Grandma Knows Best

Measurement Uncertainty in the Kitchen

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When Betty Baker planned a birthday party for her six year-old son Tommy, she knew he wanted the party food to include chocolate chip biscuits just like those he had enjoyed at a friend's party. Including Tommy, there would be nine children at the party. Betty planned to provide two biscuits per child as part of the birthday feast. Fortunately she remembered that her baking folder contained an old recipe from her grandmother simply headed "Giant Chocolate Chip Biscuits (kids love them!)"

Giant Chocolate Chip Biscuits

6 oz butter
2 fluid ounces milk
2/3 cup sugar
2 eggs
3 cups self–raising flour
2 handfuls chocolate chips
Half teaspoon rum flavouring

Melt butter in large saucepan.
Add sugar, stir to dissolve.
Cool and beat in eggs and milk.
Add flour, chocolate chips and rum flavouring.
Lightly mix by hand.
Place heaped tablespoonfuls on greased tray and bake in pre–heated moderate oven for 20 minutes.

But there was no indication of how many biscuits the recipe would make. How could she be sure that she would end up with at least 18 biscuits?

Being an analytical chemist prior to raising a family, Betty decided to use her training to estimate the number of biscuits she would produce by following the recipe in the same manner as her grandmother. She was aware that her estimate would have a degree of uncertainty associated with it but wanted to be 95% confident of producing a minimum of 18 biscuits. She knew that the uncertainty could be reduced if she simply weighed out most of the ingredients and portions but she became excited by the challenge to do things just like grandma and estimate the uncertainty accordingly[1].

Betty proposed a simple equation:

$$number\_of\_biscuits\_(n) = \frac{total\_mass\_of\_mix\_(W)}{mass\_of\_heaped\_tablespoonsful\_of\_mix\_(T)}$$

No doubt grandma would have used the term 'weight' but Betty determined to be faithful to SI terminology and refer to 'mass'.

Betty scratched her head to try and remember the GUM[2] / QUAM[3] approach to estimating measurement uncertainty:

1. specify the measurand;
2. identify major sources of uncertainty;
3. quantify uncertainty components;
4. combine significant uncertainty components;
5. expand combined uncertainty to provide a result with a stated level of confidence; and
6. review estimates.

The measurand was clearly the 'number of biscuits'. The task was to identify and quantify the sources of uncertainty. The equation was a good place to start. What items contributed to the 'total mass of mix'? What were the uncertainties associated with each item? Betty defined terms for the items she needed to consider:

Recipe	Mass	Uncertainty
6 oz butter	b	u_b
2 fluid ounces milk	m	u_m
2/3 cup sugar	s	u_s
2 eggs	e	u_e
3 cups self–raising flour	f	u_f
2 handfuls chocolate chips	c	u_c
Half teaspoon rum flavouring	r	u_r
Losses during process	l	u_l
Heaped tablespoonful mixture	T	U_T

Betty's equation could now be expressed in more detail:

$$n = \frac{W}{T} = \frac{b + m + s + e + f + c + r - l}{T}$$

6 oz butter

Betty guessed that her grandma would have simply taken three quarters of a half-pound block of butter. She knew that the present-day 250g blocks were not equivalent to a half-pound but nevertheless decided to use three quarters of a 250g block in her mix. She reckoned she could do this to within ±2.5mm that equated to ±6g. However she considered that she would be more likely closer to the exact three quarter mark than the extremities of this range.

Mass of butter, b, = 187.5 ± 6g
Standard uncertainty in mass of butter (u_b)

Assuming triangular distribution, $u_b = \frac{6}{\sqrt{6}} = 2.45g$

Betty noted that this estimate assumed that purchased blocks of butter all weigh exactly 250g. While she was certain there would be some tolerance allowed for the mass of blocks, she decided to neglect the tolerance in her calculations, assuming it to be small compared to the uncertainty associated with her quartering procedure.

2 fluid ounces milk

What in the !##! were fluid ounces? Reference to conversion tables in her much-loved copy of the "Rubber Handbook"[4] revealed that 2 fluid ounces was approximately 60ml. Betty used a small measuring jug to measure out this volume. The graduations on the jug were clearly marked at 10ml intervals and Betty judged that she could measure the volume within ±5ml. To her surprise the jug, one of a set given to her by a metrologist as a wedding present, was stamped to indicate a certified accuracy of ±3ml with a 95% level of confidence.

Volume milk = 60ml

Standard uncertainty of jug calibration = $\frac{3}{2} = 1.5ml$

Standard uncertainty of aliquot volume (assuming triangular distribution) = $\frac{5}{\sqrt{6}} = 2.0$

Possible temperature effects on this measured volume were disregarded as relatively insignificant.

Combined uncertainty in volume of milk = $\sqrt{(1.5)^2 + (2.0)^2} = 2.5ml$

Betty again referred to the "Rubber Handbook" to find the density of milk listed at 1.028–1.035g/ml. She used the factor of 1.032 to convert her volume to mass.

Mass milk, m = 60 x 1.032 = 61.9g

Betty used the quotient rule for combining relative standard deviations (RSDs) in order to estimate the uncertainty associated with this calculated mass.

Factor = 1.032 ±0.004g/ml; standard uncertainty = $\frac{0.004}{\sqrt{3}} = 0.0023$

RSD = $\frac{0.0023}{1.032} = 0.0022$

Volume = 60ml; standard uncertainty = 2.5ml, RSD = $\frac{2.5}{60} = 0.042$

Applying quotient rule,

Uncertainty in mass milk, $u_m = 61.9\sqrt{(0.0022)^2 + (0.042)^2} = 2.6g$

Betty was not surprised that the uncertainty associated with the conversion factor was insignificant compared to the uncertainty of the volume measurement.

2/3 cup sugar

Betty supposed her grandmother would have simply used a kitchen cup to measure out this quantity; possibly a 'special cup' reserved for cooking purposes to ensure consistency between batches. Even so, Betty thought judging two-thirds of a cup was likely to be imprecise. She tested her own ability by two-thirds filling one of her kitchen cups with water and measuring the volume in her calibrated jug. Her measurements ranged from 136–179ml. She used a teacup for this trial. The tapered shape of the cup meant that a small change in water level resulted in a large change in volume. Nevertheless she thought her grandmother would have used a similar cup, rather than a cylindrical mug, and used the collected data as follows:

Measured volume of two-thirds of a cup = 157.5 ±21.5ml

Standard uncertainty (rectangular distribution) = $\frac{21.5}{\sqrt{3}} = 12.4ml$; RSD= $\frac{12.4}{157.5} = 0.08$

Betty then used the same cup to measure out two-thirds of a cup of sugar and weighed the measured quantity on her kitchen scales. The mass of sugar was 115g. Betty assumed that this quantity would be subject to (at least) the same level of uncertainty that applied to the volume of water measured in the same way.

mass sugar, s = 115g

estimated standard uncertainty of the mass of sugar, u_s = 0.08 x 115 = 9.2g

2 eggs

Betty used two eggs from a 600g carton of a dozen eggs. (each egg nominally 50g). Noting that 550g and 700g cartons were available, Betty assumed that eggs in a 600g carton were likely to weigh between 48g and 54g (i.e. 51 ±3g).

Mass eggs, e, = $e_1 + e_2 = 51 + 51 = 102g$

Standard uncertainty mass of each egg, assuming rectangular distribution, $= \frac{3}{\sqrt{3}} = 1.73$

Uncertainty in total mass of eggs, $u_e = \sqrt{{u_{e1}}^2 + {u_{e2}}^2} = \sqrt{(1.73)^2 + (1.73)^2} = 2.4g$

3 cups self-raising flour

Betty estimated the uncertainty associated with this quantity from seven repeated operations whereby she weighed 3 cups of flour, measured out in a cup from her kitchen crockery set, on her kitchen scales. The mean mass was 410g and the standard deviation was 15g. Betty weighed a 500g standard mass (a 'souvenir' of her days in the lab, now used as a paper weight) on her kitchen scales a number of times and concluded that the uncertainty of the actual weighing part of the operation was insignificant.

Mass of flour, f, = 410g

Standard uncertainty in mass of flour, u_f = 15g

2 handfuls of chocolate chips
Betty considered this to be a very subjective quantity. However she found that a 100g pack of chocolate chips fitted neatly in the palm of her hand. She therefore added two packs in her biscuit mix. She assumed a 2% (2g) tolerance on the pack contents for this type of product and calculated the uncertainty of the mass of chocolate chips, u_c, in the same way she calculated u_e.

Mass chocolate chips, c, = 200g
u_c = 2.8g

half teaspoon of rum flavouring
Betty knew that although this ingredient was important to the taste of the biscuits, the quantity added, irrespective of the uncertainty associated with the quantity, would have virtually no impact on the weight of the total mixture.

Accordingly, the mass of rum flavouring, r and u_r were not included in Betty's estimate.

Losses during process
Betty figured there would be some losses during the preparation process. For instance she knew that no matter how hard she tried there was sure to be some residue left in the mixing bowl, the tablespoon and her fingers after spooning the mix onto the baking trays. She estimated this to be about 2 teaspoonful; about 30 ±15g.

Mass of lost mix, l = 30g
Standard uncertainty $u_l = \frac{15}{\sqrt{3}} = 8.7g$

Uncertainly in total mass of mix, u_W
$W = b + m + s + e + f + c - l$ = 187.5 + 61.9 + 115 + 102 + 410 + 200 - 30 = 1046.4g

$$u_W = \sqrt{(u_b)^2 + (u_m)^2 + (u_s)^2 + (u_e)^2 + (u_f)^2 + (u_c)^2 + (u_l)^2}$$

$$= \sqrt{(2.45)^2 + (2.6)^2 + (9.2)^2 + (2.4)^2 + (15)^2 + (2.8)^2 + (8.7)^2}$$

= 20.3g

Uncertainty in mass of heaped tablespoon of mix, u_T
Betty found this item difficult to quantitate prior to making the biscuits. Using judgement based on her cooking experience she based her estimate on measurements made on the dough prepared for one of her weekly batches of fruit scones. She weighed several individual heaped tablespoonfuls of this mixture to find the average mass of 50g with a standard deviation of 3.7g. Whilst appreciating there may be differences between the scone and biscuit mix she decided to use this best available data for her calculations.

Mass of heaped tablespoonful of mix, T, = 50g
Uncertainty in mass of heaped tablespoonful of mix, u_T = 3.7g

Uncertainty in the number of biscuits, u_n
Keeping in mind her original equation, Betty used the quotient rule to calculate u_n:

$$n = \frac{W}{T} = \frac{1046.4}{50} = 20.9 \text{ (say 21 biscuits)}$$

$$u_n = 20.9\sqrt{\left(\frac{u_W}{W}\right)^2 + \left(\frac{u_T}{T}\right)^2}$$

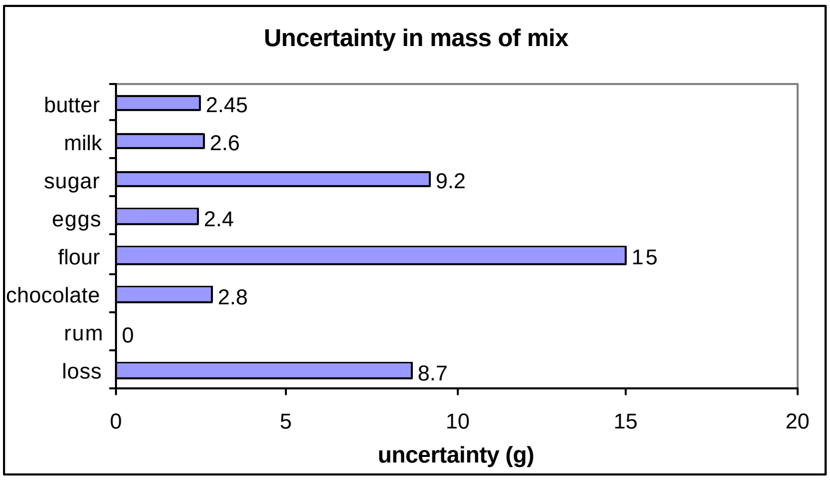
$$u_n = 20.9\sqrt{\left(\frac{20.3}{1046.4}\right)^2 + \left(\frac{3.7}{50}\right)^2}$$

$= 1.60$

Expanded uncertainty (95% confidence range) = 2 x 1.60 = 3.2

Betty felt she could be 95% confident that if she followed the recipe as planned she would produce 21 ±3 biscuits. Therefore she was 95% confident to produce at least the 18 biscuits required.

It was interesting to note the different components of uncertainty. Uncertainties associated with the mass of flour, sugar and losses were major contributions to the uncertainty in the mass of the mix. However the uncertainty in the mass of a heaped tablespoonful of mix accounted for 93.6% of the uncertainty in the number of biscuits. Clearly, if Betty wanted to be more certain about the number of biscuits produced by the recipe it would be best to concentrate on reducing the uncertainty of the tablespoonful quantity. Perhaps some simple mechanical device could be used to 'standardise' the quantity of mix dispensed onto the oven tray.



Putting theory to the test
Encouraged by her calculations, Betty baked the biscuits as planned and was delighted to find the mix made 21 biscuits. Was this good luck or good management? Subsequent experience proved it to be good management. Betty baked several batches of the biscuits to give to friends at Christmas and found the number of biscuits per batch to range between 19 and 23. The mean of a total of six batches in all was 21.0 and the standard deviation 1.4.

Grandma knows best
Betty proudly explained her calculations to her grandmother when presenting her with a gift pack of biscuits. Granny wasn't impressed: "I don't know about your arithmetic but I've followed that recipe more than a hundred times and nearly always ended up with two dozen biscuits — never more than one out".

Accepting grandma's technique as the 'standard method' producing the true result, Betty realised she must have introduced a systematic error into her baking procedure, resulting in a negative bias. Her result needed to be corrected for bias in order to produce a result closer to the true result. Alternatively, but less preferred, the estimate of uncertainty could be expanded to account for uncorrected bias. Since she could not be certain of the actual bias, the uncertainty associated with her estimate of bias would need to be considered in both cases.
But where did the bias come from? Perhaps her tablespoon was too 'heaped', the cup used to measure the sugar and flour too small, her arbitrary handful of chocolate chips too small, or a combination of these items served to introduce the bias observed. Betty thought she had part of the answer when she saw her grandma's large hand brush biscuit crumbs from her chin as she remarked, "These biscuits are pretty good but they're a bit light on chocolate".

Notes

[1] This is a common phenomenon with chemists. Caution is advised to prevent the estimation of MU becoming an all–consuming passion.
[2] *Guide to the Expression of Uncertainty*, First Edition, ISO, Geneva, 1995.
[3] *Quantifying Uncertainty in Analytical Measurement*, Guide CG4, second edition, Eurachem/CITAC, 2000.
[4] *CRC Handbook of Chemistry and Physics*, 55th Edition, 1974–75.