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SIMULATION OF ORGAN PIPES' ACOUSTIC BEHAVIOR BY MEANS OF VARIOUS NUMERICAL TECHNIQUES

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The sound generation of an organ pipe is a very complex physical process, since the acoustical phenomena take place coupled with fluid flow effects. Even so, by modeling the organ pipe merely as an acoustic resonator, one can predict several key parameters of the sounding with sufficient accuracy. As these parameters are highly affected by even small changes of the pipe's geometry, the resolution of the numeric model should be adequately fine, which means that computational time and effort will raise enormously. The aim of our work is to develop a simulation program, which provides the chance to accurately predict acoustic properties of organ pipes. At the same time, the obtained results can serve as guidelines for scaling and intonating the pipes. Taking into consideration that an organ consists of thousands of pipes, an efficient simulation method would greatly aid the work of organ builders, by speeding up the industrial procedure of organ fabrication and intonation. Int he course of the work reported herein we modeled organ pipes by means of various numerical techniques (such as FEM, BEM, coupled FEM/BEM, etc.). Commercial and self-developed software packages were used and the obtained data were compared analytic solutions and measurement results. It was shown that by using these techniques one can approximate key acoustic parameters of the pipe. We have also examined, how certain approximations and neglects can affect the accuracy of simulation.

1. Introduction

Scaling of organ pipes is still performed according to the rules, laid down in the 19th century. These rules prescribe pipe dimensions for the desired sounding, but in some cases changing the traditional geometry parameters is inevitable (for aesthetic and practical reasons). Then the organ builder can only rely on his experience, attempting to tune the sounding parameters of the pipe.

The aim of applying numerical techniques for organ pipe simulation is twofold. On the one hand to speed up the scaling and tuning process, saving quite some time for organ manufacturers as an organ consists of thousands of pipes. On the other hand it will hopefully help developing new scaling methods. The purpose of the latter is to predict, how the traditional organ sounds can be preserved with changed geometrical parameters, and how new sounding characteristics can be achieved.

In this paper we will show that certain numerical modeling techniques can indeed be applied to determine key information on the sounding.

2. Limitations of acoustical modeling

The sound generation of organ pipes is a very complex physical process. As the wind from the windchest reaches the languid of the pipe, an oscillating air jet evolves in the windway (which is in the foot part of the pipe) and this produces the excitation for the air column resonating inside the pipe body. The examination of this phenomenon in full detail would require the analysis of a coupled non-linear acoustic and fluid flow model. At the same time, some key parameters of the sounding can be predicted if the pipe is regarded merely as an acoustic resonator. This way the transfer function of the pipe can be calculated, though the transient response cannot be taken into consideration. The following key information on sounding characteristics can be obtained.

- **Fundamental frequency**

  Fundamental frequency is the first resonant frequency of the pipe. Even though, other harmonics can be more dominant during transient attacks (see [6, 11]), it is the fundamental frequency which determines the tone of the pipe.

- **Frequencies of harmonic partials**

  As will be shown later on, in case of an organ pipes the eigenresonances are not exact harmonics of the first resonance. The frequencies of these modes are slightly stretched. This effect can be understood, if one recalls that the specific acoustic impedances of the pipe terminations are frequency dependent.

- **Q-factors of eigenresonances**

  Finite values of enclosing radiation impedances also mean that the resonance peaks are not infinitely sharp. Q-factors are higher (peaks are sharper) in case of the first few harmonics and lower (wider peaks) for the further harmonics.

- **Cut-off frequency**

  Since the diameter (or depth, e.g. in case of wooden pipes) of an organ pipe is much smaller than its length, pure longitudinal eigenmodes appear at lower frequencies. The frequency, where transversal resonances start to appear, is called cut-off frequency as the sound spectrum above this frequency shows irregularities compared to the slightly stretched harmonic peaks at lower frequencies. These irregularities are caused by the combined excitation of longitudinal and transversal modes.

In figure 1 typical transfer function of an organ pipe can be seen, demonstrating the mentioned specialties of the spectrum. In the following section we will discuss how can transfer functions be calculated by numerical means to predict these characteristics.

3. Numerical techniques

Simulating the acoustic behavior of an organ pipe resonator means that we seek the solution for the following problem. A resonator object (the pipe body) is placed into a free sound field, together with an external source placed in its vicinity, providing the excitation. One has to calculate the generated sound field in and around the object. As we want to determine stationary spectra, it is obvious to solve the problem in the frequency domain. This way we will get the transfer function of the resonator.

Starting from the simplest analytic formulae we will show the complexity of the problem and consider, how certain numerical techniques can be applied to solve it.

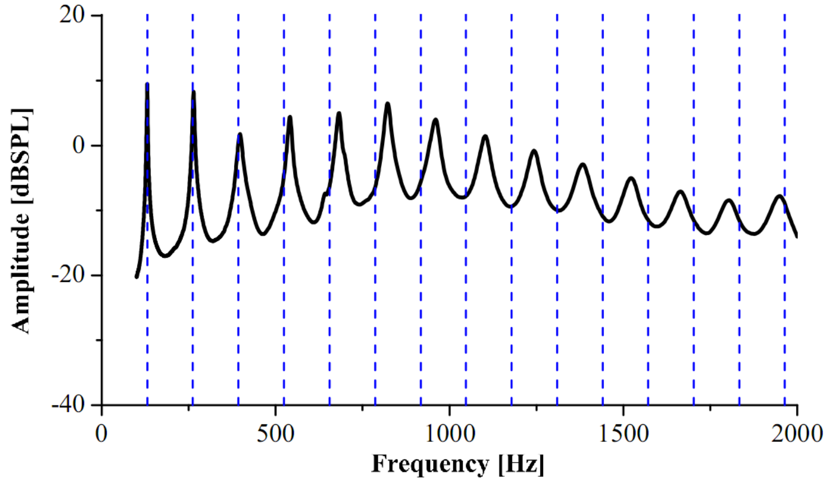


Figure 1. Typical organ pipe transfer function.

3.1 Analytic solutions

The simplest formula to calculate eigenfrequencies of a vibrating air column with both ends opened (or both ends closed) is the well-known relationship

$$f_n = n\frac{c}{2l} \qquad n = 1; 2; \dots \tag{1}$$

where f_n is the n-th eigenfrequency, c is the speed of sound and l is the length of the air column vibrating. This formula neglects the specific acoustic impedances at the enclosures and works only if there is no coupling between the interior and the exterior sound field (e.g. if the air column is bounded by perfectly rigid walls).

The effects of the terminating impedances can be taken into account by using an end correction formula, which prescribes a correction on the effective length of the pipe. The eigenfrequencies of a cylindric tube become

$$f'_n = n\frac{c}{2l_{eff}}; \qquad l_{eff} = l + 0.31d \tag{2}$$

where d is the diameter of the tube. This relationship gives a more accurate result for the fundamental frequency in case of cylindric pipes, but the frequency dependence of the enclosing impedances is still neglected.

The radiation impedance of a long tube can be analytically computed, but only for the case of a simple plane piston moving in the tube (see [1]). Besides that the solution is only computed for cylindric tubes, these computations do not take into account that the impedance values are varying over the cross-section. Comparison of analytic and simulation results of enclosing impedances can be seen in figure 2, while the distribution of impedance values over the cross-section is shown in figure 3.

It is possible to use analytical relationships as boundary conditions in a numerical (finite element) model. Earlier simulations (see [11]) showed that the results were not satisfactory, hence these methods are not discussed here. In the following we will discuss pure numerical solutions.

3.2 Uncoupled indirect BE method

As described in [4], the indirect boundary element method is able to solve simultaneously the internal and external acoustic radiation and scattering problem. This indirect representation uses layer potentials that are the differences between the outside and inside values of pressure and its normal derivative.

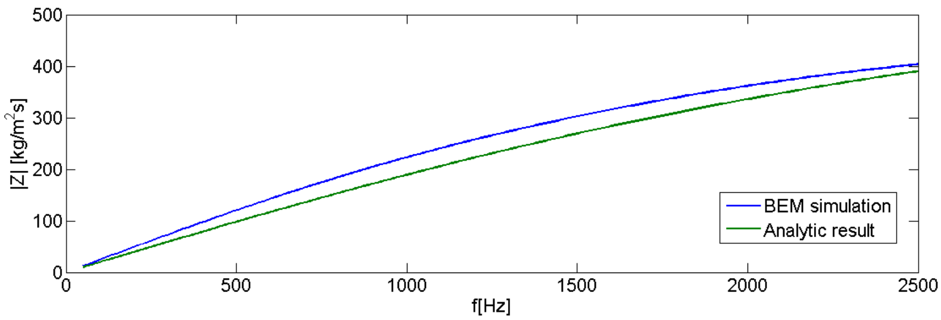
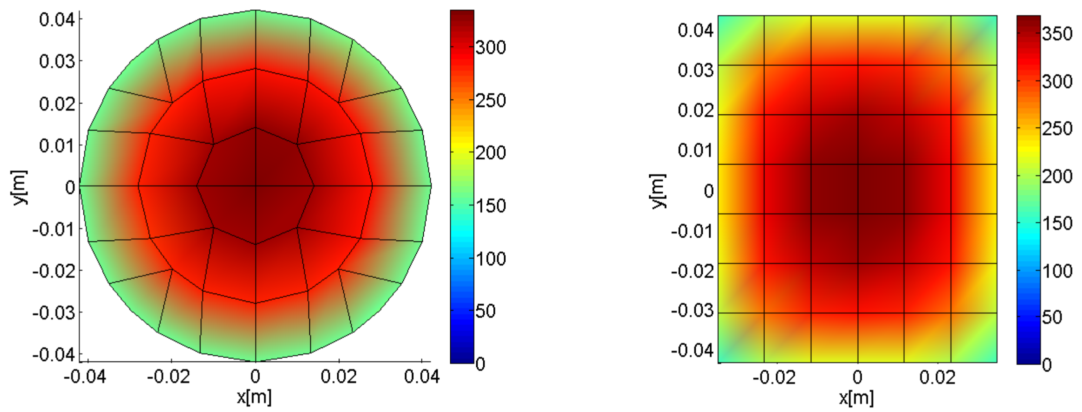


Figure 2. Comparison of enclosing impedances according to BEM simulation and analytic computation



(a) Cylindric pipe

(b) Rectangular pipe

Figure 3. Distribution of impedance values at the open pipe end at 1 kHz.

In case of organ pipe simulation it is obvious that the internal acoustic field is the sound field inside the pipe, while the external field is the radiated field outside. The connection and continuity between these two fields are described with boundary conditions, namely that the jump of pressure is zero at the boundaries (i.e at the free edges of the mesh).

In this method the mesh is generated as a surface (2-D) model, which means that the storage size of the geometry is $O(n^2)$, where n is the average number of nodes in a unit length. This yields that the matrices describing the discretized equations are of the size $O(n^4)$. These matrices are frequency dependent, dense matrices, hence they have to be recalculated for every testing frequency.

3.3 Coupled FE/BE method

Using this method we split the investigated domain to an interior (inside the pipe) and an exterior space (outside the pipe), just like we did for the indirect BEM method. The interior space is modeled as a volumetric geometry (3-D) model and the exterior domain as an infinite volume. This is a coupled problem because of the interaction between the evolving sound fields in these two domains.

The sound field that evolves is the superposition of the incident field generated by the external source and the reflected field, scattered from the pipe body. For this superimposed sound field the Helmholtz equation is solved by means of the finite element method in the interior domain. The coupling between the two sound fields is taken into consideration by setting up the adequate boundary conditions. In the simplest model consisting of perfectly rigid pipe walls this means the following:

1. **At the openings:** Impedance boundary condition can be applied by using BEM to solve the scattering problem for the reflected field in the exterior domain. ($p = Zv_n$)

2. **At pipe walls:** Normal particle velocity is zero. ($v_n = 0$)

The characteristics of the incident field (pressure and particle velocity at any point) can directly be calculated from the parameters of the source. The reflected field is a solution of an exterior domain scattering problem which can be described by the direct boundary element method, see for example [4].

On the surface points of the pipe, the relation between the pressure and normal particle velocity of the reflected field is determined by the BEM matrices. As the pipe walls are regarded as being perfectly rigid, the normal particle velocity is zero everywhere on the boundary, except for the points at the opened parts. Hence BEM matrices can be rewritten using Schur's complement form, and the boundary conditions can be substituted to the FEM equation. This way the impedance matrix, that describes the coupling of the interior and exterior field, is much smaller than the original BEM matrices.

Since BEM matrices are slowly varying with respect to the frequency, this implies, that the impedance matrix also has this property, which means that it is no need to calculate it for every distinct testing frequency. This way the calculation can be sped up using an interpolation formula (spline for example) to generate the impedance matrix.

As mentioned above, the geometry is given here as a (3-D) mesh, which requires a storage size of $O(n^3)$. This means that the size of FEM matrices are $O(n^6)$. Fortunately, these matrices (the acoustic stiffness and mass matrices and the excitation matrix) are frequency-independent sparse matrices and are to be calculated just once in the course of the whole simulation. The impedance matrix is much smaller (and requires a storage size of $O(n^4)$) but has to be recalculated (with interpolation) for each testing frequency.

3.4 PML method

In this method both the interior and the exterior domain is modeled as a 3-D geometry. Since the exterior domain is infinite, only a part of it is built with meshing elements. As there should be no reflected waves from the free field, the built part of the exterior domain is bounded with an absorbent, non-reflective layer. In this layer the outgoing waves are heavily damped and this way the effects of reflections are negligible. It is also important that there should be no reflections on the boundary of damping and non-damping elements. If these conditions apply then the damping layer is a perfectly matched layer (PML), see [7].

The Helmholtz equation is solved in the whole domain with the finite element method. The stiffness and mass matrices of the damping layers are frequency dependent which means that they have to be recalculated for each testing frequency.

PML was first formulated by Berenger in computational electromagnetics (see [2]) but can also be applied in numerical acoustics (see [3, 7, 9]). As there are no commercial software available that handles acoustic PML, our own simulation program for the PML method is currently under development. Results will be shown in the oral presentation.

4. Measurement setup and simulation model

The transfer function measurement of an organ pipe resonator can be done in the following way. A loudspeaker is placed in the longitudinal axis of the pipe near the open pipe end. It produces a wide band signal (e.g. a logarithmic sweep) and the response is recorded by microphones placed at the pipe mouth and at the open end of the pipe. The signals recorded by the microphones is analyzed by using FFT. The whole setup, which can be seen in figure 4, is placed in an anechoic room.

In the simulation model the loudspeaker is substituted by a point source of given amplitude for each testing frequency. Microphones are represented as field points. The mesh consists of perfectly rigid (and infinitely thin) walls and openings. The geometry discretization is performed by means

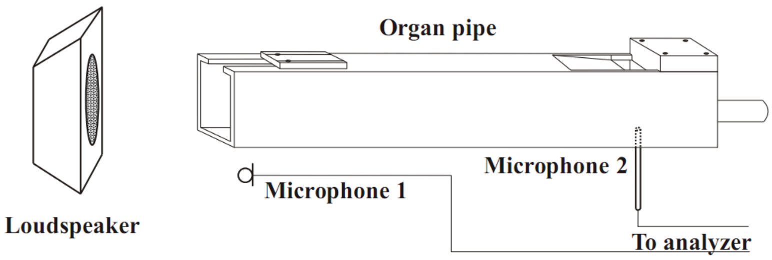


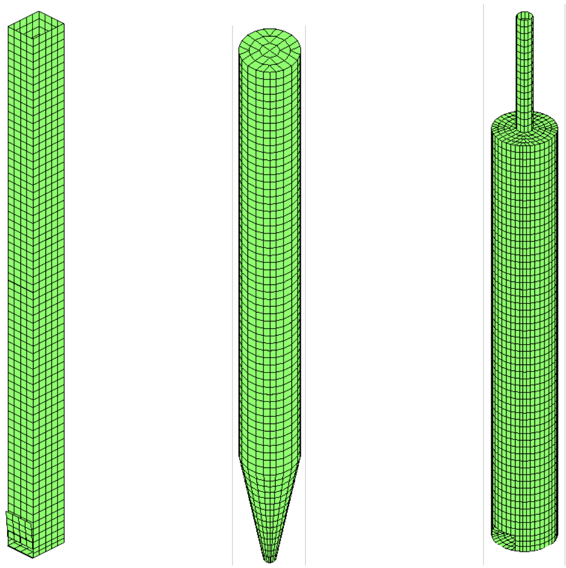
Figure 4. Measurement setup

of an algorithm that produces a symmetric mesh with quadratic elements on the surface and hexa elements for the volumetric part of the model.

The validity range of the simulation is dependent on the element size. The relation between maximal element sizes (l_x, l_y and l_z) and upper frequency limit (f_{max}) is given as:

$$f_{max} \approx \frac{c}{8 \max\{l_x, l_y, l_z\}} \tag{3}$$

Simulations with the uncoupled indirect BE method were run by using the `LMS Sysnoise` software package, while the coupled FE/BE method was tested by using `AcouBEM` software and `AcouFEM` toolbox (see [5]) under `Matlab` environment. Geometry meshes and simulation command files were generated by script programs in `Matlab`. Figure 5 shows pipe meshes used for simulations.



(a) Wooden pipe (b) Cylindric pipe (c) Chimney pipe

Figure 5. Meshes of pipe resonators of various types.

5. Results

The first series of simulations were run on wooden pipes, which were already built and measured. According to measurement results, cut-off frequencies of the pipes were between 1.5 and 2 kHz. Taking this into consideration the maximum element size was chosen to be smaller than 1.75 cm. Thus, one gets $f_{max} \approx 2.5$ kHz from (3), and the meshes consisted of approximately 1500 nodes for the structural mesh and 2500 nodes for the acoustical volume. Simulations were run with $\Delta_f = 1$ Hz resolution on the 50 to 2500 Hz frequency domain.

Table 1 shows the exact dimensions of one pipe from these series. (4/16 mean mouth width to circumference ratio.) Comparison of simulation results and measurement data can be read in table 5. Simulated transfer functions can be seen in figure 6.

Pipe	Length	Width	Depth	Mouth height	Mouth width
4/16	1180	69.80	86.87	19.87	68.64

Table 1. Pipe dimensions given in mm.

Pipe: 4/16	**Measurement**		**Indirect BEM**		**Coupled FEM/BEM**	
Harmonic	Freq. [Hz]	Stretch	Freq. [Hz]	Stretch	Freq. [Hz]	Stretch
1. (Fundamental)	129.87	1.000	131	1.000	128	1.000
2. (Octave)	261.76	2.016	263	2.008	253	1.977
3.	396.45	3.053	397	3.031	388	3.031
4.	536.98	4.135	531	4.053	522	4.078
5.	677.62	5.218	667	5.092	660	5.156
Cut-off [Hz]	1987		1987		2008	

Table 2. Comparison of measurement and simulation results for the 4/16 pipe.

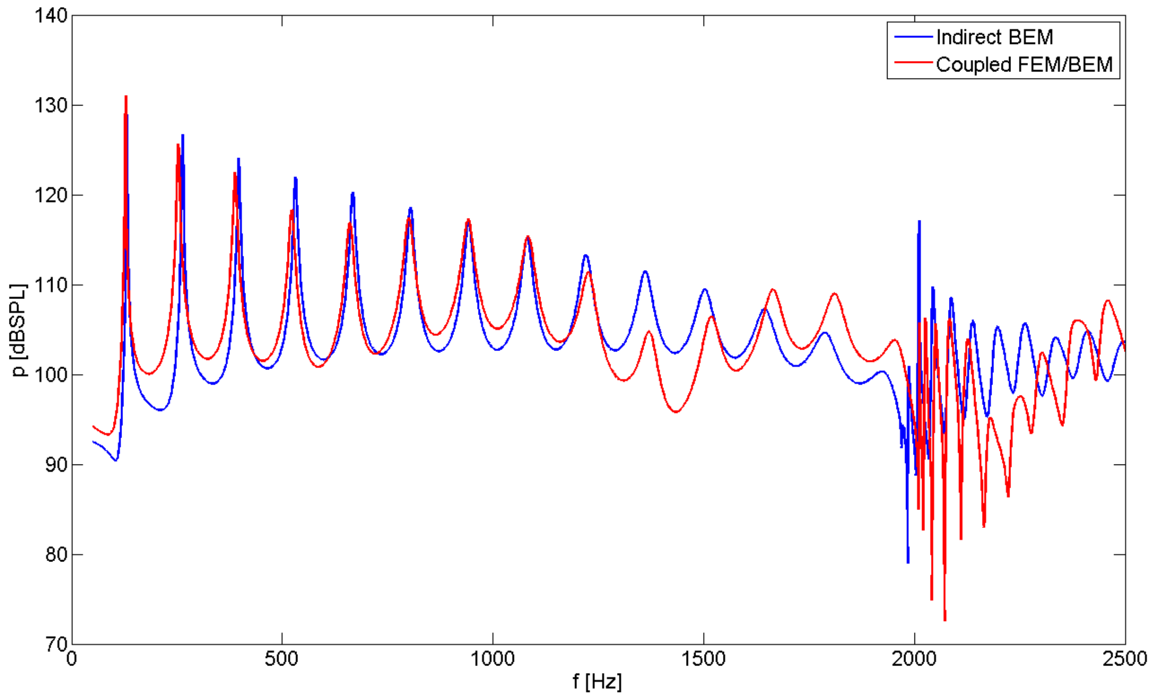


Figure 6. Comparison of simulated spectra at pipe mouth.

As seen, simulation results of the two implemented techniques approximately check up with measurement data. The fundamental and cut-off frequencies were predicted within 1.5% error range. Frequencies of further harmonics and stretching values were approximated with a maximum of 3.5% error. Q-factors of eigenresonances qualitatively match with expected characteristics (compare figure 1 to figure 6), but for their exact calculation a frequency-dependent damping model of air should be applied. In case of the coupled method, some minor irregularities (such as the disturbance of the spectrum around 1.5 kHz, which can be seen in figure 6) are the subjects of further investigations.

6. Summary and future work

It was shown in this paper that certain numerical techniques can be applied to obtain information on organ pipe sounding characteristics. The indirect BEM and the coupled FEM/BEM technique were implemented for the simulation of a series of pipes. Comparison with measurement results showed that some key parameters on the stationary spectra can be predicted with sufficient accuracy using the simplest geometry models.

Besides the implementation of PML simulations, our future plan is to extend the pure acoustic model by further physical effects, in order to get a better insight into the sound generation mechanism and to be able to simulate other key phenomena such as transient response of the pipe. Finally, a long-term plan would be the examination of the problem by means of a coupled acoustic and fluid flow model.

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