Bandpass Sampling for Software Radio

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ABSTRACT

The principal idea behind the design of a software radio is to place the analog-to-digital and digital-to-analog converters as near to the antenna as possible, such that most of the radio functionalities can be implemented on a digital signal processor. One way to achieve this is by direct bandpass sampling of the desired RF signal band to baseband frequency. However, the design of a software radio receiver becomes more complicated when signals are to be received from multiple distinct RF bands. The traditional approaches for this case have been to bandpass sample a continuous span of spectrum containing all the desired RF signals. The disadvantage with this approach is that the required sampling rate depends upon the span of spectrum, instead of the information bandwidths of the signals. In this paper, we present an efficient algorithm to compute the minimum bandpass sampling frequency for direct downconversion of two distinct RF signal bands simultaneously. The novelty of this algorithm is that, at each iteration, it not only checks for a valid sampling frequency, but also determines the next possible minimum value of sampling frequency.

INTRODUCTION

The design of a software radio is based on two simple design goals [1]. First, the analog-to-digital converter (ADC) should be placed as near to the antenna as possible in the chain of RF front-end components. Second, the resulting samples should be processed *softly* in a reconfigurable digital domain using digital signal processors or field programmable gate arrays [2]. Generally, a RF front-end consists of multiple stages of amplification, filtering, and downconversion to process a single RF transmission, whereas in the direct digitization configuration, the RF signal is sampled directly without any downconversion. However, for most radio applications, the required sampling rate for direct downconversion would be impractically high if Nyquist sampling [3] is employed. One alternative could be to include multiple frequency translation stages, however, that would add additional hardware between the antenna and ADC. The other alternative is the utilization of bandpass sampling. Bandpass sampling is a special form of undersampling that translates a high frequency bandpass signal to baseband frequency [4]. The required sampling frequency depends on the signal bandwidth, rather than on its highest frequency component. The main advantage of this is, therefore, the reduced requirement of the sampling frequency and of the associated signal processing capability.

In this regard, D. M. Akos et al. [2] proposed a method to compute the bandpass sampling frequency for direct downconversion of multiple RF signals. This method is, however, computationally intensive as it requires an exhaustive test of all frequencies up to the Nyquist rate, and at each such frequency we need to check $2N + {}^{N}C_2$ constraint equations. This computational complexity can be alleviated to a certain extent using the graphical approach of N. Wong et al. [5], where the search is restricted within the intersections of the valid ranges of sampling frequencies of individual signal band. C. H. Tseng et al. [6] proposed another method based on all the possible orders of spectral replicas of the sampled signal. However, the successful implementation of this algorithm lies in determining all the possible orders of spectral replicas, which varies as $2^N \times N!$ with the number of RF signals. In this paper, an efficient algorithm is proposed to compute the *minimum* sampling frequency for direct downconversion of two distinct RF signal bands. In this method, we start with the theoretical minimum sampling frequency. If it is not found to be a valid sampling frequency, then we determine the next minimum value of sampling frequency, based on the given band specifications, and repeat the check for validity.

PROPOSED APPROACH

In this section, we present an algorithm for determining the *minimum* bandpass sampling frequency for direct downconversion of two distinct RF signal bands. For this we consider the bandpass signals $f_i(t)$ (i = 1, 2… N), with f_{l_i}, f_{u_i} and B_i representing lower bound, upper bound and bandwidth of signal $f_i(t)$, respectively.

Algorithm of minimum f_s for two signal bands

The algorithm for two signal bands is as follows,

1. Select initial sampling frequency as $f_s = 2(B_1 + B_2)$.
2. Check out whether any integer multiple of the chosen $f_s/2$ falls within any of these two bands. If *yes*, then increase the sampling frequency by Δf_s and repeat this step, *otherwise* move on to next step.
3. Perform bandpass sampling operation of both the bands, with the chosen sampling frequency of step 2.
4. Check out whether the bands overlap over each other in the sampled bandwidth ($0 - f_s/2$). If *yes*, then again increase the sampling frequency by Δf_s and go back to step 2, *otherwise* the chosen sampling frequency represents one of the valid sampling frequencies for direct downconversion of two RF signals.

The efficiency of this algorithm is solely dependent on the value of Δf_s that needs to be chosen in step 2 and 4. Hence, we need to develop some analytical formulations to specify the values of Δf_s in step 2 and 4.

Determining Value of Δf_s for Step 2

In step 2, we actually try to find a sampling frequency such that none of the bands alias with itself. To avoid aliasing with itself each individual band needs to satisfy two constraint equations [2, Eq. (2), (3)]. We can combine these two constraints into a simple constraint as $rem(f_u, f_s/2) > B$. If this constraint is not satisfied then one of the integer multiple of the chosen $f_s/2$ would lie within the signal band, i.e., $f_1 < n f_s/2 < f_u$ where n is an integer. In such a case, we need to increase the sampling frequency to $f_s^{'}$ such that the nth multiple of this new $f_s^{'}/2$ would lie beyond f_u, i.e. $n f_s^{'}/2 \geq f_u$. This logic is schematically depicted in Fig. 1. For two-band case this can be written as

$$f_s + \Delta f_s \geq \max\left[\left(2f_{u_1}/m\right), \left(2f_{u_2}/n\right)\right] \tag{1}$$

where

$$m = floor\left[f_{u_1}/\left(f_s/2\right)\right],\ n = floor\left[f_{u_2}/\left(f_s/2\right)\right] \tag{2}$$

are two integer values. While computing the *minimum* sampling frequency we must use (1) with the equality condition.

Determining Value of Δf_s for Step 4

In step 4, we try to compute a sampling frequency such that the two bands do not overlap over each other within the sampled bandwidth ($0 - f_s/2$). For that, we need to consider eight different spectral orientations of these two bands, as presented in [6]. However, we group these orientations in four different subgroups depending on whether m and n are even or odd,

$$m = floor\left[f_{l_1}/\left(f_s/2\right)\right],\ n = floor\left[f_{l_2}/\left(f_s/2\right)\right] \tag{3}$$

When both m and n are even, there are two possible spectral orientations, as shown in Fig. 2. The alias version of signal $f_1(t)$ resides within (a, b), and that of signal $f_2(t)$ within (c, d) in the sampled bandwidth. For the orientation of Fig. 2(a), let us compute the band overlap Δ, at the chosen sampling frequency f_s, as

$$b = f_{u_1} - m\left(f_s/2\right),\ c = f_{l_2} - n\left(f_s/2\right) \quad \text{and} \quad \Delta = b - c$$

Now let us increase the sampling frequency to ($f_s + \Delta f_s$), and compute the band overlap Δ' as

$$\Delta' = b' - c' = \Delta + (n - m)\left(\Delta f_s/2\right)$$

As from (3) we have n >m, and the chosen value Δf_s must always be a positive quantity, we get

$$\Delta' > \Delta \tag{4}$$

Hence, band overlap increases for any increase of f_s for this spectrum orientation. Let us denote this condition as 'no further improvement' case. Similarly, for the band orientation of Fig. 2(b), we compute the band overlap Δ' as

$$\Delta' = d' - a' = \Delta - (n - m)\left(\Delta f_s/2\right) \tag{5}$$

To make the band overlap (Δ') zero, we need to choose the next modified sampling frequency as

$$f_s + \Delta f_s = \left(2/(n-m)\right)\left(f_{u_2} - f_{l_1}\right) \tag{6}$$

Performing similar analysis for other cases, we get eight different expressions for ($f_s + \Delta f_s$), all of which are tabulated in Table 1. The overlap conditions are expressed in terms of (a, b) and (c, d).

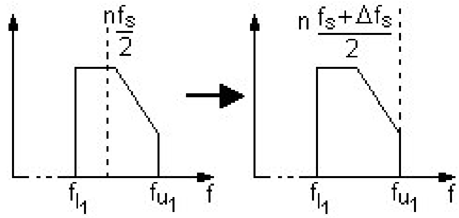

Fig. 1. Choosing Δf_s in step 2 for single band

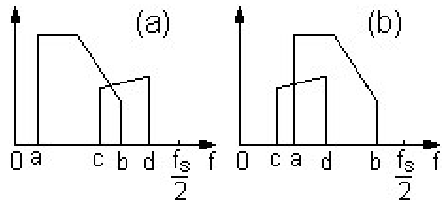

Fig. 2. Band overlap when both m and n are even

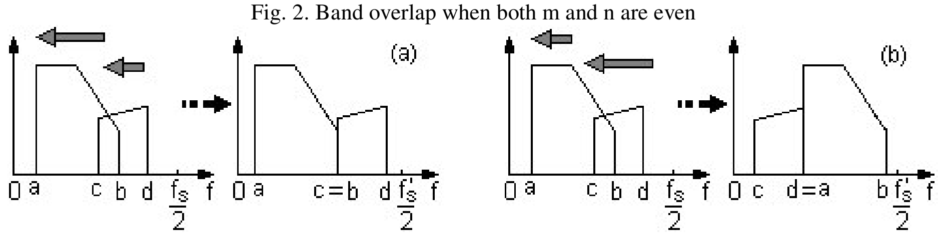

Fig. 3. Changes in band orientations (m even, n even)

Table 1. Expression for $f_s + \Delta f_s$ in step 4

m and n	**Band overlap a<c<b<d**	**Band overlap c<a<d<b**
m even n even	No further improvement	$2\left((f_{u_2} - f_{l_1})/(n-m)\right)$
m even n odd	$2\left((f_{u_1} + f_{u_2})/(n+m+1)\right)$	No further improvement
m odd n even	No further improvement	$2\left((f_{u_1} + f_{u_2})/(n+m+1)\right)$
m odd n odd	$2\left((f_{u_2} - f_{l_1})/(n-m)\right)$	No further improvement

Table 2. Modified expression for $f_s + \Delta f_s$ in step 4

m and n	**Band overlap a<c<b<d**	**Band overlap c<a<d<b**
m even n even	$2\left((f_{u_2} - f_{l_1})/(n-m)\right)$	$2\left((f_{u_2} - f_{l_1})/(n-m)\right)$
m even n odd	$2\left((f_{u_1} + f_{u_2})/(n+m+1)\right)$	$2\left((f_{u_1} + f_{u_2})/(n+m+1)\right)$
m odd n even	$2\left((f_{u_1} + f_{u_2})/(n+m+1)\right)$	$2\left((f_{u_1} + f_{u_2})/(n+m+1)\right)$
m odd n odd	$2\left((f_{u_2} - f_{l_1})/(n-m)\right)$	$2\left((f_{u_2} - f_{l_1})/(n-m)\right)$

Table 3. Final expression for $f_s + \Delta f_s$ in step 4

m and n	**n-m**	**Expression for $f_s + \Delta f_s$**
both even or both odd	even	$2\left((f_{u_2} - f_{l_1})/(n-m)\right)$
one is even and other is odd	odd	$2\left((f_{u_1} + f_{u_2})/(n+m+1)\right)$

Decision Making at 'No Further Improvement' Cases

In this subsection, let us address the issue that when one of such 'no further improvement' situations arises what the algorithm should do, as it can not take any random decision about the value of Δf_s, even at the 'no further improvement' cases. We discuss these, as before, depending on whether m and n, defined by (3), are even or odd.

When both m and n are even, for any increase in the value of f_s, all the values (a, b) and (c, d) decrease, which is depicted in Fig. 3. Hence, ultimately we can have two different band orientations having zero band overlap. However, if we consider the incremental changes in the values of (a, b) and (c, d), due to an incremental change in sampling frequency (Δf_s), we get

$$\Delta a = \Delta b = -m\left(\Delta f_s/2\right),\ \Delta c = \Delta d = -n\left(\Delta f_s/2\right)$$

Since n > m, from (3), therefore $\Delta c, \Delta d > \Delta a, \Delta b$. This suggests that orientation of Fig. 3(a) is infeasible. Then considering the band orientation of Fig. 3(b), we see that we need to choose the next sampling frequency ($f_s + \Delta f_s$) in such a way that the band positions 'a' and 'd' become the same. Equating the expressions of 'a' and 'd' we get

$$f_s + \Delta f_s = \left(2/(n-m)\right)\left(f_{u_2} - f_{l_1}\right) \qquad (7)$$

Performing similar analysis for the other 'no further improvement' cases, as mentioned in Table 1, we get Table 2 with modified expressions of ($f_s + \Delta f_s$). But because of the similarities between 1st and 4th row and between 2nd and 3rd row, we can represent these eight conditions by two simple cases, as presented in Table 3, depending on (n-m).

SIMULATION RESULTS

To get a better feeling of the effectiveness of this algorithm, let us consider a hypothetical situation where we try to incorporate the GSM and IS-95 CDMA standards over a single system. The GSM standard operates over (890 - 915)/ (935 - 960) MHz [8], whereas the IS-95 CDMA operates over (824 - 849)/ (869 - 894) MHz [9]. We applied this algorithm for both uplink and downlink bands separately. After nine iterations, f_s =117.6 MHz is found to be the *minimum* bandpass sampling frequency for direct downconversion of the uplink bands, and for the downlink bands the same is found to be f_s = 120 MHz, after eleven iterations.

CONCLUSION

While expanding the digital signal processing boundary toward the antenna, in a software radio implementation, the application of bandpass sampling technique can be very helpful to achieve the design goal. In this paper, we have presented an algorithm to determine the minimum bandpass sampling frequency for direct downconversion of multiple distinct RF signals. The essence lies in the determination of a sampling frequency closest to the theoretical lower limit $2\left(B_1 + B_2\right)$. This minimization, indirectly, is also critical in estimating the computational requirements, one of the primary bottlenecks in software radio design.

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