

## Risk analysis lab 2014. 11. 03. (Portfolio risk)

1. Load the supplied data from the binary (`x.mat`), the variable `x` will be a  $T \times N$  matrix containing daily returns for  $N$  asset and  $T$  days. Calculate its mean vector as  $\mathbf{m} = \{E(x_1), E(x_2), \dots, E(x_N)\}$ , and its covariance matrix  $\mathbf{K}$ .

2.

- a) Generate a random portfolio vector  $\mathbf{w} = \{w_1, w_2, \dots, w_N\}$ , where  $\|\mathbf{w}\|_1 = \sum_{i=1}^N |w_i| = 1$  and  $-1 \leq w_i \leq 1$  (short selling is allowed).

- b) If we define  $p(t) = \sum_{i=1}^N w_i x_i(t)$ , then  $p \sim N(\mu, \sigma)$  (CLT), where  $\mu = \mathbf{w}^T \mathbf{m}$  and  $\sigma^2 = \mathbf{w}^T \mathbf{K} \mathbf{w}$ . Calculate  $u$  to fulfill  $P = P(p < u) = 0.01$ .

Display

- the expected daily return
- and the minimal daily return (with 1% uncertainty)

of the given portfolio.

```
[u, mu, sigma] = minrisk(w, m, K, P)
```

3. Determine the optimal portfolio to minimize the risk:

$$\mathbf{w}_{opt} := \min_{\mathbf{w}} \frac{\mathbf{w}^T \mathbf{K} \mathbf{w}}{\mathbf{w}^T \mathbf{w}}.$$

The optimal portfolio can be calculated as that eigenvector ( $\mathbf{v}_N$ ) of matrix  $\mathbf{K}$  which belongs to the smallest non-zero ( $> 10^{-6}$ ) eigenvalue ( $1/N$ ). Print the expected daily return and the minimal daily return (with 1% uncertainty) of the optimal portfolio.

```
function [lN, vN] = mineig(K)
```