Risk analysis lab 2014. 10. 06. (Chernoff bounds)

1. Implement the followings into `chernoff.m`:

   ```
   function P = chernoff(p, C)
   ```

   a) Calculate the Chernoff bound $P\left(\sum_{j=1}^{J} X_j > C\right) \le min\left(e^{\left(\sum_{j=1}^{J}\mu_j(s_{opt})\right)-s_{opt}C};1\right)$, where $s_{opt}:\inf_s\left(\sum_{j=1}^{J}\mu_j(s)\right)-sC$ and $\mu_j(s)=\log\left(\sum_{l=1}^{L}e^{sl}p_l^{(j)}\right)$.

   (First, solve the optimization problem with exhaustive search on the $0<s\le 1$ interval.)

   b) Extend the comparison plot, and compare the Chernoff bound with the analytical probability calculated by convolution and the Markov bound in case of different *C* parameters.

2.

   a) Let h_i denote the amount of deposit belongs to each client ($1\le i\le N$) with $p_i=P(X_i=h_i)$ ($x_i\in\{0;h_i\}$) probability of withdrawal. In a new MatLab script (`lab5.m`) generate these vectors randomly as $h_i\sim N(\mu=2000,\sigma=500)$ and $p_i\in[0;1]$ (*N*=20).

   b) Let $y_i\in\{0;1\}$ stands for the event when the *i*th customer withdraws their deposit, while $\xi\in\{0;1\}$ denotes the event that the bank exceeds its cash *C*.
   Calculate $E(\xi)=P\left(\sum_{i=1}^{N}y_ih_i>C\right)$ analytically (sum $\xi\prod_{i=1}^{N}p_i^{y_i}(1-p_i)^{(1-y_i)}$ over all possible $\bar{y}$ vectors).

   ```
   function P = analytical(h, p, C)
   ```

   c) Calculate an upper bound for the analytical probability by modifying the Chernoff method.

   ```
   function P = chernoff2(h, p, C)
   ```

3. Now, determine C as a function of P using any of the previous methods. (Note: `fun` will be a function valued parameter, e.g. `@analytical`.)

   ```
   function C = optcash(h, p, P, fun)
   ```

   Call `optcash` with both analytical and Chernoff function, and compare their results and their running time for different values of N (hint: use `tic` and `toc`).

4. (Extra) Improve the performance of the optimization in the Chernoff function.