Correspondence

Comments on "The Modulo Time Plot: A Useful Data Acquisition Diagnostic Tool"

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In this paper[1], the use of a reordered sample set is very useful for quick visual inspection of system performance. This artificial oversampling algorithm unrolls spectra as well [1]. However, the question is how to reorder the samples.

To answer that, one must recall the conditions for *uniform permutation* of samples [2], [3]: the sample set of a sine wave with a period $T = 1/f$ and sampling frequency $f_s = 1/\Delta t$.

$$x[i] = \cos(\omega i \Delta t) = \cos(2\pi i f / f_s), \qquad i = 0, 1, \ldots, N-1$$

is *periodic* only if the value of numerical (normalized) frequency

$$f/f_s = m/N, \qquad 1 \le m < (N/2)$$

where m is prime to N, and N is the numerical period: $x[i] = x[i+N]$. This is the case of *coherent sampling*; N unique samples from m periods of the signal, $N \cdot \Delta t = m \cdot T$. Such a raw sample set ($m > 1$) can be reordered into a *single* period by an index transformation

$$j = (Ji) \bmod N$$

where J is a unique integer and must be calculated from the relation

$$(mJ) \bmod N = 1$$

i.e., J is multiplicative inverse of m.

Fig. 1 shows a simple but informative example of transposition, where $m/N = 7/32$, so that $J = 23$. Some short notes on the simulation:

1) use DFT if N is not a power-of-2;
2) n may be noninteger, and then it is the effective number of bits;
3) the common *sinc* interpolation formula [5] can be rearranged into a *finite sum* in case of *periodic* sample sets;
4) spectrum levels are in dB (voltage ratio);
5) with $N = 32$, values of J for other possible frequencies

$$\begin{array}{lcccccc} m = & 3 & 5 & 9 & 11 & 13 & 15 \\ J = & 11 & 13 & 25 & 3 & 5 & 15 \end{array}$$

6) since the numeric frequency is a ratio of small integers, there is a massive superposition of the aliased harmonics.

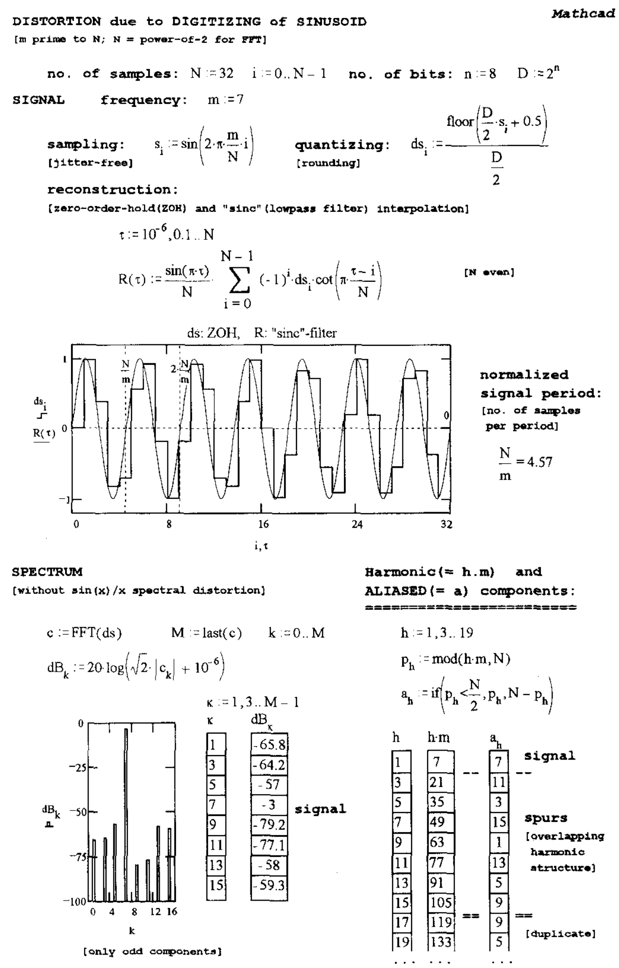

Fig. 1. *Mathcad* simulation: spurious components due to quantization.

Simulation parameters may be easily modified, i.e., in [1] $m/N = 1501/4096$, so that $J = 2677$, or, with reindexing from zero in [5, Fig. 2] the numeric frequency is $m/N = 4/19$, so that $J = 5$.

The technique of the uniform permutation can be used for other desirable waveform or spectrum modifications.

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