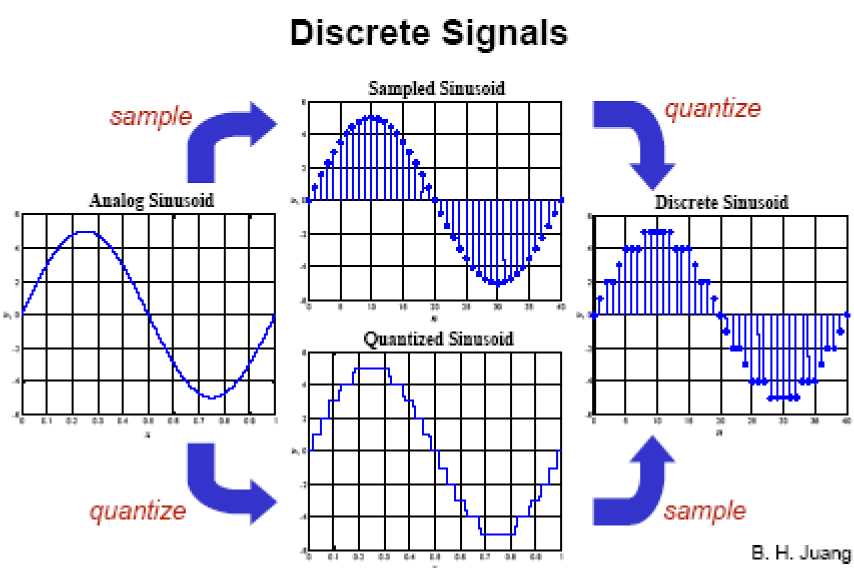
II. Spectra of Quantized Sinewaves

ADCs usually consist of a sampler before the quantizer, as shown in Fig. 1(a). For the purposes of analysis, the actions are commutative, that is, it does not matter if the ADC quantizes a signal before sampling [17], as shown in Fig. 1(b). With the order reversed, we need only focus on the quantization of continuous input signals and then impose the well-known aliasing effects. Therefore, we will use the terms "ADC" and "quantizer" interchangeably.

We start by describing the output spectrum of an ideal quantizer that is digitizing a FS sinewave. This is a sinewave whose peaks reach the extreme thresholds of the ADC. An n-bit quantizer is called ideal in this paper when its $N = 2^n$ output levels $y_i, i = 1, 2, \ldots, N$, are centered between adjacent quantization thresholds x_i and x_{i+1}, which themselves are uniformly distributed over the input FS, with equal step size $s_i = x_{i+1} - x_i = \mathrm{FS}/N$. The difference between the quantizer input–output characteristic $y = q(x)$ (Fig. 1) and the straight line $y = x$ defines the quantization error $\varepsilon(x)$. For an ideal quantization staircase



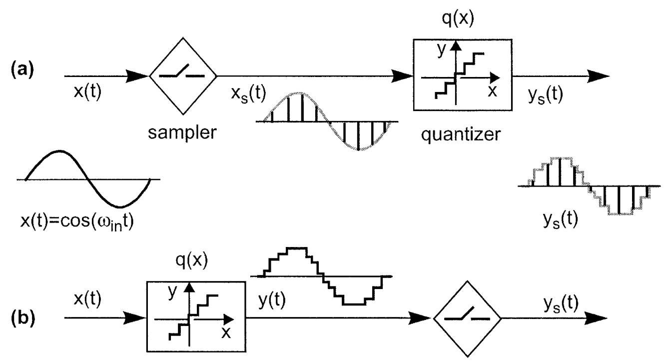


Fig. 1. ADC modeled as (a) a sampler preceding a quantizer or (b) a sampler following a quantizer [17].

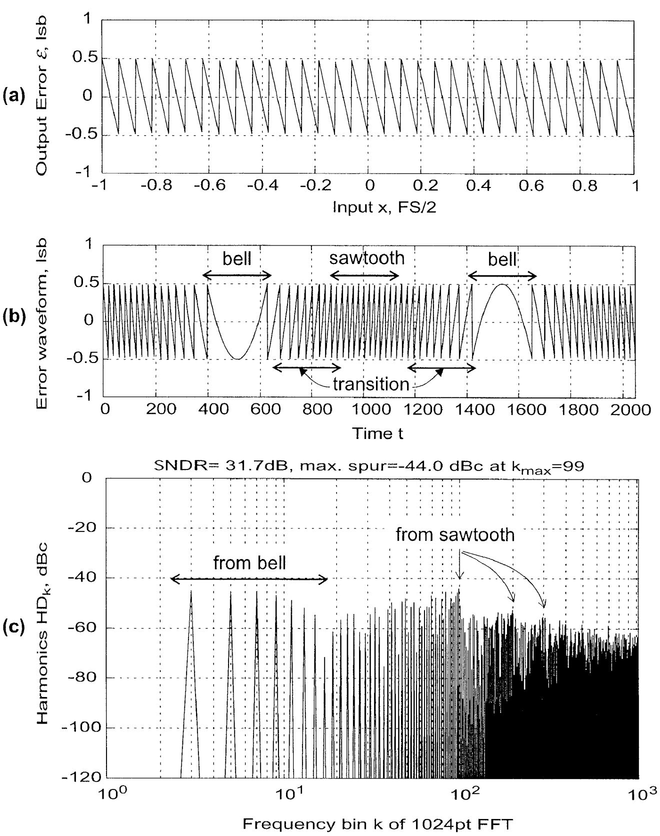


Fig. 2. A 5-bit quantizer ($n = 5$) digitizing a FS sinewave, simulated in MATLAB. (a) Sawtooth input–output error characteristic. (b) Error waveform. (c) Output spectrum.

function, the error is a sawtooth. The error in x_i is expressed in units of the least significant bit $(\mathrm{LSB}) = \mathrm{FS}/N$ as integral nonlinearity (INL). In this paper, we assume $\varepsilon(x)$ is on the order of one LSB.

Before going into details of the spectral content, let us first summarize the asymptotic properties of quantization error.

A. *Asymptotic Properties of Quantization*

An n-bit ideal quantizer divides the signal FS into 2^n uniform bins. Over time, a signal spanning the FS lands in each bin with some probability density function (PDF). Accruing the PDFs over the $N = 2^n$ identical bins into a distribution over a single bin, $[-\mathrm{LSB}/2, +\mathrm{LSB}/2]$, yields the PDF of the quantization error ε [13]. As long as the signal PDF is continuous over a given FS—which is almost always true for real-life signals—the individual and aggregate PDFs both approach uniform distribution as the resolution n goes to infinity (see [12, p. 21] for a derivation). The uniform distribution of ε over $[-\mathrm{LSB}/2, +\mathrm{LSB}/2]$ results in the well-known mean-square quantization error, $\mathrm{LSB}^2/12$, asymptotically independent of the signal. For an FS sinusoid, the signal power is given by $(\mathrm{FS}/2)^2/2$, where $\mathrm{FS} = 2^n \times (\mathrm{LSB})$. Divided by the quantization error power, this leads to an expression for the signal-to-noise-and-distortion ratio (SNDR)[1] given as follows:

$$\mathrm{SNDR} \approx 6.02n + 1.76\,(\mathrm{dB}). \tag{1}$$

Equation (1) is usually derived assuming that the "quantization noise" uniformly distributes over $[-\mathrm{LSB}/2, +\mathrm{LSB}/2]$ [10], [11]. However, this assumption is true under certain statistical conditions [13] or asymptotically for very large n. A periodic signal passing through the quantization staircase actually suffers deterministic distortion, which appears in the output spectrum as harmonics, not random noise. We will discuss this next.

B. *Error Waveform and Spectrum*

Fig. 2 shows the quantization error as a function of input x and time t and the corresponding spur spectrum for a 5-bit ideal quantizer digitizing an FS sinusoid. The spectrum is obtained numerically by the (long-length) fast Fourier transform (FFT) of the quantized output or analytically by a Fourier series expansion, which leads to the spur amplitude expressed in terms of Chebyshev polynomials or Bessel functions [17]–[19]. The error waveform in Fig. 2(b) can be divided into three portions: sawtooth, bell, and transition. The sawtooth portion arises from quantization around the zero crossing, where the sinewave is ramp-like. More precisely, this is the region where the input sinewave $x' = (\mathrm{FS}/2)\sin(\omega_{\mathrm{in}}t)$ stays within $\pm\mathrm{LSB}/2$ of the linear ramp $x = (\mathrm{FS}/2)\omega_{\mathrm{in}}t$, that is,

$$\begin{aligned} |x' - x| &= \left|\left(\frac{\mathrm{FS}}{2}\right)\sin\left(\frac{2x}{\mathrm{FS}}\right) - x\right| \\ &\approx \left(\frac{2}{\mathrm{FS}}\right)^2\left|\frac{x^3}{6}\right| < \frac{\mathrm{LSB}}{2} \Rightarrow |x| < \frac{0.91}{(2^{n/3})}\mathrm{FS}. \end{aligned}$$

The period of this approximate sawtooth is $\mathrm{LSB}/((\mathrm{FS}/2)\omega_{\mathrm{in}}) = T_{\mathrm{in}}/(2^n\pi)$, where $T_{\mathrm{in}} = 1/f_{\mathrm{in}}$ is the period of the input sinewave. An ideal sawtooth waveform of this period corresponds to a series of harmonics in the spectrum with the fundamental located at $(2^n\pi)f_{\mathrm{in}}$. The sawtooth portion of the error repeats every cycle of the sinewave, resulting in spectral energy at the sawtooth fundamental and its harmonics, surrounded by skirts of lower tones spaced by f_{in}. As is shown in Fig. 2(c), the sawtooth fundamental constitutes the highest spur whose harmonic index is given by

$$k_{\max} \approx 2^n\pi. \tag{2}$$

[1]This is referred to as SNR in the literature where quantization errors have been overwhelmingly treated as noise, but we think SNDR is a better term, especially in this paper which deals with quantization spurs.

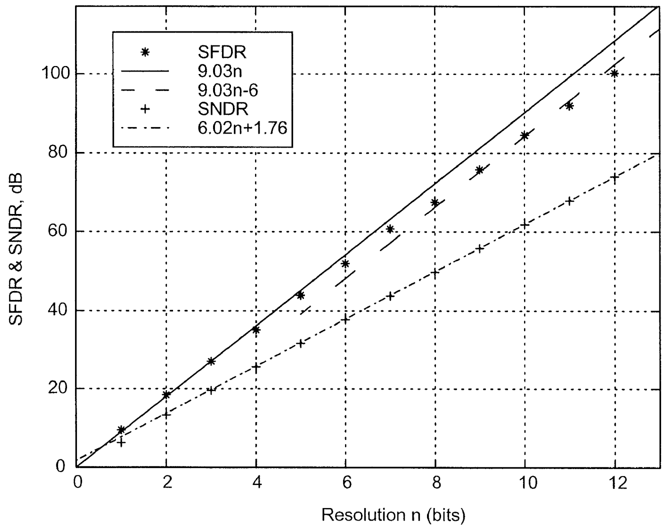


Fig. 3. SFDR and SNDR versus resolution n for ideal quantizers digitizing an FS sinusoid, simulated in MATLAB.

Quantization at the peaks of a sinewave produces errors in the form of bell-like pulses. The pulses are periodic at the sinewave frequency and therefore contribute low-order harmonics. Since the error pulses are small and narrow compared to the period, the corresponding harmonics are low and flat in the spectrum, resembling the spectrum of a train of impulses. It has been shown that, as n increases, these harmonics at low indexes approach a level of $-9.03n$ (dB) relative to the fundamental of the quantized sinewave at 0 dBc [17], [19]. Finally, the transition region in the error curve between the sawtooth and the bell induces a wide band of harmonics that fill in the frequencies between the low-index harmonics and the high-index peaks. Fig. 2(c) shows the signature spurs corresponding to the three portions of quantization error.

C. SFDR and Energy Conservation

Single-tone SFDR is usually defined as the difference in decibels (dBc) between the fundamental and the largest spur of a quantized sinewave. By default, we assume that the input sinewave covers the ADC FS and the SFDR is simply specified in decibels. Fig. 3 plots the numerically simulated SFDR of ideal quantizers digitizing an FS sinusoid with a resolution of n bits. For n below 4, the SFDR follows a $9.03n$ (dB) asymptote, where low-index harmonics dominate; for $n > 4$, it retreats to an asymptote of $(9.03n - 6)$ dB, where the high-index spurs dominate. On this basis, we postulate an expression [18] for SFDR as follows:

$$\mathrm{SFDR} = 9.03n + c(n)\ (\mathrm{dB}) \tag{3}$$

where $c(n)$, an empirical quantity, ranges from 0 to -6 over the span $n \in [1, 12]$.

The $9n$ term in (3) can be justified using energy conservation [19]. As ADC resolution n increases by one bit, the *amplitude* of $\varepsilon(x)$ drops by $2\times$ or 6 dB, as (1) also indicates. However, the sawtooth *periodicity* of $\varepsilon(x)$ doubles, which means that, according to (2), the index of the largest harmonic is pushed out $2\times$. Also, there are now twice as many harmonics that fill the gap between the fundamental and the largest harmonic. With half of the error distributed across twice as many harmonics, the height of each harmonic must go down, and therefore the SFDR rises by 9 dB. On the other hand, when the input amplitude halves, the asymptotic error power remains unchanged, but the periodicity of the error waveform goes down $2\times$ because the input sinusoid traverses half of the quantization thresholds. There are now half as many significant harmonics sharing the same amount of power as before. The spurs must rise by 3 dB, and the SFDR worsens by 3 dB FS (or 9 dBc). This trend is the opposite to what happens in continuous-time nonlinear systems, where we are accustomed to improvements in linearity with smaller inputs. This shows why quantizer SFDR must be specified as a function of the input magnitude.

D. Spectrum Aliasing

Sampling the quantized signal aliases high-order spurs into the Nyquist band $[0, f_s/2)$, where f_s is the sample rate. If the sample rate is an integer multiple of the input frequency, aliased spurs will coincide in frequency with unaliased low-order spurs and add to them as phasors, worsening the SFDR. This can confuse the interpretation of our numerical experiments, which is why we use a very large number of points in numerical FFTs—up to 16×2^n—and place the input sinusoid in the lowest FFT frequency bin. In ADC testing, the input frequency is usually chosen to lie in a prime numbered bin and the number of FFT points is set to some power of two. In real applications, careful frequency planning makes it unlikely that harmonics will clump together.

E. Signal Dependence

Equations (2) and (3) would not be very useful if they predict a peak spur which is sensitive to small perturbations in the phase, offset, or amplitude of the input sinusoid. Let us examine these one by one. Phase shift in the sinusoid has no effect on the spur spectrum, except to induce an identical phase shift in the output. On the other hand, as the input amplitude falls below FS, the bell portion of the error waveform shrinks in width and magnitude, creating smaller harmonics at low indices [19], [33]. If the input is offset from a zero baseline, it breaks the odd symmetry in the quantization error, causing even harmonics to appear. Although the average spur level goes down by up to 3 dB because now there are roughly as many even harmonics as there are odd, the maximum spur, which arises from the sawtooth portion of the error waveform, remains almost unchanged.

Quantization of nonsinusoidal inputs may create markedly different patterns of spurs. It is impossible to analyze all possible signals, but we should be able to construct some worst-case signals for which the ADC output error energy, which is asymptotically $\mathrm{LSB}^2/12$ for ideal quantization, collapses onto just a few dominant spurs that, for reasons of energy conservation, should be of the order $-6n$ (dB FS).

One such case is a periodic sawtooth input, whose quantization error waveform is also a sawtooth. The spurs consist *only* of the harmonics of the sawtooth error waveform. A second case is that of a small input sinewave whose amplitude is lower than one LSB. Now the quantizer degenerates into a one-bit slicer, whose entire error waveform is a square wave toggling between $-\mathrm{LSB}/2$ and $+\mathrm{LSB}/2$.

Two equal tones closely spaced in frequency create a sinewave fully modulated in amplitude at the beat frequency. As the sawtooth portion of the error waveform shrinks with the modulated amplitude, the bell-like portion expands. As a result, error energy moves from high-index to low-index spurs and simulations show that the SFDR in dB FS is not much affected. Generally, the signal that consists of M ($>$ 3) tones evenly spaced at a small frequency step and identical in amplitude and phase is a sinewave at the center frequency modulated by sharp pulses which are M times the magnitude of the individual tones and separated by ($M - 2$) small ripples of amplitude comparable to the individual ones. When $2 \times M$ exceeds the number of quantization thresholds, N, that are distributed over the full range of the multitone signal, the ripple amplitude reduces below one LSB, causing the aforementioned worst-case spurs.

In some instances, this dependence of the quantized spectrum on input waveform can be used to advantage. As we will describe in Section IV, noise accompanying the signal [13] scrambles the error waveform and converts the spurs into a near-continuous spectrum resembling white noise

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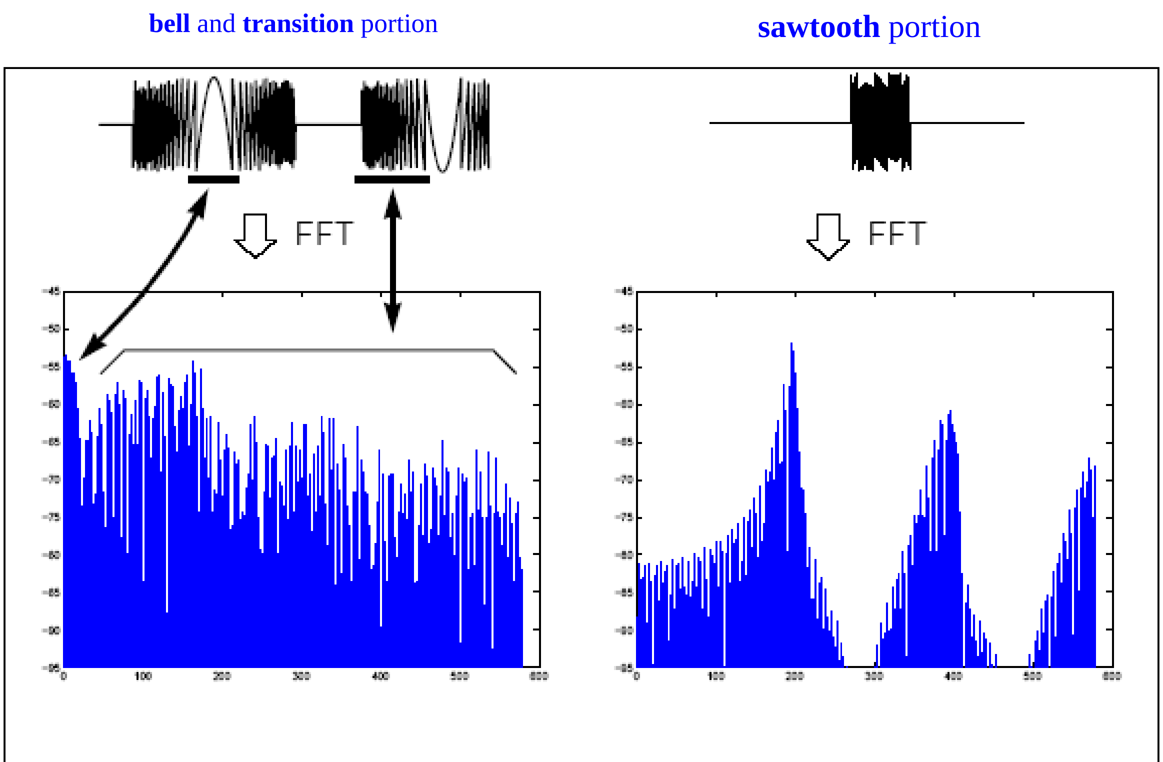

Decomposed error waveforms and spectra (n=6).